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Vehicle Routing for Medical Supplies in Large-Scale Emergencies*

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Abstract A typical Vehicle Routing Problem (VRP) is to design the least cost routes for a vehicle fleet to supply goods from inventory to demanding customer locations. In this paper, we are interested in routing vehicles to minimize unmet demand and time delays. An important application of the presented model is to distribute medical supplies to response to large-scale emergencies, such as natural disasters, decease outbreaks, or acts of terrorism in which the supplies must be sent to cover all demands in the recommended response time. In this situation, transportation cost is the least important because it is unmet demand and/or time delay in an emergency situation that result in loss of life.

We formulate a new model that describes the vehicle routing problem for large-scale emergency scenario. A quick heuristic algorithm is designed to obtain a fleet dispatching plan. This algorithm can be very useful for emergency responder to best use the available vehicles in case of emergencies.

Keywords Vehicle Routing Problem; Large-Scale Emergency; Heuristics

1 Introduction

Large-scale emergencies are of high-consequence, low-consequence (HCLP) events, such as substantial acts of nature, large human-caused accidents, and major terrorist attacks like September 11^{th}, 2001 that may result in loss of life and severe property damage. In recent years, developing decision-oriented operations research models to improve preparation for and response to major emergencies has been drown more and more attention[Larson, et al, 2006, Altay, et al, 2005].

Due to the scarce resources and overwhelming demands occurs during an emergency, careful pre-planning and efficient execution in responding to a large-scale emergency can save many lives. A key factor in an effective response to an emergency is the prompt availability of necessary supplies at emergency sites. Therefore

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the efficient emergency logistics becomes important in addressing and optimizing the complex distribution process.

Traditional Vehicle Routing Problem (VRP) is to design the least cost routes for a vehicle fleet to supply goods from inventory to demanding customer locations. The problem was fist introduced by Dantzig and Ramser (1959) to solve a real-world application concerning the delivery of gasoline to service stations. A comprehensive overview of the VRP can be found in Toth and Vigo (2002) and other general surveys on the deterministic VRP also can be found in Laporte (1992). Various specific VRP models, e.g. with time windows, multiple depots, dynamic routes, and stochastic customer demands, etc. were published in the literature(). Almost all the VRP models and algorithms are for "normal operation" that minimize cost represented by travel distances or travel times and applied in daily operating logistic systems.

However in the large-scale emergency circumstances, the highly unpredictable nature of large-scale emergencies leads to significant uncertainty both in demand and travel time. For example, in certain emergency cases, medication or antidotes must be applied within a specific time limit from the occurrence of the event to maximize their effectiveness to save lives. Requirement for the medication may change rapidly along with the case development and is hard to be predictable. Traditional pharmaceutical supply chains are no longer adequate to provide the rush demand. In this case the so called Strategic National Stockpile, a large managed inventory from manufacturers, may be used to in case of emergencies. Vehicle fleet size can be unsure according to emergency calls. The vehicles may load supplies from multiple depots (e.g. airports) and may not return to the original depot location. Travel times of transporting the medication from the central supply to the demand population areas also become uncertain in case of emergency because of sudden road congestion and panic, or because of strict traffic control. Finally the objectives of the VRP for response to emergency are to minimize both the unmet demand and delay time. Finally, an efficient algorithm to find a good solution is very important for emergency operation managers.

All these uncertain factors must be considered by an emergency operation manager in dispatching vehicles to effectively deliver the life-saving demands to the needed. Due to the characteristics of uncertainty of large-scale emergency, a dynamic VRP can be stated as follows:

1. when an emergency occurs, with reported demand calls, a responder must evaluate the demand pattern, including locations, quantity, and time requirement for the deliveries.
2. organize the supplies and routing the available vehicles to meet the emergency requirements in an efficient way to minimize the unmet demand and the total time delay.
3. with the updated demand information, relocate medical supplies and vehicles, route and dispatch next fleet with the same objective.
4. keep evaluating the updated demand and routing further vehicles, until all the demand is met.

This work focuses on modeling and solution framework for the VRP in response to a large-scale emergency. An efficient heuristic algorithm is designed for the proposed model. Numerical experiments and a case simulation demonstrate that the model and algorithm can be very useful as a decision tool for emergency responders.

2 A Deterministic VRP Model

In this paper, we consider a situation that a fleet of vehicles send emergency supplies from a single depot (e.g. airport or central inventory) to demanding locations (e.g. hospitals or triage stations), and return to the depot after deliver all the supplies. Objectives of the model is to minimize the unmet demand and the total time delay.

Shen et al (2006) studied a stochastic VRP model with time window that minimize unmet demand for large-scale emergencies. In their paper, vehicle time delay is not allowed when visit a demand node. However this strict limitation may be unreasonable because in emergency situations even the urgency of need for medical supplies may not be met from the time perspective, the dispatcher will still send the supplies to save as many as lives in a way of the least time delays.

In the model presented in this paper, the total time delays are explicitly expressed in the objective function as the most important factor in the solution method.

Decision Variables and Coefficients:

We consider a fleet set K of vehicles, a set D represents demand nodes. We denote node 0 as depot. The node set is expressed as$C = D \cup \{0\}$. Suppose from each node i to any another node j there is a route, or an arc (i, j). Therefore a transport network can be expressed by the node set $C = \{0\} \cup D$ and arc set $\{(0, j), (j, 0), (i, j), i, j \in D, i \neq j\}$. Define:

n: number of available vehicles;

s: Total available supply at depot;

$c_k \in Z^+$: the maximum load of vehicle k;

dl_i: the latest arrival time required by demand node i, or the expected deadline for node i;

τ_{ijk}: the estimated time to traverse arc (i, j) for vehicle k; it is set to ∞ for non-existent links;

ζ_i: amount of commodity needed at node i;

X_{ijk}: a binary flow variable, equal to 1 if (i, j) is traversed by vehicle k and 0 otherwise;

Y_{ik}: delivery by vehicle k to the demand node i, integer value is assumed;

U_i: amount of unsatisfied demand at node i;

T_{ik}: time that vehicle k arriving at node i, we neglect the unload time by consider it a zero or small constant; and

δ_{ik}: delay time happened when vehicle k sends supply to node i. If k arrives i later than dl_i, then $\delta_{ik} > 0$.

A deterministic VRP model for emergency medical supply is formulated as follows:

$$\text{Minimize} \quad z = \sum_{i\in D} U_i + \kappa \sum_{i\in D, k\in K} \delta_{ik} \tag{1}$$

subject to

$$\sum_{j\in D}\sum_{k\in K} X_{0jk} \leq n \tag{2}$$

$$\sum_{j\in C} X_{ijk} = \sum_{j\in C} X_{jik} \leq 1 \qquad (\forall i \in C, k \in K) \tag{3}$$

$$T_{0k} = 0 \qquad (\forall k \in K) \tag{4}$$

$$0 \leq (T_{ik} + \tau_{ijk} - T_{jk}) \leq (1 - X_{ijk})M \qquad (\forall i \in C, j \in D, k \in K) \tag{5}$$

$$0 \leq T_{ik} - \delta_{ik} \leq dl_i \sum_{j\in C} X_{ijk} \qquad (\forall i \in D, k \in K) \tag{6}$$

$$\delta_{ik} \leq \sum_{j\in C} X_{ijk} M \qquad (\forall i \in D, k \in K) \tag{7}$$

$$s - \sum_{k\in K}\sum_{i\in D} Y_{ik} \geq 0 \tag{8}$$

$$c_k \geq \sum_{i\in D} Y_{ik} \qquad (\forall k \in K) \tag{9}$$

$$Y_{ik} \leq c_k \sum_{j\in D} X_{ijk} \quad i \in D, k \in K \tag{10}$$

$$\sum_{k\in K} Y_{ik} + U_i - \zeta_i \geq 0 \qquad (\forall i \in D) \tag{11}$$

$$X_{ijk} \geq 0, Y_{ik} \geq 0, \text{binary}; U_i \geq 0, \delta_{ik} \geq 0 \tag{12}$$

The objective of the model is to minimize the total unsatisfied demands over all demand points and the total time delay of the vehicles arriving demand nodes. Constraints (2) specifies that the number of vehicles to service must not exceed the available fleet size. Constraint (3) is a equilibrium constraint that each vehicle visits one demand point only once at most and the vehicle must leave out from the demand node without staying there. This feasible route constraint allow split delivery. Constraints (4)-(7) are time-window constraints that guarantee schedule feasibility with respect to time considerations. Once a vehicle arrives a demand point later than the required deadline, a penalty $\delta_{ik} > 0$ appear. (8)-(11) state the construction on the commodity flows, while constraint (12) specifies the binary and integer variables.

3 A global search solution method

The formulation has two objectives: unmet demand and total time delay. We firstly look at $\min\{\sum_{i\in D} U_i\}$。

Define a supply capability:

$$M = \min\{s, \sum_{k\in K} c_k\}$$

we have

$$\sum_{i\in D} U_i = (\sum_{i\in D} \zeta_i - \sum_{i\in D,k\in K} Y_{ik}) \geq \max(\sum_{i\in D} \zeta_i - M, 0)$$

The above equation gives a lower bound to the unmet demand objective, which can be used as a criterion for the optimal solution.

We have three situations:

1. When $\sum_{i\in D} \zeta_i - M > 0$, this represents there is not a sufficient capacity to deliver all the commodity to the demanding nodes, and reaches its minimum when equation holds.
2. When $\sum_{i\in D} \zeta_i - M = 0$, there is a balanced capacity for supply and demand. The optimal $\sum_{i\in D} U_i = 0$. In the case of 1 or 2, all the vehicles return the depot with empty load.
3. When $\sum_{i\in D} \zeta_i - M < 0$, there is still a surplus capacity. We can send out part of the fleet，or send all the vehicles with partial load ($< c_k$)。

Now for the second objective $\sum_{i\in D,k\in K} \delta_{ik}$, we can apply a greedy heuristic to prioritize the most urgent demanding points to minimize the delay of the tight deadline values. In this way we make the objective as low as possible.

Before presenting our algorithm, we assume that $|K| < |D|$, i.e. the number of vehicles is less than the number of demand nodes, otherwise the problem will become easier. We also assume that the routes distance (or travel time) follow the triangle inequality, i.e. the direct distance or travel time between any two points is less than that through the third point.

Let a set $P = \{p_k\}$ specify the current position $(i \in D)$of vehicle $lk \in K$. Before the search , let $U_i = \zeta_i, i \in D$ and let $p_k = 0, k = 1, 2, \ldots, K$ to specify all the vehicles depart from the depot. $T_{0k} = 0, OPT = \max(\sum_{i\in D} \zeta_i - M, 0)$.

Search Algorithm

Step 1: Find

$$q \in \{i | dl_i = \min_{j\in D, U_j>0} dl_j\},$$

a node with the earliest deadline with unmet demand. Compute

$$p \in \{j | T_{jk} + \tau_{jqk} = \min_{p_k\in P, p_k\neq q, c_k>0} T_{p_k k} + \tau_{p_k q k}\}$$

Suppose the corresponding vehicle is k, then we have found the vehicle k located at node p_k, which is closest to node q and can service the point.

Update

$$\begin{aligned} X_{pqk} &:= 1 \\ p_k &:= q \\ Y_{p_k k} &:= \min\{U_{p_k}, c_k\} \\ c_k &:= c_k - Y_{p_k k} \\ U_{p_k} &:= U_{p_k} - Y_{p_k k} \end{aligned}$$

Step 2: Check if $\sum_{i \in D} U_i = OPT$, then $X_{p_k 0k} = 1, Y_{0k} = c_k, \forall k \in K$, stop; otherwise back to Step 1.

4 Effectiveness of the algorithm

According to the stop criterion, $\sum_{i \in D} U_i = OPT$, the numet demand is minimal, because otherwise based on the search condition of the step 1 there must be a pair of p, q such that $U_q > 0, c_k > 0$. Let $X_{p_k qk} = 1, Y_{qk} = \min(U_q, c_k)$ so that $\sum_{i \in D} U_i$ can be further smaller.

Next we prove that the algorithm will not recycle. After a vehicle k traverse node i, there can only be two cases:

1. $U_i \geq c_k$. In this case, the vehicle k delivers all its load to the node i. According to the step 1 for p, the vehicle k can only travel back to depot but not any other demanding node. Therefore it will not be possible to go back to i.
2. $U_i < c_k$. In this case, after k delivers U_i to i, $U_i = 0$ at the next iteration for i. Based on the equation for q at the step 1, the node will not require any demand as a destination. Therefore vehicle k will not return to i.

Finally since the algorithm will always prioritize the most urgent demanding point, the routing policy will make the total delay as little as possible. This follows the general of large-scale emergency response principle.

5 Case Simulation

Based on the above model and algorithm, we simulate an emergency situation when a pandemic disease (e.g. SARS) happens in Beijing, China and a certain quantity of medication need to be delivered from the airport to major downtown hospitals as soon as possible. The distance between the airport to the hospitals and between the hospitals are as the following table.

Suppose we have a fleet of 8 identical trucks to do the delivery. Other data for the simulation are as follows:

Number of trucks: 8. Total supply of the medication of 200 units medication is to be sent to the 16 hospitals with demand (23, 19, 20, 21, 17, 6, 7, 16, 14, 2, 3, 7, 11, 12, 23, 15), totaled 216 units. The required deadline of the deliveries for these hospitals are generated randomly between 40 and 90 to indicate the urgency of the demand as (77, 62, 89, 44, 68, 55, 83, 57, 75, 43, 58, 65, 62, 69, 71, 46). This simulation result is as figure 1. The solution gives 0 unmet demand and 16 total time delay. With different coefficients data input we have simulated cases of severe supply shortage, tight deadline, large fleet, and large number of randomly generated demand nodes. All these results show that the polynomial time algorithm is very efficient and can be a very useful tool in routing vehicles during a large-scale emergency scenario.

label	hospital
1	Peking Union Medical College Hospital (PUMCH)
2	China-Japan Friendship Hospital (CJFH)
3	Beijing Tongren Hospital (BTrH)
4	Beijing Ditan Hospital (BDH)
5	Beijing Chaoyang Hospital (BCH)
6	Beijing Obstetrics and Gynecology Hospital (BOGH)
7	Peking University Third Hospital (PUTH)
8	Peking University First Hospital (PUFH)
9	Peking University People Hospital (PUPH)
10	Beijing Ji Shui Tan Hospital (BJSYH)
11	Beijing Shijitan Hospital (BSH)
12	Beijing Tiantan Hospital (BTtH)
13	Beijing Friendship Hospital (BFH)
14	China Rehabilitation Research Center (CRRC)
15	Beijing Youan Hospital (BYH)
16	Beijing Hui Long Guan Hospital(BHLGH)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Airport
PUMCT	0	8	1.2	10.6	5	2	12.8	5.7	8.4	7.5	9.1	6.3	5.4	10.4	10.1	21.94	26.36
CJFH	8	0	8.8	4.4	7.8	8.2	7.7	8.8	9.8	8	16.6	13.8	13	21.6	18.4	16.63	22.67
BTrH	1.2	8.8	0	12	5.8	3	13.8	6.6	9.5	8.4	9.4	5	4.4	9	9.5	24.26	27.51
BDH	10.6	4.4	12	0	7.1	4.4	7.8	5.2	6.2	4.3	13	12.5	11.2	20	14.9	17.15	23.91
BCH	5	7.8	5.8	7.1	0	5.1	14.4	7	9.8	8.8	13.8	11.2	10.4	15.3	16.1	19.32	23.73
BOGH	2	8.2	3	4.4	5.1	0	11	3.7	6.5	5.4	9.2	6	5.3	10.8	11.2	21.21	29.14
PUTH	12.8	7.7	13.8	7.8	14.4	11	0	7.5	5.8	6	12.6	17	14.3	17.8	16	11.83	27.72
PUFH	5.7	8.8	6.6	5.2	7	3.7	7.5	0	2.8	1.7	8.5	8	7.6	11.1	9.34	17.73	27.73
PUPH	8.4	9.8	9.5	6.2	9.8	6.5	5.8	2.8	0	3.4	6.8	11.6	9	13.6	8.77	15.66	28.94
BJSYH	7.5	8	8.4	4.3	8.8	5.4	6	1.7	3.4	0	9.4	8.2	8.8	16	9.94	16.49	26.81
BSH	9.1	16.6	9.4	13	13.8	9.2	12.6	8.5	6.8	9.4	0	12	8.7	13.1	7.2	24.65	35.79
BTtH	6.3	13.8	5	12.5	11.2	6	17	8	11.6	8.2	12	0	1.8	4.9	4.6	26.86	32.63
BFH	5.4	13	4.4	11.2	10.4	5.3	14.3	7.6	9	8.8	8.7	1.8	0	6.5	4.88	24	32.12
CRRC	10.4	21.6	9	20	15.3	10.8	17.8	11.1	13.6	16	13.1	4.9	6.5	0	4.8	27.58	37.24
BYH	10.1	18.4	9.5	14.9	16.1	11.2	16	9.34	8.77	9.94	7.2	4.6	4.88	4.8	0	24.56	37.29
BHLGH	21.94	16.63	24.26	17.15	19.32	21.21	11.83	17.73	15.66	16.49	24.65	26.86	24	27.58	24.56	0	32.34
Airport	26.36	22.67	27.51	23.91	23.73	29.14	27.72	27.73	28.94	26.81	35.79	32.63	32.12	37.24	37.29	32.34	0

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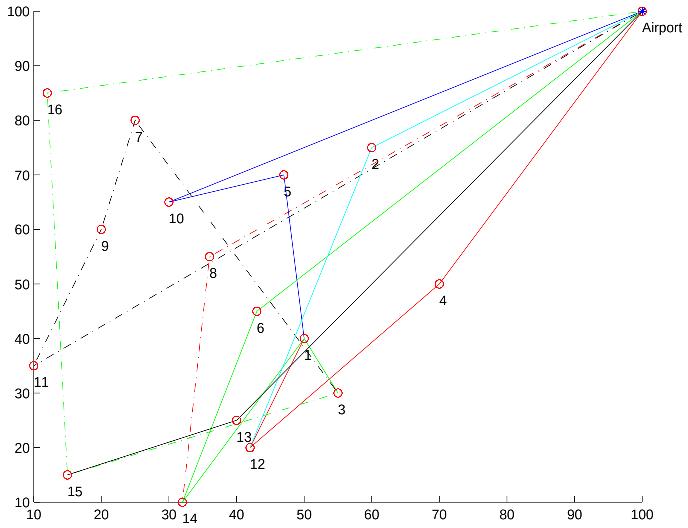

Figure 1: Simulation Results

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