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The application of fuzzy programming to the aggregate production planning-markdown pricing problem

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Abstract It is wildly believed that the information sharing and cooperation become important now for improving the performance of the retailer and the manufacturer of the short lifcycle product. This paper suggests a way to enhance the competency of short lifecycle products by integrating the markdown pricing policy into the aggregate production planning.An Aggregate Production Planning -Markdown Pricing Model (APP-MP) has been developed in this paper,which is a fuzzy nonlinear programming with fuzzy objective and fuzzy constraints. The fuzzy programming model can somewhat interpret the uncertainty of business information and be applied to practical APP-MP problem.A numerical example demonstrates the feasibility of applying the proposed model to APP-MP problem. The proposed model, which yields a compromise solution and the decision maker's overall levels of satisfaction, is proved to be effective on making more profits for the manufacturer and the retailer by a numerical example.

Keywords Short Lifecycle Product; Aggregate Production Planning; Cooperation;Fuzzy Programming

1 Introduction

In mainland China and other developing countries of Asia, we observed that the current traditional business mode is quite common: the retailer and the manufacturer run their business individually without any cooperation. Such traditional business mode exists wildly in some industries of the short lifecycle product. The retailer, even the third party who owns the market information is unable to control the production process of its perishable products. In that case, the retailer focus on the pricing strategy to optimize their revenue while the manufacturer continues to reduce the cost by different production planning and other operation management techniques.

The academic research related to linking pricing and operation management is in its early stage, either. Hence, this article is to study how significant will be manufacturer and the retailer if they consider markdown pricing and aggregate production planning together.

We believe that there is a potential effect on the decision-making of the whole system (i.e. the manufacturer and the retailer) by linking pricing information with those in manufacturing side, allowing them to refine pricing, capacity, production and inventory decisions. many inputs to the model, in the reality, are naturally imprecise. Therefore, applying fuzzy programming to the problem is significant. Fuzzy set theory was developed in [10]. Since then, fuzzy set theory has been applied to several fields including operation research and management science. In1976, [9] first introduced fuzzy set theory into conventional LP problems. That study considered LP problems with a fuzzy goal and fuzzy constraints. [2]classified fuzzy mathematical programming into the following four categories: (1) a precise objective and fuzzy constraints; (2) a fuzzy objective and precise constraints; (3) a fuzzy objective and fuzzy constraints; (4) robust programming.In this paper, we will adopted the fuzzy programming model with the fuzzy objective and fuzzy constraints.

Table 1: Notation List

Notation	Description
W_t	The workforce size
H_t	The number of employees hired
L_t	The number of employees being laid off
P_t	The quantity of units produced
I_t	The quantity of inventory
B_t	The quantity of units stocked out/backlogged
S_t	The quantity of units subcontracted
O_t	The number of overtime hours worked
D_t	The forcasting Demand
v_t	The price of the product of month t
c_r	Labor cost in regular time ($/unit)
c_o	Labor cost in overtime ($/unit)
c_h	Hiring and training cost ($/worker)
c_l	Layoff cost ($/worker)
c_i	Unit inventory holding cost ($/unit/month)
c_b	Unit marginal cost of stockout/ backlog ($/unit/month)
c_m	Unit material cost ($/unit)
c_s	Unit subcontracting cost ($/unit)
π	Required labor hours for unit product (hours/unit)
θ	The proportion of maximum capacity that can obtain from outsourcing
N_d	The number of working days in each month for one worker (days/month)
N_h	The number of working hours at regular time everyday for one worker (hours/day)
I_0	The initial inventory level
W_0	The fixed number of workers who have long-term contracts from the manufacturer
W_{max}	The maximum workforce size level in each month
γ	The ratio of salvage value to the price
λ	Unit of overtime hour for each worker

2 Literature Review

In light of recent works [2], [3], [4], [5], [6], [7], [8], a conclusion can be drawn that the pricing policy and the APP were generally studied individually. In the meanwhile, fuzzy programming was not adopted extensively on the problem of linkage of operation and pricing. In order to study the integration of pricing policy and APP models in uncertain business environment, this research intends to develop the APPMP model to be a fuzzy programming model.

3 Aggregate Production Planning-Markdown Pricing Model

As mentioned before, this paper introduces a cooperative business mode for the perishable product. In order to exchange their individual information, the retailer and the manufacturer could have a meeting to simultaneously make the markdown pricing policy and production plan. The retailer the manufacturer should provide the marketing information (demand model) and the production cost information respectively. Therefore, Aggregate Production Planning- Markdown Pricing Model (APPMP)is to model such an cooperative mode mathematically. The demand function and markdown pricing model will be introduced first. Integrating the markdown pricing model into a classical aggregate production planning model, we can propose APPMP model and develop it to a fuzzy programming model by making the objective function and some constraints fuzzy.

3.1 Demand model and markdown impact

Various formulations of demand function related to price have been proposed to forecast the future demand. We found that the demand model in [1] described in the above feature without considering the timing and effect of markdown price. Hence, we enhance their model as follows:

$$D_t = f(\alpha_t, v_t, \beta, t) = \alpha_t (a + b v_t) \exp^{-\beta |t - t_{peak}|} \tag{1}$$

In that model, the formulation $a + bv_t$ is the deseasonalised demand of month t which can be determined by v_t, where a and b are constant. α_t is a calendar factor and $\exp^{-\beta|t-t_{peak}|}$ is an exponentially decaying age factor The estimation of all the parameters in this paper refers to [1].

It is noted that the promotion or markdown will influence the price sensitivity of consumers, especially for the fashionable goods or short lifecycle products. Hence the price sensitivity is adopted. Obviously, higher price sensitivity indicates that little amount markdown magnitude would bring more demand. To reflect this impact on the demand model, the coefficient ęÁt is required to change once one-off markdown happens so that stimulation of each time markdown is quantified

$$\Delta = \frac{v_0 - v_t}{v_0} \tag{2}$$

$$\alpha_t' = \alpha_t + \Delta \tag{3}$$

which represent the impact of markdown on the demand model when the markdown strategy is implemented in t^{th} month. It is actually a percentage of changed magnitude to original price. So, after markdown time, the demand model will be revised to:

$$D_t = f(\alpha_t', v_t, \beta, t) = \alpha_t' (a + b v_t) \exp^{-\beta |t - t_{peak}|} \tag{4}$$

3.2 Markdown Pricing Model

The retailer employs markdown itself which leads them to pursue the maximum revenue by making the optimal markdown pricing decisions and ignores the production cost during the planning horizon. The price parameters $v_t = (v_1 v_2, , v_T)$ are considered as decision variables in the MP model. Particularly the price-demand function is introduced to the MP model and add some constraints to assure the price is reduced gradually (markdown).The markdown model for retailers is presented as follow: Objective function: The objective of the retailer is to maximize the revenue:

$$\sum_{t=1}^{T} v_t D_t \tag{5}$$

Markdown constraints: The price must be reduced and never rises again provided that these reductions occur, which is given by:

$$v_t \leq v_{t-1}, t = 2, 3, ..., T \tag{6}$$

where the initial price v_1 is set in advance.

Demand constraints: Demand function (1)is set as the constraints of demand which reflect the deterministic relationship among time, price and demand The impact on the demand pattern will follow the equation (2) and (3) when the one- off markdown occurs. From that month, Demand will follow such a new pattern in (4)

Capacity constraints: the retailer orders products from the manufacturer which should not exceed the maximum capacity of the manufacturer.

$$\sum_{t=1}^{T} D_t \leq T(\frac{N_d N_h}{\pi} W_{\max} + \frac{\lambda W_{\max}}{\pi})(1+\theta), \tag{7}$$

Consequently, this markdown pricing (MP) model is formed as follow:

$$\max(5)$$

$s.t.$

$$(1) \sim (4), (6), (7); v_t > 0$$

From this nonlinear programming model, the optimal pricing and the markdown magnitude can be discovered and, furthermore, the corresponding adjusted demand for each month.

3.3 APPMP:a fuzzy programming model

The APP-MP model, a fuzzy nonlinear programming model, is an integration of the classical APP whose demand pattern is given first and MP models. Especially, the price becomes a decision variable since the MP model is incorporated into APP model so that the demand pattern will be updated correspondingly due to implementing markdown. All the constraints in MP model are combined with production constraints from APP and the objective is maximizing their total profits (revenue from retailer minus cost from manufacturer). Hence the fuzzy APP-MP model is defined as follow.

$$\begin{array}{l}\widetilde{\max} F = \sum_{t=1}^{T} v_t D_t + I_T \gamma v_1 - (\sum_{t=1}^{T} N_d N_h c_r W_t \\ \sum_{t=1}^{T} c_o O_t + \sum_{t=1}^{T} c_h H_t + \sum_{t=1}^{T} c_l H_t + \sum_{t=1}^{T} c_i I_t \\ + \sum_{t=1}^{T} c_i I_t + \sum_{t=1}^{T} c_b B_t + \sum_{t=1}^{T} c_m I_P + \sum_{t=1}^{T} c_s S_t)\end{array} \tag{8}$$

$s.t.$

$$W_t = W_{t-1} + H_t - L_t, t = 1,2,...,T \tag{9}$$

$$W_0 \le W_t \widetilde{\le} W_{\max}, t = 1,2,...,T \tag{10}$$

$$P_t \le \frac{N_d N_h}{\pi} W_t + \frac{O_t}{\pi}, t = 1,2,...,T \tag{11}$$

$$I_{t-1} + P_t + S_t = D_t + B_{t-1} + I_t - B_t, t = 1,2,...,T \tag{12}$$

$$O_t \le \lambda W_t, t = 1,2,...,T \tag{13}$$

$$S_t \widetilde{\le} S_{max} = \widetilde{\theta}(\frac{N_d N_h}{\pi} W_{\max} + \frac{\lambda W_{\max}}{\pi}), t = 1,2,...,T \tag{14}$$

$$(1) \sim (4), (6), (7), t = 1,2,...,T$$

Wt, Ht, Lt are positive integers; $P_t, I_t, B_t, S_t, O_t, v_t > 0$. $I_0 = 0; B_T = 0; B_0 = 0$

$I_T \gamma v_1$is the salvage value income of the final inventory by the end of the planning horizon. The interpretation of all constraints are presented as follows:

(9) Workforce size, hiring and layoff balance constraint;

(10) Workforce size constraint: the workforce size must be in this interval due to the long-term contract to workers W0 and further the maximum size of worker cannot exceed a certain level;

(11) Capacity constraints: the quantity produced in each month cannot be larger than the available capacity within the industry;

(12) Inventory balance constraints: this constraint balances inventory at the end of each month;

(13) Overtime constraints: no employee is allowed to work more than hours of overtime each month;

(14) Subcontracting constraints: we assume that the outsourcing amount is a fraction of the maximum capacity within the plant.

Additionally the initial inventory I_0 must be zero in the model: $I_0 = 0$ Another constraint is that there must be no any backlog at the end of the planning horizon: $B_T = 0$ and initially: $B_0 = 0$. All the variables are nonnegative here.

Fuzzy Programming can be developed to model the APPMP problem in which the objective function can be consider as the estimation of profits for this planning horizon and constraints (10), (14) are fuzzy that denotes the maximum workforce and the maximum available subcontract capacity are uncertain and set to be fuzzy number. The membership function (15) reflects the decision maker (DM)'s level of satisfaction with the consumption of workeforce. As the consumption of workforce increases, DM's satifaction level decreases, and the DM is becoming dissatisfied totally when the consumption reaches the tolerance point $w_1 + p$. The membership function of the maximum available subcontract capacity (16) has the similar interpretation.

$$\mu_{w_{\max}}(y) = \begin{cases} 1, & \text{if } y \leq w_1 \\ \frac{w_1+p-y}{p}, & \text{if } w_1 < y \leq w_1 + p \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

$$\mu_{S_{\max}}(y) = \begin{cases} 1, & \text{if } y \leq s_1 \\ \frac{s_1+p-y}{p}, & \text{if } s_1 < y \leq s_1 + p \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

For the objective function, we establish the linear membership function for the objective function in which F_0 is the expected profits with the lowest satisfaction level. Thus, the objective function is considered as an new constraint without difference with other constraints. Satisfaction level increases linearly as the profits earned increase so that the membership function is defined as follows:

$$\mu_f(z) = \begin{cases} 1, & \text{if } z \geq F_0 + p \\ \frac{z-F_0}{p}, & \text{if } F_0 < z \leq F_0 + p \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

The tolerance level p of each function is predetermined by the decision maker. The expected interval of profits can be determined by asking the DM. In that case, APPMP model is a fuzzy programming model which can be transformed to be a crisp one for searching the optimal solution. This paper introduces a compromising satisfaction level L based on the Zemmermann's approach, see [10], using Bellman and Zadeh's max-min operator. Then, the fuzzy programming model is transformed to be the following crisp nonlinear programming model.

$$\max L \tag{18}$$

$s.t.$

$$\mu_{W_{max}}(W_t) \geq L \tag{19}$$

$$\mu_S(S_t) \geq L \tag{20}$$

$$\mu_f(F) \geq L \tag{21}$$

$$(1) \sim (4), (6), (7), (8), (9), (10), (11), (12), t = 1, 2, ..., T$$

W_t, H_t, L_t are positive integers; $P_t, I_t, B_t, S_t, O_t, v_t > 0$. $I_0 = 0; B_T = 0; B_0 = 0$

This crisp model is to balance all fuzzy constraints and to maximize the overall satisfaction level L. The optimal solution can be obtained by several algorithms for the crisp programming model.

4 Numerical Example

The feasibility of applying this proposed models is demonstrated by the following numerical example which is a hypothetical one-product and six-month problem (T=6). The product here is defined as the short lifecycle one. The coefficient within the model is presented in Table 2 and 3. Table 2 shows the initial parameters of the demand function while Table 3 presents the parameters of the classical APP model.

Table 2: Parameters (1)

a	b	α_1	β	t_{peak}
2000	25	1	0.1	3

Table 3: Parameters (2)

v_0	c_r	c_o	c_h	c_l	c_i	c_b	c_m	c_s	π	N_d	N_h	I_0	W_0	γ	λ
70	4	6	300	500	2	5	10	40	4	20	8	0	8	0.1	10

The fuzzy number is fomulated as $[z_0, z_1 = z_0 + p]$. Therefore,the decision maker estimates the profits F interval will be [60000,80000]. The fuzzy maximum workforce W_{max} is considered as [10,12] while the subcontract proportion parameter θ is [0.2, 0.3]. LINGO 10.0 is capable of solving APPMP model. Thereby details about the solution techniques will not be discussed here and only the result of fuzzy APPMP model will be presented below. The markdown time is assumed in the 4th month.

Table 4 represents the optimal solution of the new crisp model for APPMP model. It can be seen that the overall satisfaction level is 0.8852 with which the total profit is \$77704. Specifically, the markdown magnitude is around \$17 from \$70 to approximately \$52.72 and the demand pattern is updated by markdown, either. The model finds the compromising optimal solution on the satisfaction level 0.8852. After all, we can conclude that, the model considers the uncertainty of cost information to provide a flexible plan based on the satisfaction level of the decision maker. It is realistic for the real world problem.

Table 4: Result of APPMP model

	Production plann								Markdown	
t	P_t	S_t	I_t	B_t	W_t	H_t	L_t	O_t	v_t	D_t
1	320	0	94	0	8	0	0	0	70	226
2	400	0	244	0	10	2	0	0	70	250
3	400	0	418	0	10	0	0	0	70	226
4	400	83	205	0	10	0	0	0	52.72	696
5	400	86	61	0	10	0	0	0	52.72	630
6	423	86	0	0	10	0	0	86	52.72	570
L=0.8852										

5 Conclusion

This paper developed an APPMP model to link the markdown pricing strategy with the Aggregate Production Planning to achieve the goal of information sharing. It is believed that the APPMP model developed an approach to help the retailer and the manufacturer to have a cooperative planning: the retailer provides the demand information while the manufacturer provides the operation cost information. Then they make the decision related to markdown policy and aggregate production planning together. Such a business mode is different from the traditional one and can be applied to the supply chain.

Proposed APPMP model yields a compromising optimal solution and the DM's overall levels of satisfaction.In terms of their membership functions, the fuzzy parameters are characterized by fuzzy sets. The solution can be determined by transforming the model to a crisp one. The outcome is more appropriate for a realistic APPMP problem. Employing fuzzy programming model for APPMP problem, the manufacturer and retailer can model the problem according to the information available.

The limitation of the proposed model concerns the assumption of the fuzzy number membership function since its value is probably predetermined by DM based on their experience. It is necessary to modify the model to be better suited to the practical APPMP application.

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