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Improving Supplier's Performance Using Common Replenishment Epochs in a Vendor-Managed Inventory System

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Abstract This paper studies a single-product supply chain, in which the supplier needs to manage its own inventory to fulfill requests from a set of heterogonous retailers. The retailers' requests are determined by the supplier in a fashion of vendor-managed inventory (VMI). Under a VMI arrangement, the supplier agrees to pay all the order setup costs and a portion of unit holding costs for the retailers. The objective of the system is to minimize the total relevant cost of the supplier. This paper compares the supplier's performance under the VMI strategy and an uncooperative supply chain in which the retailers decide replenishment requests based on their order setup and holding costs. To further improve the supplier's performance, a VMI/CRE strategy, which applies the strategy of common replenishment epochs (CRE) under the VMI system, is utilized to save the order processing costs of the supplier. However, the supplier is required to provide a price discount to compensate retailers' loss for fitting ordering schedules with CRE scheme. Computational experiments are conducted and the performances of the VMI and VMI/CRE strategies are quite satisfied.

Keywords supply chain; vendor-managed inventory; common replenishment epochs

1 Introduction and Literature Review

In order to enhance competitiveness, coordination initiatives are utilized in supply chain to reduce waste and redundancy. Vendor-managed-inventory (VMI) program, in which the supplier is authorized to manage the retailers' inventory, has been wildly used as a coordination initiative for supply chains in various industries since the early adoptions by Wal-Mart, Kmart and JC Penney (Emigh, 1999; Dong et al., 2007). Due to the popularity of VMI, several literatures are written around the theme, e.g., Dong and Xu (2002), Piplani and Viswanathan (2003), Yao et al. (2007), etc. However, most literatures consider supply chain inventory models with single supplier and single buyer. Nevertheless, the single-vendor multi-buyer models represent most of the practical cases (Lu, 1995) and deserve more research attention. For a single-vendor multi-buyer system, Nachiappan and

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Jawahar (2007) formulate a model and solve the problem by a heuristic based genetic algorithm. Note that in the VMI models of Dong and Xu, Yao et al., and Nachiappan and Jawahar, the supplier absorbs the retailers' (or retailer's) order costs and inventory holding costs and the objective is to maximize the profit of the supply chain. Under VMI, the retailers may remain the owners of their inventory or there is still some inventory cost incurred at the retailers. Therefore, this study assumes that the supplier bears the entire order costs and partial holding costs of the retailers due to its responsibility to manage retailers' inventory under VMI.

Note that the three mentioned VMI models assume the supplier's order quantity as an integer multiple of the retailer's replenishment quantity. With defining a unit length of a period in this study, all retailers' requests occur only at discrete time epochs. Hence, the demand of the supplier which constituted from the aggregate requests of all retailers may vary at different time epochs. The inventory problem of the supplier facing discrete time-varying demand is known as a dynamic lot sizing problem (Jans and Degraeve, 2008). Therefore, we utilize Silver-Meal algorithm (Silver and Meal, 1973), one of the lot sizing heuristic commonly used in practices, to determine the replenishment quantities of the supplier.

Although VMI may benefit the whole supply chain, several studies have revealed that most benefits of VMI are flowing to the retailers (Dong and Xu, 2002; Yao and Dresner, 2008). To set up VMI system, the supplier may install IT facilities or allocate resources to monitor and control retailers' inventory. Hence, the supplier's costs may increase for providing VMI services. A survey has shown that only few suppliers have received a costs decrease for VMI arrangement and most suppliers have responded that their participation in VMI was demanding by their customers (Baljko, 2003). However, the suppliers have realized the trend and are looking ways to further improve their performance (Yao and Dresner).

Besides VMI, there are several coordination initiatives aimed to improve supply chain efficiency. Viswanathen and Piplani (2001) investigate a coordination initiative in which the supplier offers a price discount to attract retailers to replenish following its required common replenishment epochs (CREs). Subsequent researches have revealed the benefits of CRE strategy, e.g., Mishra (2004). In this study, we incorporate CRE strategy in a VMI model to further improve the supplier's performance. Therefore, the objective is to minimize supplier's total cost. Three models presented in the study include an uncooperative supply chain, a VMI supply chain, and a VMI/CRE supply chain. Performances of these three models are compared in the numerical experiment.

2 Model Formulation and Descriptions

Consider a supply chain system with a single supplier and m heterogeneous retailers, in which the supplier is responsible for replenishing a single item product to all the retailers with an identical price P. The demand of the system only occurs at the retailers with constant demand rate λ_i for retailer $i(=1,\ldots,m)$. All the demands must be satisfied with no backlogging and the deliveries are assumed instantaneous. For any excess inventory on hand, retailer i needs to pay a unit holding cost h_i. A retailer i also occurs a fixed cost k_i for any order placed. The supplier incurs a delivery cost A_i for distributing individual order to retailer i. In addition, the supplier costs a fixed value of A_s for processing simultaneous orders of all retailers. While the supplier manages its own inventory, it incurs a unit holding cost H per unit time and a setup cost K for an order released to its upstream vendor.

The objective of the supply chain system is to minimize the total relevant cost per unit time of the supplier. This paper proposes three models to manage the system. The first model is an uncooperative one, where the supplier and all the retailers manage their own inventories independently. The second model is based on VMI, in which the inventories at the retailers are managed by the supplier. In order to achieve a VMI agreement, the supplier needs to bear the retailers' ordering costs and a certain portion of retailers' inventory holding costs. The third model aims to improve the performance of VMI strategy by further employing CRE to synchronize the replenishment processes of all the retailers. In the following, the system is described and analyzed according to the three models.

2.1 Uncooperative Model: The system with independent policies

Facing a constant demand rate, a retailer i decides its inventory policy based on the classic EOQ model with order interval $t_i = \sqrt{2\lambda_i/k_i h_i}$ and total cost $g_i = \sqrt{2\lambda_i k_i h_i}$. Total relevant cost of the supplier can be divided into two parts. The first part is the cost for responding order requests from all the retailers, and the second part is the cost to maintain its own inventory. The costs for responding retailers' order requests include the fixed costs for processing and delivering orders. The supplier's total delivery cost simply depends on the retailers' order patterns, and can be obtained easily as $\sum_{i=1}^{m} A_i/t_i$. The supplier's total processing cost is affected by the interrelationship among the retailers' order patterns, i.e., by the correlation among the retailers' order intervals. To illustrate the correlation of t_i's, let $y$be the largest rational number such that $t_i/y \in N$(natural numbers), for all i. That is, t_i can be represented by $l_i y, l_i \in N$, Note that t_i can be considered as a rational number without severe damaging the optimality of g_i due to the robustness of the classic EOQ model. With frequent orders by the retailers, it is reasonable to approximate the total order processing cost per unit time as $A_s/\ y$. Therefore, the cost per unit time of responding the order requests from all the retailers is $A_s/y + \sum_{i=1}^{m} A_i/t_i$.

The analysis of the supplier's inventory costs is also beneficial from the definition of y. Hence, the order lot size of retailer i can be expressed as $\lambda_i l_i y$ units. With y as the unit length of a period, periods t = 1,2,... in a time horizon represent the time points y,2y,.... The supplier's unit inventory holding cost H is now converted to Hy representing the unit holding cost per period. Assume that all the retailers place their first orders simultaneously at the start of the planning horizon at t = 1. The time points indexed by t represent all the possible occasions for any retailers to place their orders. Hence, the aggregate demand of the supplier at period $t$is denoted by

$$\Lambda_t = \sum_{i=1}^{m} \delta_i(t)\lambda_i l_i y,$$

where

$$\delta_i(t) = 1 - \lceil (t-1)/l_i \rceil + \lfloor (t-1)/l_i \rfloor \tag{1}$$

The symbol of $\lceil z \rceil$ denotes the smallest integer not smaller than z and $\lfloor z \rfloor$ the largest integer not larger than z. Note that $\delta_i(t)$ = 1 indicates that retailer i places a request at period t and $\delta_i(t)$ = 0, otherwise.

Observe that the request pattern of a retailer i resumes every l_i periods. Define the least common multiple (LCM) of all l_i's as L. Thus, the behavior of the request sequence (Λ_t) will repeat every L periods. This phenomenon allows us to approximate the inventory policy at the supplier every L periods. From now on, denote the request sequence from periods 1 to L as Λ_L. Given the unit holding cost per period Hy, the problem of determining a best inventory policy in periods 1 to L is known as a dynamic lot sizing problem. This research will utilize the lot size rule of least period cost in Silver-Meal algorithm to determine the supplier's replenishment lot sizes in periods 1 to L with the total cost G. Then, given Λ_L and Hy, the supplier's minimum inventory cost per unit time using Silver-Meal algorithm is $G(\Lambda_L)/Ly$. Thus, the supplier's minimum total relevant cost becomes

$$MinTRC = \frac{G(\Lambda_L)}{Ly} + \frac{A_S}{y} + \sum_{i=1}^{m} \frac{A_i}{t_i} \tag{2}$$

In the later models, the order interval of retailer i will be influenced by VMI or VMI/CRE strategies and changed from t_i to t_i^v, t_i^{vc}, respectively. Corresponding to y, L, Λ_t, G, TRC in Uncooperative model, let's define y^v, L^v,, G^v, TRC^v in VMI model and y^{vc}, L^{vc},, G^{vc}, TRC^{vc} in VMI/CRE model.

2.2 VMI Model: The system with a VMI strategy

In general, VMI strategies allow suppliers to take charge of managing inventories for retailers. By adopting VMI strategy, the supplier is authorized to make replenishment decisions for the retailers. In order to do so, the supplier is asked to bear the ordering costs for the retailers, and share partial inventory costs for the retailers. The holding cost shared by the supplier is represented by an identical proportion r for all the retailers. That is, it costs the supplier rh_i for each unit of inventories holding for retailer i, and the rest of the proportion $(1\text{-}r)h_i$ is still the expenses of retailer i.

In literature, there are various methods for a supplier to determine inventory policies for both retailers and its own under a VMI system. Basically, with 100% visibility of the demand and inventory information of retailers, the supplier will consider all the costs in the system to decide the replenishments of all retailers and its own. Under VMI strategies, some of the supplier's costs dedicated to retailer i, can be clearly identified, e.g., A_i, k_i, and rh_i. On the other hand, other costs of the supplier such as A_s, H and K, can still be identified to a particular retailer if the system contains only a single retailer—the particular one (e.g., Dong and Xu, 2002; Yao et al., 2007). It is clearly that in a multi-retailer system, the supplier's costs of making its own replenishment decision are relevant to all the retailers but not for any particular ones. Thus, the supplier in VMI model will consider the costs which can be recognized for serving specific retailers while deciding the inventory policy for the retailers. Hence, under VMI arrangement, the order interval for retailer i becomes $t_i^v = \sqrt{2\lambda_i/(A_i+k_i)\,rh_i}$ with inventory cost $\sqrt{2\lambda_i rh_i(A_i+k_i)}$ born by the supplier. Without paying the order setup cost, retailer i incurs only partial holding cost $g_i^v = \lambda_i t_i^v(1-r)h_i/2$. Based on for all i and a similar procedure in Uncooperative model, a set of y^v, L^v,Λ_t^v and G^v can be obtained for VMI model. Therefore, the minimum total relevant cost of the supplier using VMI is

$$MinTRC^{v}=\frac{G^{v}\left(\Lambda_{L^{v}}^{v}\right)}{L^{v}y^{v}}+\frac{A_{s}}{y^{v}}+\sum_{i=1}^{m}\sqrt{2\lambda_{i}rh_{i}\left(A_{i}+k_{i}\right)} \tag{3}$$

2.3 VMI/CRE Model: The system with a VMI/CRE strategy

In VMI/CRE model, the supplier intends to further improve its performance under VMI arrangement by adopting CRE strategy. The CRE strategy implemented in VMI/CRE model is based on the framework of Viswanathan and Piplani (2001). A set of preferable CREs offered by the supplier is labeled as X, and T represents a nonspecific CRE in X. Notice that all the retailers' order intervals are previously determined by the supplier in VMI model. Instead of allowing the retailers to accept–CRE strategy according to their decisions as in the model of Viswanathan and Piplani, the CRE strategy in VMI/CRE model allows the supplier to further adjust the order intervals for all the retailers. Based on the supplier's expenses on the order setup cost k_i and partial unit holding cost rh_i, the order interval for retailer i following CRE T is $t_i^{vc}=n_iT$, where $n_i=\left\lfloor 0.5+\sqrt{1+8k_i/\lambda_i rh_iT^2}/2\right\rfloor$ by a similar analysis of Viswanathan and Piplani. The inventory cost shared by the supplier for retailer i in VMI model becomes $\left(A_i+k_i\right)/n_iT+\lambda_in_iTrh_i/2$.

Under VMI/CRE model, the total cost of retailer i becomes $g_i^{vc}=\lambda_it_i^{vc}\left(1-r\right)h_i/2$. The cost increase of retailer i, $g_i^{vc}-g_i^{v}=\lambda_i\left(t_i^{vc}-t_i^{v}\right)\left(1-r\right)h_i/2$, should be compensated by the price discount offered by the supplier under CRE strategy. Thus, the minimum discount accepted by retailer i is $d_i=\left(g_i^{vc}-g_i^{v}\right)/\lambda_iP$. To avoid violation of trade laws in many countries, the discount should be identical for all the retailers (Mishra, 2004). Therefore, d = $\max_i\{d_i\}$ is defined as the discount for all the retailers to join the CRE strategy. Based on t_i^{vc} for all i, a set of y^{vc}, L^{vc}, $\Lambda_t^{vc}$and G^{vc} can be obtained for VMI/CRE model. The minimum total relevant cost of the supplier using VMI/CRE is

$$\underset{T\in X}{Min}\,TRC^{vc}=\frac{G\left(\Lambda_{L^{vc}}^{vc}\right)}{L^{vc}y^{vc}}+\frac{A_s}{T}+\sum_{i=1}^{m}\left(\lambda_iPd+\frac{A_i+k_i}{n_iT}+\frac{\lambda_in_iTrh_i}{2}\right) \tag{4}$$

s.t. $d\geq\left(1-r\right)\left(n_iT-\sqrt{2\lambda_i/\left(A_i+k_i\right)rh_i}\right)h_i/2P$ and $n_i\in N$, for i= 1,..,m.

The CRE strategy in VMI/CRE model differs from the work of Viswanathan and Piplani in three ways. First, the objective function in (4) considers not only the costs toward the retailers but also the costs to control the supplier's own inventory by relaxing Lot-for-Lot policy in Viswanathan and Piplani. Second, the supplier needs to incorporate retailers' setup costs and partial holding costs into (4) under VMI arrangement. Finally, the CRE compensation through the price discount is only referred to the retailers' partial holding costs.

3 Algorithms and Numerical Experiments

Algorithms for the three models are given below.

1. Algorithm for Uncooperative model:
   Step 1. Establish the order interval $t_i=\sqrt{2\lambda_i/k_ih_i}$ for all i.
   Step 2. Compute the unit length of a period $y$such that $t_i/y=l_i\in N$ for all i.

Step 3. Determine the aggregate demand Λ_t from (1) and $L \quad$ = LCM of l_i for all i.
Step 4. Decide the unit holding cost Hy and the request sequence $\Lambda_{L.}$
Step 5. Use Silver-Meal algorithm to obtain G(Λ_t)/Ly.
Step 6. The minimum total relevant cost of the supplier TRC is obtained from (2).

2. Algorithm for VMI model:
Step 1. Calculate the order interval $t_i^v = \sqrt{2\lambda_i/(A_i+k_i)\,rh_i}$ for all i.
Step 2. Repeat Steps 2-5 in the algorithm of Uncooperative model by replacing t_i by t_i^v and then y^v, L^v, $G^v\left(\Lambda_{L^v}^v\right)/L^v y^v$,are obtained.
Step 3. The minimum total relevant cost of the supplier TRC^v is obtained from (3).
3. Algorithm for VMI/CRE model:
Step 1. Select a T from X.
Step 2. Determine $t_i^{vc} = n_i T$, where n_i = $\left\lfloor 0.5+\sqrt{1+8k_i/\lambda_i rh_i T^2}/2\right\rfloor$ for all i.
Step 3. Repeat Steps 2-5 in the algorithm for Uncooperative model by replacing t_i by t_i^{vc} and then y^{vc}, L^{vc},, G^{vc} are obtained. Clearly, $y^{vc} = T$ and L^{vc} = LCM of all n_i's.
Step 4. Decide $d_i = (1-r)\left(n_i T - \sqrt{2\lambda_i/(A_i+k_i)\,rh_i}\right) h_i/2P$ for all i and d = $\max_i\{d_i\}$.
Step 5. The total relevant cost TRC^{vc} given T is found by the right hand side of (4).
Step 6. Go to Step 1 to select another T and repeat Steps 2-5 until the minimum of TRC^{vc} is obtained.

Consider a supply chain with single supplier and five retailers, the numerical experiments design two scenarios, labeled as EX 1 and EX 2, to demonstrate the three proposed models. The cost parameters for the two scenarios are identical with P = 1, K = 100, H = 0.075, k_i = 58, h_i = 0.15, for all i. The value ranges for other cost parameters are A_s = 100, 500, 1000, A_i = 50, 100, and r = 0.3, 0.5, 0.9. With various values of A_s, A_i and r, there are 18 instances each for EX 1 and 2 and a total of 36 instances for the entire experiment.

The demand rates of the retailers in EX 1 are 2080000, 2080000, 520000, 520000, 130000, and 2080000, 520000, 130000, 32500, 8125 in EX 2. The various demand patterns of the retailers with identical cost parameters result in different timings of the retailers' ordering requests, e.g., the standard deviations of order timings of EX 1 and EX 2 are 0.023 and 0.118 respectively.

By utilizing the algorithms given in this section, the minimum total relevant costs of the supplier for EX 1 and 2 can be obtained from (2)-(4). The savings of VMI model and VMI/CRE model over Uncooperative model are given in percentages as shown in Table 1. The differences of savings in VMI model and VMI/CRE model are given in the last few rows of EX 1 and 2 in Table 1, i.e., the savings of VMI/CRE model subtract the savings of VMI model.

Based on Table 1, applying VMI or VMI/CRE generates positive savings over Uncooperative model in 33 or 34 out of total 36 instances, respectively. By adopting CRE after VMI, VMI/CRE shows advantages over simple VMI in 29 out of 36 instances. It seems that the savings of VMI increases as r, the proportion of holding cost shared by the supplier, decreases. On the contrary, as r increases, the savings of VMI/CRE increases. The later trend is similar to the saving differences of VMI/CRE over VMI. There are instances where the savings of VMI is negative. This indicates as the supplier overpays the retailers' inventory holding costs, the benefits from VMI may not cover what it costs. However, as r is getting large, the improvement made by VMI/CRE over VMI is significant to the supplier. This indicates the best timings to incorporate CRE under VMI strategy, i.e., when the supplier has to bear most of the retailers' holding costs. In EX 1, VMI/CRE makes improvement over VMI in all instances, whereas there are two instances where VMI/CRE results in inferior solutions in EX 2. The inferior cases under VMI /CRE might be explained by the supplier's lower processing costs which results in fewer CRE benefits that cannot cover the compensation paid for CRE. However, the numerical experiment indicates the opportunities to utilize VMI/CRE model for improving supplier's performance, i.e., when the variation of retailers' order timings is smaller.

Table 1: The savings of VMI and VMI/CRE models over Uncooperative model

	Model	A_s		100		500		1000	
		A_i		50	100	50	100	50	100
EX 1	VMI	r	0.3	31%	42%	52%	59%	56%	63%
			0.5	11%	25%	38%	47%	43%	52%
			0.9	-19%	-1%	17%	29%	23%	35%
	VMI/CRE	r	0.3	31%	31%	78%	76%	87%	86%
			0.5	23%	25%	76%	74%	85%	85%
			0.9	13%	17%	71%	69%	82%	81%
	VMI/CRE-VMI	r	0.3	-1%	-11%	26%	18%	31%	24%
			0.5	12%	1%	38%	27%	43%	33%
			0.9	31%	18%	54%	41%	58%	46%
EX 2	VMI	r	0.3	39%	48%	55%	62%	57%	64%
			0.5	22%	33%	41%	50%	45%	54%
			0.9	-5%	10%	22%	33%	26%	38%
	VMI/CRE	r	0.3	-25%	-83%	67%	48%	82%	71%
			0.5	14%	0.04%	77%	71%	87%	84%
			0.9	38%	34%	80%	78%	87%	86%
	VMI/CRE-VMI	r	0.3	-64%	-131%	12%	-14%	25%	7%
			0.5	-7%	-33%	35%	20%	42%	30%
			0.9	43%	24%	58%	44%	61%	48%

4 Conclusions

This paper proposes a single-supplier multi-retailer VMI model, in which the supplier shares retailers' unit inventory holding costs and decide retailers' replenishment from the supplier's perspective. The objective is to improve the supplier's total cost, including the costs of filling up the requests of the downstream retailers and the costs of managing its own replenishment to an upstream vendor. The performance of the VMI model is further enhanced by using CRE strategy to save joint order processing cost.

The results of the numerical experiment show that both VMI and VMI/CRE models yield significant improvements over Uncooperative model. In comparing VMI and VMI/CRE models, VMI/CRE strategy results further savings for most cases with higher joint order processing costs. For lower joint order processing costs, the extra savings made by VMI/CRE may become negative, especially for cases with higher deviation of order timings. As the supplier shares more holding costs of the retailers, the supplier's performance suffers under VMI arrangement, whereas it progresses under VMI/CRE.

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