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A Purchasing Decision Support System for a Rental Company

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Abstract The equipment rental problem is a difficult, but essential issue for Universal AV Services Ltd., a company that provides rental equipment and technical support for events. It is essential to ensure that the equipment inventory level fulfils customer requirements and meanwhile minimises the costly capital, storage and maintenance costs associated with large stocks of infrequently used equipment. For equipment only used for a few jobs annually, sub-hiring will be more cost effective than purchasing, considering capital investment, management fees, and limited warehouse space. A purchasing decision support system (PDSS) is proposed in this paper to optimise overall level of rental capacity and to fulfil the procurement decisions. With the aid of the proposed models, the purchasing problem has been successfully quantified, which demonstrates clear advantages over the previous systems which were based on purely qualitative decisions.

Keywords Decision support system; Rental; Sub-rental; Forecasting; Exponential smoothing; Regression

1 Introduction

Decision support systems (DSSs) have been proven of great importance since the 1970s, especially recently in various business and industrial areas [1]. Purchasing DSS has been introduced to make decisions on choosing suppliers based on cost and lead time, as well as on assess purchasing vs leasing problem with consideration on financial aspects [2][3]. The equipment rental problem focuses on yield management, which has common objectives such as maximising capacity utilisation, maximising revenue, and minimising lost customer goodwill [4]. The problem in this paper is raised by an event solution company, which provides rental equipment and technical support all over the UK. It is essential to ensure that the equipment inventory level fulfils customer requirements and minimise holdings of infrequently used equipment stock. If a piece of equipment is only used for few jobs annually, it would be better to sub-hire than to purchase it. Related research problems in this area focus on hotel room booking and airline seat reservation [5] [6] with limited resources. In this research, a way of increasing the rental inventory is considered by "sub-hiring" from other suppliers.

The system structure for the proposed PDSS is shown in Fig.1, based on a classic DSS structure suggested by Sprague with a model base, a database and a user interface

....[7]. A forecasting model is designed to perform database function, i.e., to process input data flow and generate inputs for decision. A purchasing decision model is developed to calculate the suggested equipment purchasing level. Apart from the classic structure, a data converter is designed to transform the original data inputs into the format required by the forecasting model.

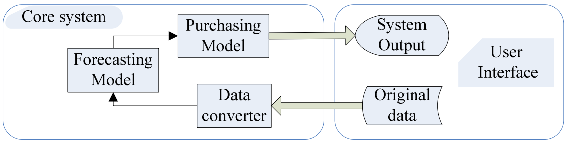

Figure 1: Structure of the proposed PDSS

2 Methodology

This research aims to fulfil three objectives as follows:

- To design an algorithm to determine whether each rental item should be owned or sub-hired;
- To determine the total number of items for an equipment that should be purchased;
- To find the most appropriate forecasting method for this situation;
- To design an algorithm to transform collected daily series into half year usage.

To achieve these objectives, forecasting experiments are designed in MatLab to evaluate error performance. The forecasting model and data converter are developed in MatLab and integrated with the purchasing model developed in C#.Net.

3 Data Collection And Recovery

One of the most commonly hired (and expensive) pieces of equipment, the laptop PC, is selected as a research object. The collected equipment data includes both company owned laptops on hire and laptops sub-hired from other suppliers. Sub-hires indicate the situation that equipment is in shortfall, which is critical to make purchasing decisions. Sub-hires are available for most cases, with extra costs, to satisfy the requirements when the requested laptops surpass the stock level.

The rental data for the laptop in previous years are recorded mainly in three ways: namely equipment logbook on paper, job records, and usage history in an information system. The earliest data record can be traced back to April 2006 when a paper logbook was introduced .[8]. Daily data are then input into a spreadsheet from the logbook by setting “1” for an item in use and “0” for an idle item. An information system was introduced to record all jobs but not equipment usage in March 2007 when the logbook was abandoned. To recover and collect the data in this period, all jobs’ start and end dates were searched and job handlers were inquired to extract the number of laptops that were used for each job. A data set is then obtained on laptop usage for the information system stage. Comparing with Miles’s data recovery method which uses another period’s data for the required period ..[9], this method takes more time, but more accurate results are obtained since actual data in the same period are used. From Oct 2007, equipment usage was recorded in a database. A report can be generated for all equipment usage, and data can be easily entered into Excel from the report. Total 761 daily usages data are collected in spreadsheet with a time span of 25 months from April 2006 to April 2008.

4 Data Converter

The required inputs for the proposed PDSS are yearly peak equipment usages. Two inputs are required for the purchasing decision: annual peak number of an equipment type in use N_{peak}; and the number of annual usage days D_i, for the number of item being greater than i (i inclusive). D_i here means the number of days having i items, i+1 items, i+2 items, $\ldots$, N_{peak} items in use.

Since the collected data are based on daily series, a data conversion model is developed to transform the daily number of items into each items' number of days in use for a period (a year or half year), using an aggregate forecasting method that forecasts the usage for a period by number of days was introduced ....[10]. To meet the requirements described at the beginning of this section, the data converter consists of two parts. In the first part, a simple calculation for maximum number of items in use is carried out. In the second part, a parameter L is used for any given level (number) of an equipment type in use in one year period. Given L from the maximum number of this type in use ($Item_{max}$.) to 0, the model calculates the number of days whose daily usage equals or exceeds L.

5 Forecasting Model

5.1 Data in use and error measurement

The input data of the PDSS are values for peak equipment usage in the coming year. In Trueman's paper on Internet firm revenue forecasting ...[11], a solution was introduced to obtain internet access usage by the sum of actual data and forecasted data, and the accumulated actual daily usage was used as the forecasting objective. Considering the usage peak values and distributions, the data from May 2006 to Apr 2008 were divided into four half-year periods in Fig.2, namely May-Oct 2006 (06-01), Nov 2006-Apr 2007 (06-02), May-Oct 2007 (07-01), and Nov 2007-Apr 2008 (07-02). One year's usage is then split into two periods with a balance of peak values and distributions, since annual peaks generally occur after Easter and summer holidays.

The four periods' daily usages in Fig.2 are then converted into four usage curves, as shown in Fig.3, by the data converter described in last section. From the past half year's usage, the designed model forecasts the next half year's usage for each item's daily usage level.

A 3 laptops' usage historical data can be obtained from the usage levels shown in Fig.3, with the four markers shared with Fig.4. From Fig.4, it can be seen that the usage has a positive trend of growth, though there is a slight drop in 06-2. Year 2008's first half year usage of 3 laptops can then be forecasted based on the data in the past periods. Then the annual usage data for the previous $1^1/_2$ years will be used to forecast the next half year's usage. To compare the forecasting method's performance, Root Mean Square Error (RMSE) is used to compare the forecasting results by different forecasting methods.

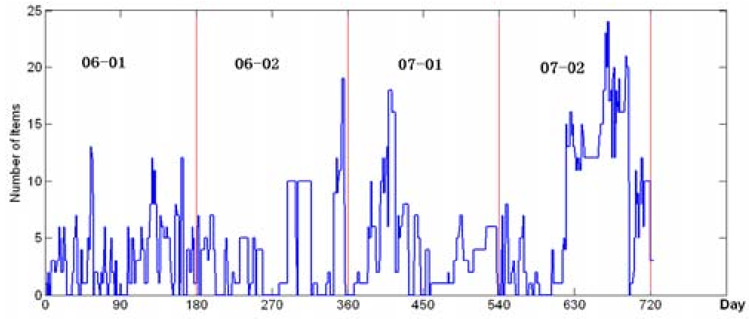

Figure 2: Two year daily usage distribution

5.2 Simulation on half year usage

There are various classic forecasting methods, such as exponential smoothing (ES), regression and ARIMA [12][13]. ES methods weigh past observations using exponentially decaying weights and have been successful in demand forecasting [6]. Regression is another class of methods that have been used on hotel and network usage forecasting [10]. Considering the similarity with relevant previous research, two ES methods, Single Exponential Smoothing (SES) for level (α) and Holt's double ES for trend (β) [12] are used from the time series class forecasting methods.

The following forecasting simulations are used to identify the forecasting method for half year usage. The model is developed and implemented in MatLab. The simulations are carried out on the two year's usage data. The lowest RMSE by using ES method occurs by using Holt's method when $\alpha = 0.6$ and $\beta = 0.4$. The RMSEs by using Holt's are significantly lower that SES's. This means that the data have trend characterisation. This forecasted results and actual data on the second half of year 2007 are compared in Fig.5. From Fig.2 it can be seen that the business has a fast growth in second half of 2007, which ES methods are not able catch up with this trend very well.

We also explored regression methods on the sets of usage data. There are two regression methods considered in this paper, which are linear regression and Logarithmic Linear Regression as suggested in Weatherford's hotel room usage forecasting paper [5]. In linear regression, each level of items D_i has had calculated a set of coefficients using Matlab function to obtain the minimum RMSE on this method, which is 9.937. The forecasted results and actual data on the second half of year 2007 are compared in Fig.6. In the logarithmic linear regression model, the nonlinear functions are transformed into linear functions for the calculation of coefficients. There are a number of ways doing this. The method used in this paper is taken from [5]. The minimum RMSE on this method is 23.454 and the forecasted results and actual data on the second half of year 2007 are compared in Fig.7. From the result view of the two regression methods, it can be seen that the linear regression performs better than the logarithmic regression.

Fig.5, 6 and 7 are forecasted results for the same period (07-2). Comparing the regression and ES methods, it can be seen that regression results are much closer to the actual curve than ES method. Using the lowest RMSE = 38.186 in SES as a benchmark,

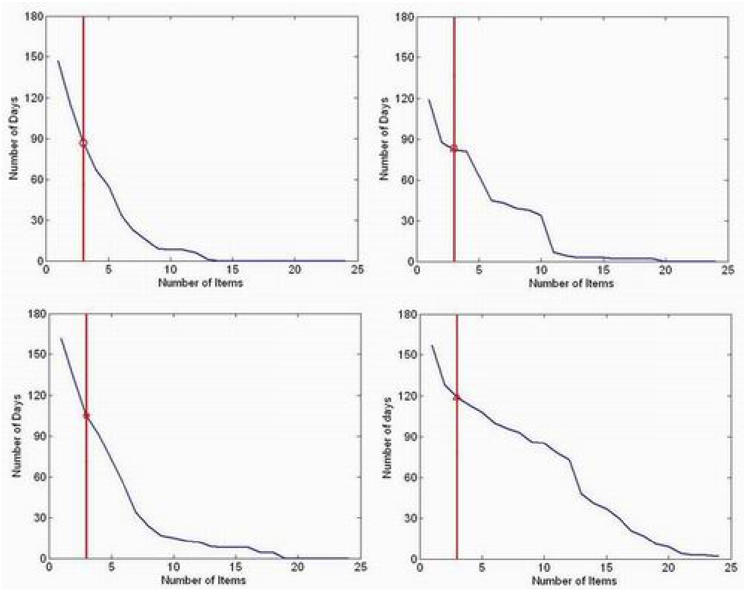

Figure 3: Half year usage curves

the Holt's method reduces the RMSE by 19.08%. Regression methods further reduce the RMSE. The logarithmic regression reduces the RMSE by 38.58%, which doubles the Holt's error performance. Furthermore, the linear regression reduces RMSE by 73.98%, which doubles the logarithmic regression's error performance, and is four times less than the Holt's one. Since the observed data has a high growth trend, the historical data are all important to extract this growth rate, for which linear regression is better than ES methods. Whereas the ES methods treat the nearer observed data more important, which influences the accuracy of the calculated growth rate. Considering the business situation in Universal ltd, the growth rate in the following year will keep high, linear regression is an appropriate method in terms of the error performance on the equipment usage forecasting problem in this paper.

6 Purchasing Decision Model

The purchasing decisions address two problems: 1) whether to own or to sub-hire for one item given the forecast peak demand; 2) the number of a type of equipment which should to be owned to minimise costs. If the number that needs to be owned is more than the current stock level, the additional number needs to be purchased; otherwise stocks can be run down over time. The two problems are explained in subsection A and B respectively. A list of symbols is given in Table 1.

6.1 Single item situation

A number of assumptions are suggested before the calculation:

Assumption 1: Only the next year's data is considered, such as days on hire, depreci-

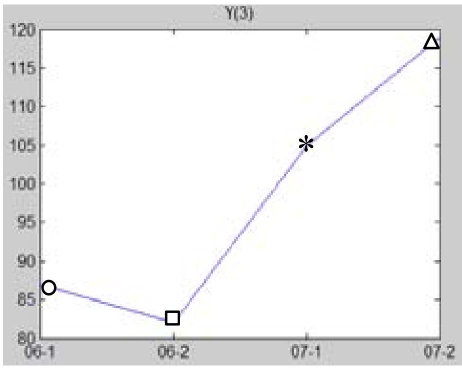

Figure 4: Three laptops' usage obversed data for forecasting

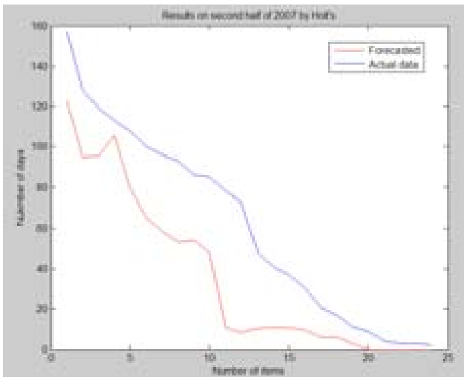

Figure 5: Usage 07-2 by Holt's

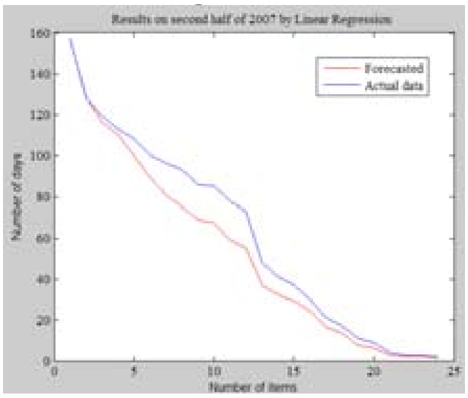

Figure 6: Usage 07-2 by Linear Regression

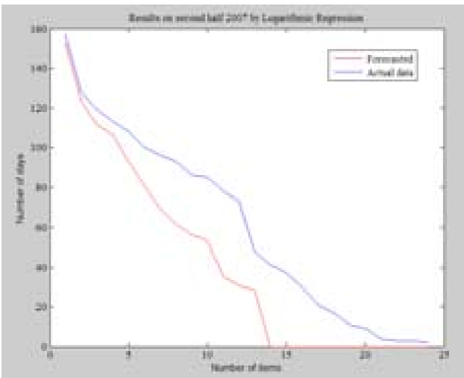

Figure 7: Usage 07-2 by Logarithmic Regression

Table 1: Symbol of input and output data

Constant inputs	P_h: Hire price per day	P_s: Sub hire price per day
	P_{pur}: Equipment's purchase price	Dep: First year depreciation %
	C_m: Maintenance cost per year	N_{own}: Number currently owned
Variable inputs	N_{peak}: Peak number of an equipment in use	D_f: Forecasted total hire days of an item
Outputs	D_{min}: Minimum days on hire to own an item	N_{pur}: Number to be purchased

ation rate and maintenance cost. The output data is the forecasted value for the next year. The data input are this year and previous year's data.

Assumption 2: Only one type of equipment is considered at one time, and its usage is considered to be independent of other equipment's usage.

Assumption 3: The sub-hire price (P_s) herein is set as a constant value (market price). In a real situation, suppliers may change the price depending on how urgent the equipment is required. Sub-hires from a competitor may be charged two or three time more than an ordinary price. Therefore the calculation of P_s uses non-uniform weighted mean average on various suppliers' price:

$$P_s = \frac{aP_{sa} + bP_{sb} + cP_{sc} + ...}{a + b + c + ...} \tag{1}$$

where weights a, b, $c\ldots$ are the number of times of sub-hires from each supplier in the past year. P_{sa}, P_{sb}, P_{sc}... are the sub-hire prices from each supplier.

Assumption 4: the purchase price is set as a constant value. It is assumed not to change from the decision making point to the purchasing date.

To answer the first problem at the beginning of section VI, 'to own or to sub-hire', a formula is designed to calculate the D_{min} based on the assumptions and given inputs:

$$D_{\min} = \frac{P_{pur} \times dep + C_m}{P_s} \tag{2}$$

D_f for this item's next year usage is obtained from the forecasting model. The decision can be suggested: if $D_f \geq D_{min}$, it costs the company less to own the equipment than to sub-hire, where "=" means that the cost to own and sub-hire are same and the system will advise to own the item. All inputs have two digit decimal accuracy. The result D_{min} is round up to the next integer as it is a number of days in the proposed system.

6.2 Type of items situation

Now our consideration is extended to a type of equipment – a group of identical items. "Identical" here means the function of each item in this equipment type is identical, although the precise specification and manufacturer may vary. Learning from the single item situation, the requirement is to obtain the number of items that have $D_f \geq D_{min}$ by comparing each item's D_f and D_{min} for this type of equipment. A simple example is given in Table 2. There are three item's $D_f \geq D_{min}$, which are the required number for this type of equipment.

For the purchasing decision, two parameters need to be focused, which are the peak number of equipment items in use N_{peak}, and the suggested number of equipment items to be purchased N_{pur}. Since the inventory pool can be instantly filled by sub-hiring items, N_{peak} may be greater than N_{own}. A list of D_f is calculated for each level of equipment in use during a year's period. It aims to find out the level of items whose D_f values are not greater than their D_{min} values. Then the level of relevant items should be owned for stock. The model is developed based on the flowchart shown in Fig.8.

Table 2: Comparison in item D_f and D_{min}

	D_{min}	D_f	
Owned Laptop 1		8	<
Owned Laptop 2		11	>
Owned Laptop 3		9	=
Owned Laptop 4		13	>
Owned Laptop 5	9	6	<
Sub-hired 1		5	<
Sub-hired 2		2	<
......		...	...

The model is versatile which can be used in purchasing decisions for either rental companies or other service business with similar decision problems. The final output N_{pur} is the suggested number of items that should be purchased for stock. If $N_{pur} \leq 0$, it means that the current stock level is more than required. The company may consider methods to reduce the stock level if the negative N_{pur} is in a large amount. As the purchasing DSS outcomes are calculated by financial data and the forecasted usage, the valid outcomes highly depend on the accuracy of forecasting results as discussed in section V, and the updated marketing data. Therefore, it is important that the marketing data described in Table 1 are precise for the system input.

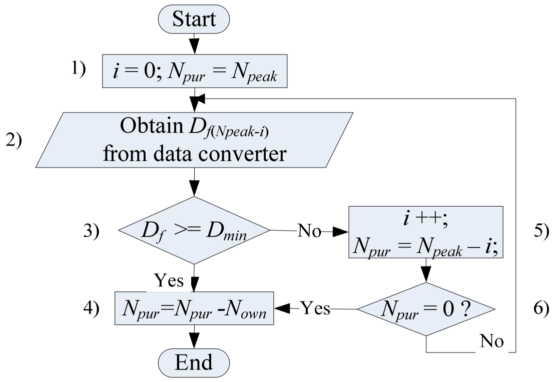

Figure 8: Purchasing model outline flowchart

7 System Test Results

A number of tests were carried out on the designed purchasing model. The model and software were validated by comparing the system output and manual calculation. One of the tests was based on the actual 2007 yearly usage. Given the current market data for P_{pur} and P_s, the system outputs are: D_{min} = 18 days and N_{pur} = -5.

Data shows that there are 23 laptops owned for stock. The suggested number of purchase is -5 means that 5 of the owned items are not frequently used and may not be necessary to be kept in stock. In Neuhaus's asset PDSS, the system was based on an Expert System, which gives 'Yes' or 'No' for asset purchasing or leasing decisions. The PDSS proposed in this paper extends the system output into a quantitative result which can give a required number on purchasing decisions [3]. According to the feedback from the company, sub-hiring is preferred to purchasing to reduce the occupying of cash flow if the former is not expensive as the latter.

8 Conclusion and Further Work

In this paper, we investigate purchasing decision support when considering sub-hires, and solution approaches are compared. A data converter algorithm is designed to transform the daily data series into usage in a required period. Experiments comparing forecasting methods using annual usage data are carried out to compare the characteristics of a number of forecasting methods for our problem. One forecasting method, linear regression, is selected for the development of the forecasting model since it has the best accuracy. The purchasing model is developed which considers both owned inventory and sub-hire volume.

Future work includes the further tailoring of the forecasting model. The full system will be evaluated in a real environment over a period of time. Issues to be considered include the potential usage, for example the data capture for work foregone due to the unavailability of sub-hired items. Moreover, in our test results, it is shown that a number of items of equipment are not frequently in use. Further decisions on how to deal with these equipments will be researched.

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