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Risk Management Analyses on Measuring the Robustness of the Water Supply Network System in Tokyo

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1 Introduction

Water has always been an essentially inevitable resource for our daily lives. "Water problems" such as shortage caused by natural phenomena, disruption of water pipelines due to "natural disasters", such as earthquakes, floods; and also by some intentional attacks, and terrorism, need to be challenged at least, even though they might not be solved completely.

Given the concentration of population in large cities, water demand in urban areas has been growing rapidly. Thus, keeping the reliability and the robustness of the water supply system high has been becoming more and more important.

Tokyo, as the exceptionally biggest city in Japan, has its modern water supply network, which was established in 1898. Nowadays, 6,859,500 $m^3$of water per day is provided for Tokyo area, excluding Hinohara village, Okutama town, and Izu Islands. These areas have their own water supplies, which are not provided by the Tokyo water supply network [1].

The Tokyo water system has 11 purification plants, for example Kanamachi, Asaka, and Misato. From the historical data, we find that the share in supply for all purification plants during the period of 1999-2006 is rather stable, and Asaka, Higashi-Murayama, Kanamachi, and Misato are the four major plants occupying almost 80 percent of the total. These water resources and supply areas for Tokyo is shown in the Figure 1.

These purification plants obtain the water from four water resources: 1) Tone/Ara River, which covers 79.9% of the total water supply in Tokyo; 2) Tama River (17.0%); 3) Sagami River (2.9%); and 4) ground water (0.2%). From this coverage rate, we can say that the water supply in Tokyo heavily depends on the Tone/Ara River, and followed by the Tama River.

Asides from that, the Tokyo area is very vulnerable to natural disasters. From 1970-2002, the natural disasters which has mostly happened in this area are earthquakes, wind

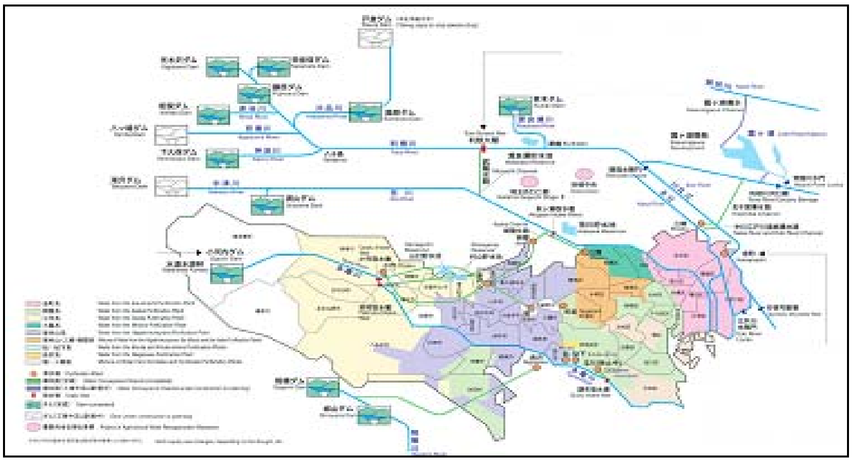

Figure 1: Tokyo Water Resources and Supply Areas

storms, waves/surges, and flood [2]. These disasters, especially earthquake, affects buildings, including the infrastructures.

For example, the Hanshin earthquake in 1995 brought serious damages to the Kobe water supply network system. Due to water supply network system disruption, around one million people had to live inconveniently, with lack-of-water lives during almost one month. Among major and essential lifelines networks, such as electricity, gas, and water; electricity network was the first to recovered from black out normally. Almost all households could use electricity in a just few days after the earthquake, while gas and water network needed almost 3 months and one month, respectively. Therefore, this study aims to investigate the robustness of Tokyo water supply network system under several disturbances, such as damaged purification plants, broken transmission pipes, and broken water resources.

In the following section, we explain our methodology based upon the Network Flow Optimization Problem, with input data used for the model analysis. Then in section 3, we show our numerical results, starting from the Standard case, followed by the simulation cases and the worst case analysis. In the last section, we derived some conclusions based on this study.

2 Methodology and Input Data

2.1 Network Flow Optimization Problem

The network flow optimization approach used in this study is a multi-source multi-sink maximum flow model, wherein sources correspond to water intake sites, while sinks are demand sites. We formulate the network flow optimization problem maximizing the total flow, under the conditions that each source node corresponding to water intake site has upper bound of water supply availability, each edge corresponding to the water pipeline has capacity indicating the upper bound flow availability, and each sink node has an upper bound of its each water demand.

We solve the network flow optimization problem for each case such that several edges are "broken" randomly, thus arbitrary number of edges are "disrupted", thus try to measure the "coverage rate" quantitatively, indicating how much of the total demand can be met.

Given an undirected network $N = (V,E)$ consisting of node set V and edge set E with $V = n$ and $E = m$, respectively, we assume each edge $(i, j) \in E$, $i, j \in V$, has its capacity u_{ij}. We partitioned the node set V into three subsets, denoted by source (S), intermediate (R), and sink (T) sets, respectively, thus $V = S \cup R \cup T$. The network $N= (V,E)$ is illustrated in Figure 2.

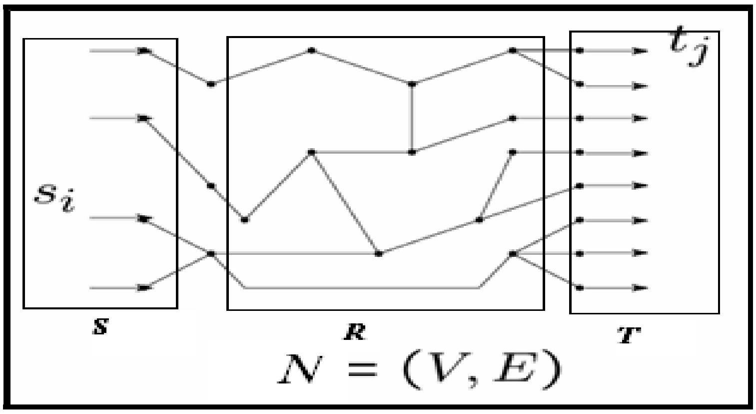

Figure 2: Multi-source Multi-sink Problem

The mathematical formulation of this network flow optimization model can be given as follows.

$$\text{Maximize} \quad Z = \sum_{i \in T} (\sum_j x_{ji} - \sum_j x_{ij})$$

$$\text{subject to} \quad \sum_j x_{ji} - \sum_j x_{ij} = \begin{cases} \geq -s_i, & i \in S \\ = 0, & i \in R \\ \leq t_i, & i \in T \end{cases} \tag{1}$$

$$0 \leq x_{ij}, x_{ji} \leq A_{ij} u_{ij}, \quad (i, j) \in E$$

Each x_{ij} is the decision variable designating the flow passing through each edge (i, j) from node i to j. The volume t_j indicates the "demand of water" at each sink node j, whilst the flow from each source node is denoted by s_i. The objective function of this model is to maximize the amount of total flow running into sink nodes. Assuming that each edge in the network may be broken with equal probability, we measure the reliability of the network system by computing the maximum flow of this network. In the formulation, we include random variables $\{A_{ij} : (i, j) \in E\}$, each of which takes the value of 0 or 1, corresponding to the case that each edge (i, j) is "broken" or "unbroken" (alive).

The capital letter Z is used here to denote the stochastic maximum flow value, since the realized maximum value of (1) naturally becomes a random variable, because it depends on the random capacity $A_{ij}u_{ij}$.

Moreover, since we assume each edge is "broken" independently, and the maximum flow of (1) including the mean value of Z under the condition that the random variables $A_{ij}$is satisfying. $\sum A_{ij} = m - k$, corresponding as the condition that k out of m edges are randomly broken, then the conditional distribution function can be defined by

$$F_k(z) = P\{Z \leq z | \sum A_{ij} = m - k\} \tag{2}$$

which provides the ratios of the networks which maximum flow value is less than or equal to z to all the $\begin{pmatrix} m \\ k \end{pmatrix}$ networks obtained by deleting k edges randomly from the original network. Thus, the conditional expected maximum flow value for this network can be defined as follows.

$$z_k = E[Z\Sigma A_{ij} = m - k] \tag{3}$$

As a result, the coverage rate (CR) or the reliability for this network can be computed as the degree of satisfying the total demand as follows:

$$r_k = \frac{z_k}{\sum\limits_{i \in T} t_i} \tag{4}$$

Therefore, the maximum and minimum of CR are given as follows.

$$\overline{r_k} = max \frac{[Z|\sum A_{ij} = m - k]}{\sum\limits_{i \in T} t_i} = \frac{\max[Z|\sum A_{ij} = m - k]}{\sum\limits_{i \in T} t_i} \tag{5}$$

$$\underline{r_k} = min \frac{[Z|\sum A_{ij} = m - k]}{\sum\limits_{i \in T} t_i} = \frac{\min[Z|\sum A_{ij} = m - k]}{\sum\limits_{i \in T} t_i} \tag{6}$$

It is almost impossible to find r_k, $\overline{r_k}$, and $\underline{r_k}$ exactly by computing Z for all $\begin{pmatrix} m \\ k \end{pmatrix}$ cases; instead we try to estimate r_k, $\overline{r_k}$, and $\underline{r_k}$ for each k ranging from 0 to m by applying Monte Carlo method.

The conditional expected maximum flow value defined by (3), and the corresponding reliability defined by (4) based upon the following Monte Carlo simulation procedure.

Computational procedure:

Step 1. Determining the set of k "broken" edges out of m edges randomly, thus obtaining a network $N_k = (V_k, E_k)$.

Step 2. Solving the flow optimization problem for the network $N_k = (V_k, E_k)$.

Step 3. Calculating the statistics for the coverage rate.

We solve the above network flow optimization problem for each k, ranging from 0 to m, at most 10,000 times, thus obtain the estimates for r_k, $\overline{r_k}$, and $\underline{r_k}$ respectively.

2.2 Input Data and Definition of Cases

In order to run the network flow optimization model described in the previous section as in (1), we used the following data, regarding the right hand side data s_i, $i \in S$ given,and t_j, $j \in T$..

Table 1: Water Supply (unit: $10^3 m^3$/day) for Standard Case

<table>
<tr><th>River</th><th>Purification Plants</th><th>Capacity</th><th>Region</th></tr>
<tr><td rowspan="5">Tone-Ara River</td><td>Kanamachi</td><td>939</td><td rowspan="2">Joto</td></tr>
<tr><td>Misato</td><td>900</td></tr>
<tr><td>Asaka</td><td>950</td><td rowspan="3">Yamate</td></tr>
<tr><td>Misono</td><td>280</td></tr>
<tr><td rowspan="2">Higashimurayama</td><td rowspan="2">843</td></tr>
<tr><td rowspan="5">Tama River</td></tr>
<tr><td>Ozaku</td><td>150</td><td rowspan="5">Tama</td></tr>
<tr><td>Sakai</td><td>120</td></tr>
<tr><td>Kinuta</td><td>0</td></tr>
<tr><td>Kinuta-simo</td><td>34</td></tr>
<tr><td rowspan="2">Other</td><td>Nagasawa</td><td>230</td></tr>
<tr><td>Suginami</td><td>4</td><td>Yamate</td></tr>
<tr><td>Total Supply</td><td></td><td>4450</td><td></td></tr>
<tr><td>Total Demand</td><td></td><td>4450</td><td></td></tr>
</table>

Input data shown in Table 1 are used as the Standard case of our computation. Thus, the other cases defined for further computation are based upon these Standard case data. Also capacity data u_{ij} in (1) are defined as follows.

Regarding the Standard case data, capacity for each edge segment (pipeline) in this case assumed to be twice of the actual capacity as they are planned. The cases conducted here are based on the assumptions that some of the pipes, whose diameters are more than or equal to 1,000 mm, are considered to be unbroken, since they are very strong. There are 56 unbroken edges in this network (24.9% of the total network). The distribution of these unbroken edges is: 35 edges in Yamate (62.5%), 15 edges in Joto (26.8%), and 6 edges in Tama region (10.7%).

Figure 3 shows the structure of the water supply network system in Tokyo [3]. The grey lines show the very strong pipes, which is considered to be unbroken, while the black lines show the usual pipes, which are considered to be equally broken in random.

The numbers of edges and nodes used in this network are shown in Table 2 (including all of originally zero capacity edges).

From Table 2, we find that most nodes and edges are located in Yamate, followed by Joto, and Tama. Besides that, there are also some edges which are connected between regions, such as between Joto and Yamate, and Yamate to Tama.

Based upon the above Standard case data, we define 3 different cases for further numerical experiments. The following cases are defined to be based upon the emergent cases, such that each of the intake sources: Tone, Kanamachi, Kanamachi and Asaka is damaged. Thus for each case, some of other purification plants increase their intake. Detailed data for the capacity of each purification plant are given in the Table 3.

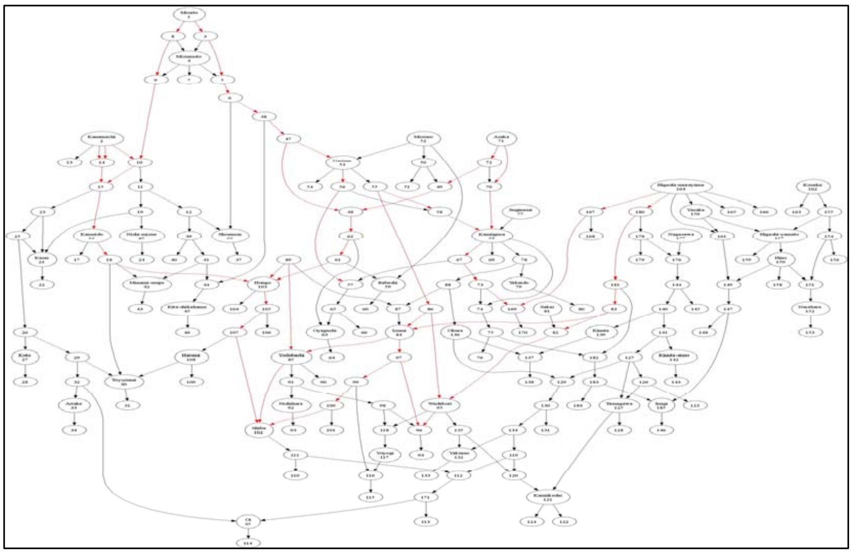

Figure 3: Water Supply Network System in Tokyo

Table 2: Distribution of Edge and Node by Region

Region	**Nodes**	**%Nodes**	**Edges**	**%Edges**
Joto	47	26.0%	57	25.3%
Yamate	101	55.8%	122	54.2%
Tama	33	18.2%	38	16.9%
Joto-Yamate	-	-	4	1.8%
Yamate-Tama	-	-	4	1.8%
Total	**181**	**100.0%**	**225**	**100.0%**

The first, case 1 [4], which corresponds to the case that the intake Tone is damaged, indicates that purification plants in the Yamate region cannot obtain water from Tone River, while other plants can take it as planned. Higashimurayama plant in the Tama region delivers water to the Asaka plant in the Yamate region, and then Asaka plant supplies it.

The second, case 2, which corresponds to the case that the intake Kanamachi is damaged, indicates that Kanamachi purification plant cannot obtain water from Tone-Ara River, while other plants can take it as planned.

The third, case 3, which corresponds to the case that the intake Kanamachi and Asaka are damaged, indicates that Kanamachi and Asaka purification plants cannot obtain water from Tone-Ara River, while other plants can take it as planned.

In the following section, we show our numerical results including Standard and other cases (Case 1 to Case 3).

Table 3. Water Supply (unit: $10^3 m^3$/day) for Three Cases

River	Purification Plants	Case 1	Case 2	Case 3	Region
Tone-Ara River	Kanamachi	1500	0	0	Joto
	Misato	1100	900	900	
	Asaka	0	950	0	Yamate
	Misono	280	280	280	
Tama River	Higashimurayama	1265	843	843	
	Ozaku	280	150	150	Tama
	Sakai	300	120	120	
	Kinuta	114	0	0	
	Kinuta-simo	70	34	34	
Other	Nagasawa	230	230	230	
	Suginami	4	4	4	Yamate
Total Supply		**5143**	**3511**	**2561**	
Total Demand		**4260**	**3511**	**2561**	

Note: *) *Based on actual data taken on 1 September 2004.*
**) *The grey figures in the table are the changed condition, compare to Standard case.*

3 Numerical Results for Edge Deletion Simulations

3.1 Standard Case Results

Firstly, we describe the results for the Standard case. Standard case here is analyzed using the maximum flow obtained under the condition that all of the water resources provided the water supply for Tokyo area as it planned.

The results obtained for the statistics after 10,000 iterations and 0-15% simulation of edge broken ratio (EBR) are shown in Figure 4. The coverage rate here is defined as CR in formula (4), but the x axis in the figures here represent the EBR, instead of k. EBR here is defined as follows:

$$\text{EBR} = \text{k} / \text{Total number of edges in the network} \quad (7)$$

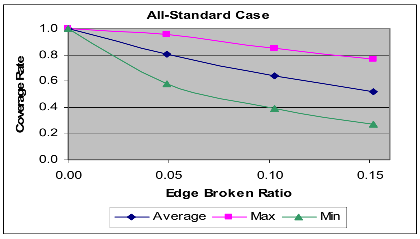

Figure 4: Statistics of Standard Case Results

The average line shows that CR decreased by 19.3% when EBR is 5%, then by 35.8% at 10% of EBR, and by 48.2% when EBR is 15%, respectively.

The maximum line shows that CR decreased by 4.8% when EBR is 5%, then by 14.9% at 10% of EBR, and by 23.6% when EBR is 15%, respectively, while the minimum line shows that CR decreased by 41.9% when EBR is 5%, then by 61.2% at 10% of EBR, and by 72.7% when EBR is 15%, respectively. These two lines show the best and the worst scenarios. The best scenario is represented by the maximum line, and the worst is represented by the minimum line. Therefore, the maximum line shows the highest line in the figure, while minimum line located in the lowest part.

The worst case scenario happens when many edges connected to source or sink nodes are deleted, which results in disabling the flow to get into the network nor reach sink nodes. In this minimum case, if EBR increased from 0 to 5%, the worst case happens when there are 11 broken pipes occurred. Broken pipes here are defined as the originally positive capacity edges which becomes zero capacity edges. In other words, the originally zero capacity edges are not included in the EBR calculation.

Furthermore, the CR between maximum and average line shows a big difference, which varies between 14.4 to 24.6% for EBR 0-15%. This big difference also occurs between the minimum and average lines, which are 22.6%, 25.3%, and 24.5% for EBR of 0-15% respectively.

For all statistics, except maximum, the graphs show that the higher EBR, the narrower the gaps are. In other words, for these statistics, the graphs are convex to the origin.

3.2 Simulation Results for Edge Deletion Cases

We show the results for all four cases in Figure 5-7. "All" here represents the condition when there is no node deleted, since we also conducted the node deletion simulations, which result is summarized in the last section.

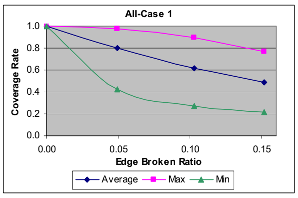

Figure 5. All-Case 1

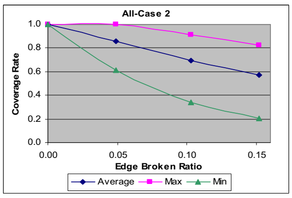

Figure 6. All-Case 2

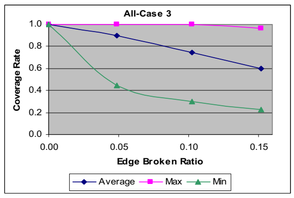

Figure 7. All-Case 3

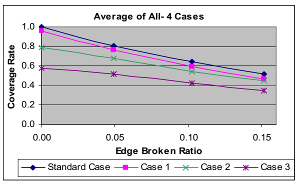

Figure 8. Average: All-4 Cases

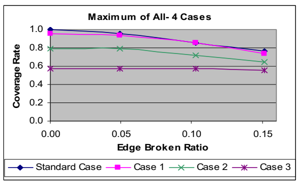

Figure 9. Maximum: All-4 Cases

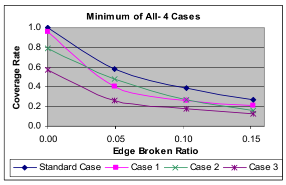

Figure 10. Minimum: All-4 Cases

In Case 2 and 3, the CR at EBR 0% is very much lower than the Standard case. This can be explained as follows. As we see from Table 2, these 2 cases have lower supply than the Standard case, because Kanamachi and Asaka are the purification plants with the biggest water supply among others. Therefore, Case 2 shows that the damaged of Kanamachi itself, without the replacement of supply from other purification plants, have resulted in decreasing the CR by more than 20 percent.

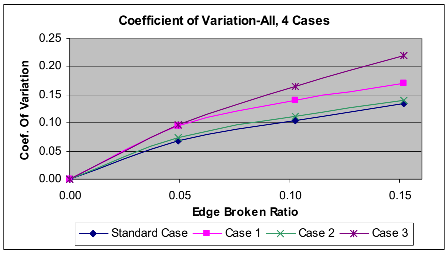

Figure 11. Coefficient of Variation

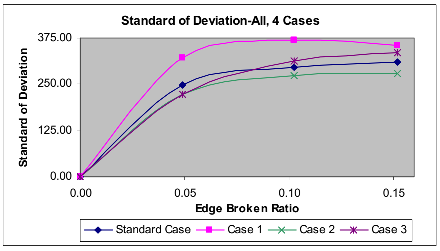

Figure 12.Standard of Deviation

As for Case 3, both of Kanamachi and Asaka are damaged, which resulted in the CR decreased by more than 40 percent at EBR of 0%. In other words, if other purification plants supplied as Standard case, the damage caused by Kanamachi or Asaka will result in a decrease in the CR by around 20% each, compare to Standard case. Moreover, the trends for maximum lines are similar, while the minimum lines have a different trend. This is caused by the number of broken pipes, which is different for the minimum line in each case.

Compared to Standard case, Case 1 have lower CR, despite having bigger supply than the Standard case. This happens because some broken edges are able to block the water supply system, which lead to some bottle necks in several areas, and results in less amount of demanded water than the supply from the system.

At 0% of EBR, the difference between Case 1 and Standard case for all statistics is 4.3%. This means the CR obtained from the system under Case 1 assumptions is 4.3% lower from Standard case. Meanwhile, the differences of cases 2, and 3 from Standard case are 21.1% and 42.4%, respectively. This differences is caused by the assumption used for each cases, where some of the water resources/purification plants are damaged (Table 3). In other words, when the intake Tone of is damaged, the CR is decreased by 4.3%, while the damage of Kanamachi purification plant, and both of of Kanamachi and Asaka purification plants, has decreased the CR by 21.1% and 42.4%, respectively.

In terms of average and minimum, Standard case has the highest CR values, while Case 3 has the lowest. The difference between Case 1 and Standard case in terms of average is between 4.3-5.1% for EBR 5-15. The difference between Case 2 and Standard case in terms of average is between 13.4-6.9% for EBR 5-15%. The difference between Case 3 and Standard case in terms of average is between 29.2-17.4% for EBR 5-15%.

The maximum lines show that Case 1 and Standard case differ by 1.9-2.8% for EBR 5-15%, while Case 2, Case 3, and Standard case differs by 16.3-11.7% and 37.6-21.1%, respectively. In other words, the CR of Case 3 is the lowest, while Standard case has the highest CR values.

The worst scenario shown in the minimum lines (Figure 10), with Case 3 as the worst one. The difference between Case 1 and Standard case in terms of minimum is between 17.5-6.3% for EBR 5-15%, while Case 2, Case 3 and Standard case differs by 10.0-11.3%, and 32.4-14.4%, respectively.

Figure 11 shows the coefficient of variation (CV) for each node deletion, which is calculated as follows:

$$\text{Coefficient of Variation} = \text{Standard Deviation} / \text{Mean} \quad (8)$$

The high value of CV shows the irregularity among the samples within the group [5]. The highest CV is shown in Case 3, while the lowest one is shown in Standard case. Case 1 shows that it's CV increased by 9.5% when EBR increased from 0 to 5%, then 14.1% at EBR of 10%, and 17.1% at EBR of 15%, respectively. Case 2 shows that it's CV increased by 7.4% when EBR increased from 0 to 5%, then 11.2% at EBR of 10%, and 14.0% at EBR of 15%, respectively. Case 3 shows that it's CV increased by 9.7% when EBR increased from 0 to 5%, then 16.5% at EBR of 10%, and 21.9% at EBR of 15%, respectively. Meanwhile, Standard case shows that it's CV increased by 6.9% when EBR increased from 0 to 5%, then 10.3% at EBR of 10%, and 13.5% at EBR of 15%, respectively. Therefore, we can conclude that Case 3 is the most dispersed to the mean value, while Standard case is the most concentrated to the mean value.

This can be explained as follows. Standard case has the smallest standard deviation (see Figure 12) and the largest mean value. Since the numerator is much smaller than the denominator, as a result, the CV of Standard case is also the lowest. On the other hand, Case 3 has the largest standard deviation and the smallest mean. Therefore, the numerator is larger while the denominator is smaller is resulting in the highest CV. The numerator and denominator in Case 2 are slightly lower than the Standard case, therefore the CV is similar with Standard case.

4 Summary and Conclusion

In order to investigate the impact of damage of water resources, broken pipes, and supply stations to the water supply system in Tokyo, the simulation of edges deletion represented by edge broken ratio (EBR) of 0%-15%, and nodes deletion under four cases: 1) Standard case; 2) Tone damaged; 3) Kanamachi down; and 4) Kanamachi and Asaka down.

The results show that from all cases, the coverage rate (CR) under Case 3 is the lowest, since Kanamachi and Asaka are the biggest purification plants which depend on Tone River, despite the assumption that in Case 3 the total flow of Tone River is not decreasing into zero. However, in the real situation, Tokyo is very much depends on it. Therefore, in case Tone water resources are damaged, the water supply system will be seriously damaged, and will not be able to provide enough water for Tokyo area. As a result, it is strongly suggested to distribute the water resources more proportionally, to prevent the high dependency on one water resource/purification plant.

Besides of the edge deletion simulations, we have also conducted the node deletion simulations, by setting one of the intermediate supply station's (Nerima, Kamiigusa, and Hongo) capacity to be zero at a time. The Standard case results show that Nerima (for 0-5% edge broken ratio) and Hongo (for 10-15% edge broken ratio) are the most important supply stations, while Kamiigusa is the least important one. Moreover, the difference between total CR obtained from deleting the supply station shows that the damage of Nerima and Kamiigusa (which has the largest installed capacity among the 3 supply stations) is giving a big impact to the total flow of water supply system in Tokyo.

Despite of that, the impact of deleting the edges for all of the simulations is very small, as shown by the convexity of average and minimum, and the concavity of maximum. In other words, the damage of water resources has resulted in the biggest impact to Tokyo water supply system, followed by intermediate supply stations, and broken pipes.

For the future research, we will analyze the robustness and reliability of water supply network system in the emergency case, which is defined as the condition after a disastrous earthquake.

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