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A case study of optimal ambulance location problems

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1 Introduction

Aiming at improving the efficiency and reliability of ambulance service, several location models for ambulance stations have been proposed in the operations research literature. Well-known approaches to this problem are coverage model and median model. Coverage model looks for the location to maximize the (deterministic or probabilistic) covered demand of ambulance calls. Hence this model can be thought of reliability oriented model. On the other hand in median model the objective is to minimize the total traveling distance of the ambulances from the station to the scene of call. This model gives more weight to the efficiency of ambulance operation. This paper gives a comparison of those optimization models through actual patient call data from Tokyo metropolitan area to show the characteristics of each model and investigate a possibility of improvement in ambulance service.

In the next section we introduce two well-known approaches to ambulance location problem, coverage model and median model. In Sec. 3 we apply these models to patient call data from Tokyo and compare the actual location and optimal solution. The result shows us the optimal solution can achieve improvement in both models, and suggests the possibility of more efficient and reliable location.

2 Ambulance location models

In this section we briefly review several optimization models for ambulance location problem following [1, 2, 6]. The most distinctive feature of the problem is its stochastic property of demand. To deal with stochastic property of the problem several models are introduced as follows. First we introduce several notations commonly used in the following.

M: set of potential ambulance station, N: set of demand point,
d_{ij}: distance between $i \in M$ and $j \in N$, p_j: demand at $j \in N$,

The **location set covering model** (**LSCM**)[8], which is an early and simple example of ambulance location problem, is formulated as follows. The model aims minimizing the number of ambulances with keeping the coverage of all the demand points. It introduces binary decision variables z_i, $i \in M$, which is equal to 1 if an ambulance is located at i and 0 otherwise. Also introduced the set of potential station covering the demand point j is $M_j = \{i \in M : d_{ij} \le D\}$, where D is the distance to be determined as a coverage standard. Then mathematical formulation of LSCM is given below.

$$\text{(LSCM)} \quad \text{min.} \quad \sum_{i \in M} z_i \tag{1}$$

$$\text{s.t.} \quad \sum_{i \in M_j} z_i \ge 1, \quad j \in N \tag{2}$$

$$z_i \in \{0,1\}, \quad i \in M \tag{3}$$

The constraints (2) mean that every demand point must be covered by at least one ambulance station.

The **maximal covering location problem** (**MCLP**) [3] is another covering type model. This model alters the objective into maximizing covered demand with fixing the number of locating ambulances to K. Newly introduced binary decision variables y_j, $j \in N$, each of which is equal to 1 if the demand at j is covered and 0 otherwise,

$$\text{(MCLP)} \quad \text{max.} \quad \sum_{j \in N} p_j y_j \tag{4}$$

$$\text{s.t.} \quad \sum_{i \in M_j} z_i \ge y_j, \quad j \in N \tag{5}$$

$$\sum_{i \in M} z_i = K \tag{6}$$

$$z_i, y_j \in \{0,1\}, \quad i \in M, j \in N \tag{7}$$

The constraints (5) mean that a demand point j is covered if and only if at least one ambulance is located in M_j. The objective (4) is the number of demand which are within the coverage standard.

The **maximum expected covering location problem** (**MEXCLP**)[4] brings the idea of queueing theory into ambulance location problem. Suppose the probability that each individual ambulance at any given time is busy is equal to a constant q, regardless of the ambulance position and time. Then, if the demand point j is covered by k ambulances, the expectation of covered demand at j is found to be $p_j(1-q^k)$ by simple calculation. A group of binary decision variable y_{jk} for the demand point j is introduced instead of y_j in MCLP, which is equal to 1 if the point j is covered by ambulances of more than or equal to k. Suppose we locate K ambulances again, then the total expectation of covered demand at point j is

$$\sum_{k=1}^{K} p_j(1-q^k)(y_{jk} - y_{j,k+1}) = \sum_{k=1}^{K} p_j(1-q)q^{k-1} y_{jk}$$

with convention $y_{j,K+1} = 0$. The formulation is given as follows.

$$\text{(MEXCLP)} \quad \text{max.} \quad \sum_{j \in N} \sum_{k=1}^{K} p_j (1-q) q^{k-1} y_{jk} \tag{8}$$

$$\text{s.t.} \quad \sum_{i \in M_j} z_i \geq \sum_{k=1}^{K} y_{jk}, \quad j \in N \tag{9}$$

$$\sum_{i \in M} z_i = K \tag{10}$$

$$z_i, y_{jk} \in \{0,1\}, \quad i \in M, j \in N, k = 1, \ldots, K \tag{11}$$

The **maximum availability location problem** (**MALP**)[7] is another approach to ambulance location problem using queueing theory. Using notation in MEXCLP each demand point j is covered by $\sum_{i \in M_j} z_i$ ambulances, then the probability that all these ambulances are busy is $1 - q^{\sum_{i \in M_j} z_i}$. We require this probability is above a reliability level α:

$$1 - q^{\sum_{i \in M_j} z_i} \geq \alpha, \quad j \in N \tag{12}$$

which is equivalent to

$$\sum_{i \in M_j} z_i \geq \lceil \log(1-\alpha) / \log q \rceil, \quad j \in N. \tag{13}$$

Set $b = \lceil \log(1-\alpha)/\log q \rceil$. The decision variables are same as MCLP, and the objective of MALP is also the covered demand.

$$\text{(MALP)} \quad \text{max.} \quad \sum_{j \in N} p_j y_j \tag{14}$$

$$\text{s.t.} \quad \sum_{i \in M_j} z_i \geq b y_j \quad j \in N. \tag{15}$$

$$\sum_{i \in M} z_i = K \tag{16}$$

$$z_i, y_j \in \{0,1\}, \quad i \in M, j \in N \tag{17}$$

A **median model** (**MM**) has a different objective than covering models. It aims to minimize the total traveling distance of ambulances. New decision variable x_{ij} designates the number of demands at the point j which is served the ambulance at station i.

$$\text{(MM)} \quad \text{min.} \quad \sum_{i \in M} \sum_{j \in N} d_{ij} x_{ij} \tag{18}$$

$$\text{s.t.} \quad \sum_{i \in M_j} x_{ij} \geq p_j \quad j \in N \tag{19}$$

$$\sum_{j \in N} x_{ij} \leq C z_i \quad i \in M \tag{20}$$

$$\sum_{i \in M} z_i = K \tag{21}$$

$$x_{ij} \geq 0, \quad i \in M, j \in N \tag{22}$$

$$z_i \in \{0,1\}, \quad i \in M \tag{23}$$

In the above model, C is the capacity of an ambulance, which is the maximum number of demands served by one ambulance for the prescribed time interval.

3 Numerical studies

In this section we apply those models described in the previous section to Tokyo metropolis data to analyze its ambulance system. Our analysis uses the data of weekday in Tokyo special ward area. This is because the number of ambulance dispatches on Sunday or holiday is smaller than weekday and it is more interesting for us to investigate the system in the busy period. Table. 1 lists some aspects of data used in our analysis. In optimization models we assume the demand point $i \in N$ is the center of town block, and ambulance can be located to any town block, i.e. $N = M$.

Table 1: Data for analysis: Ambulance dispatch of Tokyo special ward area in 2002 (weekday only, number of days is 300).

#(dispatch)	#(ambulance) K	#(town block) $\|N\| = \|M\|$
397242	145	3115

3.1 Analysis by LSCM

First analysis is done by using LSCM to find the minimum number of ambulances to cover the area with several coverage standards D. The observed ambulance vehicle speed in the Tokyo metropolis is approximately 350 m/min. To achieve five-minute arrival to the scene (this is desirable response time for the rescue of serious patient) D should be 1750m for Euclidean distance measure, or 1250m for Manhattan distance. We add D = 1500m as an intermediate case to study three cases in total by LSCM. The optimal solution is not found within 2 hours run of program. Table 2 shows the found solutions, from which we know that 85 ambulances are needed in the case the coverage standard $D = 1750$, while the actual number of ambulances is 145. When $D = 1500$ and $D = 1250$, necessary number of ambulances are 132 and 153, respectively. This result encourages us to try to find more reliable and efficient location for ambulances.

Table 2: Necessary ambulances by LSCM for different coverage standards D in Tokyo. Solutions are not optimal but found after 2 hrs. run

Coverage standard (D)/m	1250	1500	1750
minimum number of ambulances	153	132	85

3.2 Analysis by coverage models

Coverage models are applied to Tokyo data for the sake of measuring the covered demand in the optimal location and the current location. We first solve each coverage model and find the optimal solution for three coverage standards D. Then we compute the same problem with fixing z_i to actual ambulance position, i.e. set $z_i = 1$ if i is the actual ambulance station, and $z_i = 0$ otherwise. Since MALP finds the maximum covered demand under the constraints of reliability level α, which is the parameter for MALP, we set $\alpha = 0.999$ in the computation.

The result of three models is shown in each row of Table 3, where first line shows the optimal objective function value and second line shows the objective function value when the location of ambulances is fixed to actual positions.

Table 3: Objective values of optimal location and actual location in three coverage models for Tokyo.

Model		Covered demand		
		$D = 1250$	$D = 1500$	$D = 1750$
MCLP	optimal	396142	397242	397242
	actual	332531	369049	387708
MEXCLP	optimal	394788	397233	397241
	actual	332514	369036	387725
MALP	optimal	395378	397242	397242
	actual	332531	369049	387733

3.3 Analysis by median model

We apply MM to Tokyo data to find optimal location in terms of the total traveling distance. We compare the average traveling distance computed from the objective function value of MM to the actual one which is calculated based on the data in 2002 in Table. 4 Since our computed optimal average traveling distance is based on Euclidean distance, while the actual one is measured on the real road network, a straightforward comparison of them may not be permitted. However, this result implies the possibility of improving the access traveling distance to the scene by modifying the location of ambulance stations.

Table 4: Average traveling distance computed from objective value of optimal location in MM for Tokyo. Capacity C is set to 3000. Last column shows actual average traveling distance from the data in 2002.

	optimal location	actual location
Average traveling distance (m)	667	2100

4 Concluding remarks

We applied several facility location models to the ambulance system in Tokyo. The optimization result of all the models indicates the possibility of improvement of demand coverage or average access time by choosing optimal location. Since all of our analyses are "static" analyses, it is necessary to make stochastic simulation studies, like a hypercube model, to the obtained optimal location in order to know more detailed features of the system as mentioned in [5]. Those studies should be a future work for us.

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