The First International Symposium on Optimization and Systems Biology (OSB'07)
Beijing, China, August 8–10, 2007

Targeting Spatio–Temporal Patterns in Chaotic Neural Network

Manish Dev Shrimali[1,2,*] Guoguang He[1,2,3]
Kazuyuki Aihara[1,2]

[1] Aihara Complexity Modelling Project, ERATO, JST, Tokyo 153 8505, Japan
[2] Institute of Industrial Science, The University of Tokyo, Tokyo 153 8505, Japan
[3] Department of physics, College of Science, Zhejiang University, Hangzhou 310 027, China

Abstract We have studied the chaos control in chaotic neural network by limiting the phase space at varying time interval. It provides a controlled output patterns with different temporal periods depending upon the control parameters. The chaotic neural network constructed with chaotic neurons exhibits very rich dynamic behavior with a non–periodic associative memory. In the chaotic neural network, it is difficult to distinguish the stored patterns from others, because of the chaotic states of output of the network. In order to apply the non–periodic associative memory into information search and pattern identification, etc, it is necessary to control chaos in this chaotic neural network.

Keywords Controlling chaos; Chaotic neural network; Information processing.

1 Introduction

Chaos in neural network has attracted much interest in recent years because of its rich chaotic dynamics and potential applications in optimization and information processing, etc [1, 2, 3]. The chaotic neural network possesses the characters of larger memory content and good tolerance, etc as compared to the Hopefield neural network. The chaotic neural network has shown a non–periodic associative memory, but its associative memory is realized in the chaos dynamics of the network. The outputs of the network are non–periodic states which change continuously and can not be stabilized in one of its stored patterns. One therefore meets difficulties in the application of the associative memory in information processing. To achieve the information processing in the chaotic neural network, we should put the control on the network and let the network to be stable in an expected pattern.

In previous works, He *et al*. achieved control in chaotic neural network using the pinning [4] and phase space constraint methods [5]. In the phase space constraint method, the chaos in the CNN is controlled by limiting the phase space and the network converges in one of its stored patterns or their reverse with temporal period $p = 1$ which has the smallest Hamming distance with the initial state of the

*Corresponding author. Email: shrimali@aihara.jst.go.jp.

network. In this work, we have applied the phase space constraint method at varying time intervals to control the chaotic neural network. This mechanism provides a desired output patterns with different temporal periods, $p > 1$, as compared to the phase space constraint method applied at very time step having a controlled output pattern with temporal period $p = 1$. We investigate the range of spatio–temporal patterns of controlled chaotic neural network with temporal periods p for different initial patterns. In the CNN, the associative memory is represented by the chaotic wandering around all stored patterns [2], while in the controlled CNN, the associative memory dynamics has a periodic state with a desired period p and only the stored pattern related with an initial pattern and its reverse pattern appear in an output sequence. This characteristic shows that the application of the controlled CNN to information processing is feasible. The details of chaotic neural network is presented in next section followed by the control method, results and conclusions.

2 The Chaotic Neural Network Model

The chaotic neural network model is proposed by Aihara to study the chaotic response of biological neuron [1, 2]. A chaotic neural network is constructed with chaotic neurons by considering the spatio–temporal summation of both the external inputs and feedback inputs from other chaotic neurons [1]. In the chaotic neural network, the dynamics of ith chaotic neuron is described as follows:

$$x_i(t+1) = f[\eta_i(t+1) + \zeta_i(t+1)], \tag{1}$$

$$\eta_i(t+1) = k_f \eta_i(t) + \sum_{j=1}^{100} w_{ij} x_j(t), \tag{2}$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha g[x_i(t)] + a_i. \tag{3}$$

where $x_i(t)$ is the output of the neuron. $\eta_i(t)$ and $\zeta_i(t)$ are the internal state variables for feedback input from the constituent neurons in network and refractoriness at time t, respectively. $f(\cdot)$ and $g(\cdot)$ are the output function and the refractory function of the neuron, respectively. We take the output function of the neuron $f(x)$ as Sigmoid function with the steepness parameter ε, i.e., $f(x) = 1/[1 + exp(-x/\varepsilon)]$, refractoriness function as $g(x) \equiv x$. α is the refractory scaling parameter. a_i is the threshold of neuron. k_f and k_r are the decay parameters for the feedback inputs and the refractoriness, respectively. w_{ij} are synaptic weights to the ith constituent neuron from the jth constituent neuron, the weights are defined according to the following symmetric auto–associative matrix of n binary patterns:

$$w_{ij} = \frac{1}{n} \sum_{p=1}^{n} (2x_i^p - 1)(2x_j^p - 1), \tag{4}$$

where x_i^p is the ith component of the pth binary pattern. In this way, the binary patterns can be stored as basal memory patterns. We use a picture composed of 10×10 matrix to show the stored patterns of the neural network constructed with

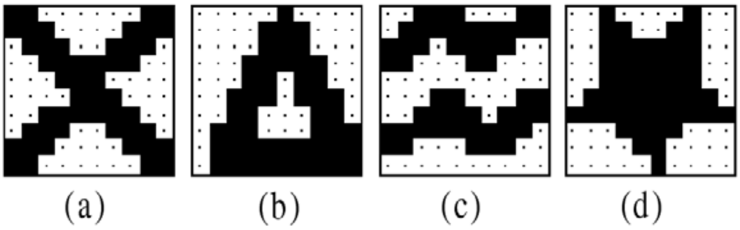

Figure 1: Four stored patterns

100 neurons. A neuron with its output x_i equal to 1, which means the neuron is "exciting", is represented by a block "■" while a neuron with its output x_i equal to 0, which means the neuron is "resting", is denoted by a dot "·".

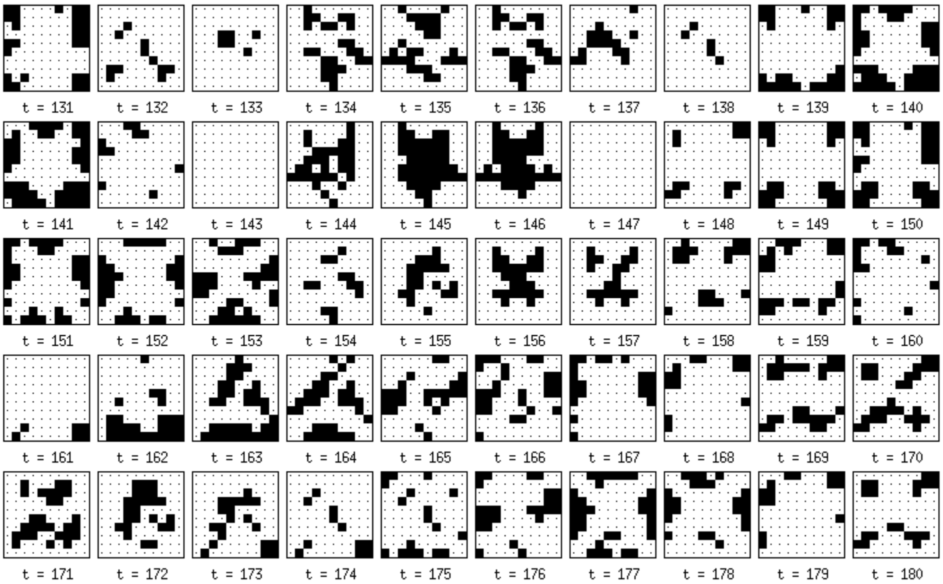

Figure 2: The sequence of the uncontrolled output patterns of the chaotic neural network with initial state of Fig. 2(a).

This chaotic neural network can recall acyclic pattern sequences by using auto–correlative associative memory [2]. When the decay parameters of the network are set to certain values, the network generates non–periodic sequential patterns including the stored ones as its output sequence. Four stored patterns studied in this paper are shown in Fig. 2. The sequence of the output patterns for the network is shown in Fig. 2 with a initial state of Fig. 2(a). The parameters of the network are taken as $\alpha =$

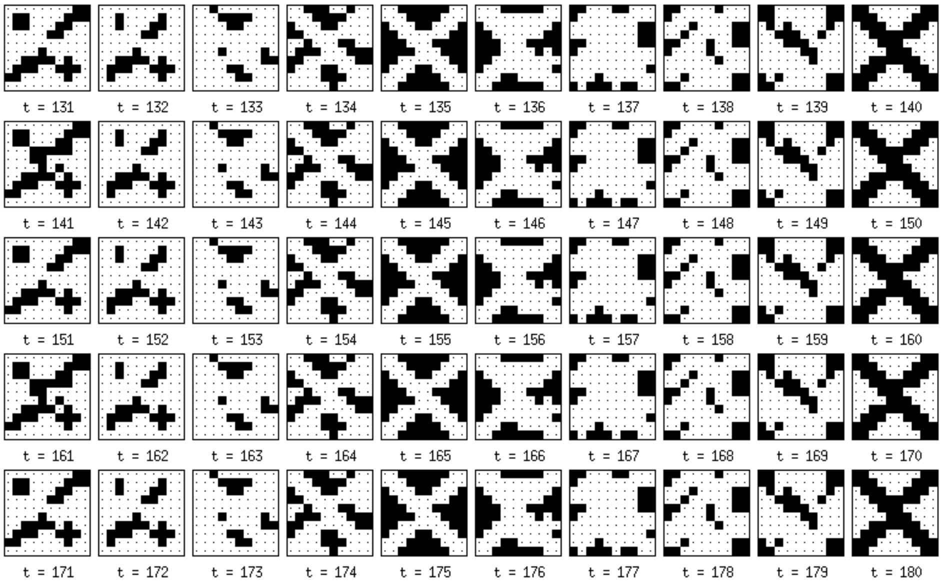

Figure 3: The sequence of the controlled output patterns of the chaotic neural network of initial pattern Fig. 2(a) with $\zeta^* = 5$ and $t_c = 5$.

$10.0, a_i = 2.0 (i = 1, 2, \ldots, 100), k_f = 0.20$ and $k_r = 0.95$. The output of the network exhibits the complex dynamic behavior in time and space [2]. Though the stored patterns are included in the outputs, the network can not be stabilized in one stored pattern or near it. It is therefore impossible to carry out the information processing in the network. In order to search the stored patterns involved in the network, one needs to control the chaos dynamics in the chaotic neural network.

3 Control Method

In 1990, a chaos control procedure is proposed by Ott *et al*. [7]. This method, known as the OGY method, consists on stabilizing a desired unstable periodic orbit embedded in a chaotic attractor by using only a tiny perturbation on an available control parameter. Another chaos control strategy was proposed by Pyragas [8]. In this case the method implementation requires a delay feedback signal. Several another control methods proposed are chaos synchronization [9], delayed self-controlling feedback [6], pinning control [4] and phase space compression[5], etc. There are two ways to make nonlinear systems converge on a stable or periodic state from a chaotic state: a feedback control and a non–feedback control.

In previous work by He *et al*. [5] the controlling aim was achieved by limiting existent space of the chaos. The dynamic structure of the chaotic neural network is changed by constraining the existent area of its states. By this method, the chaotic

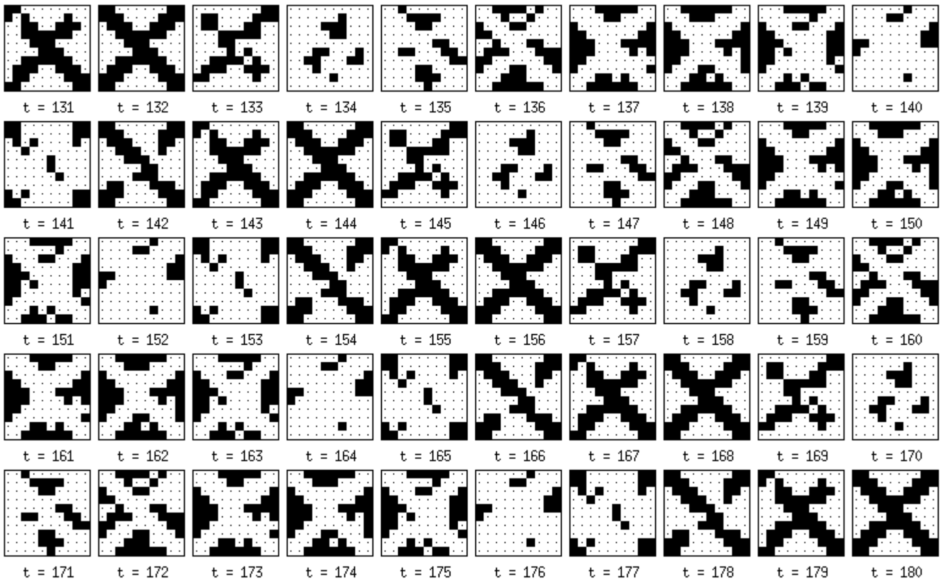

Figure 4: The sequence of the controlled output patterns of the chaotic neural network of initial pattern Fig. 2(a) with $\zeta^* = 5$ and $t_c = 6$.

motion can be controlled with a desired output pattern with temporal period $p = 1$. In the present study, we apply a phase space constraint method at varying time interval to control chaos in chaotic neural network. By varying the control time interval, we can target a controlled output pattern with desired temporal periods $p > 1$. This method provides a controlled chaotic neural network without any loss of network dynamics as compared to the fixed point solution of previous results.

For the chaotic system described by Eqs.$(1) \sim (3)$, the feedback input variable η and the refractoriness variable ζ constitute its phase space. One can apply the control on either the refractoriness or the feedback input variable. In the Ref. [5], chaos is controlled in the chaotic neural network by limiting the freedom phase space of the internal states of the network. That is, the thresholds ζ_{max} and ζ_{min} is set. If the $\zeta_i(t) > \zeta_{max}$ or $\zeta_i(t) < \zeta_{max}$, and let the $\zeta_i(t) = \zeta_{max}$ or $\zeta_i(t) = \zeta_{max}$, respectively. The excess values of the parameter ζ are removed from the particular neuron, which provide a desired controlled state of the neural network.

Here, the control is triggered when the absolute value of variable ζ of a particular neuron exceeds the critical value ζ^* at certain time $t_c = 1, 2, 3, 4, ...$, i.e. when $|\zeta_i(t_c)| > \zeta^*$. The Eqs. $(1) \sim (3)$ together with following equation (Eq. 5) constitute the dynamic model of the controlled chaotic neural network.

$$\zeta_i(t_c) \to \pm\zeta^* \tag{5}$$

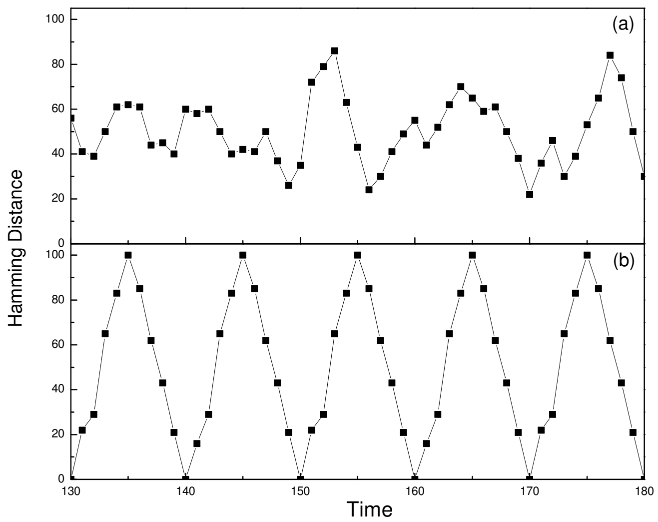

Figure 5: The Hamming distance between the (a) uncontrolled and (b) controlled output pattern of the network and the stored pattern of Fig. 2(a) for $\zeta^* = 5$ and $t_c = 5$.

The results with $t_c = 1$ corresponds to the previous work by He *et al*. [5]. Here we study the chaos control in chaotic neural network with $t_c > 1$ to get a controlled output pattern with temporal period $p > 1$. The controlled chaotic neural network can be stabilized in one of the stored patterns with temporal period p when ζ^* and t_c are suitably chosen. In the numerical simulation, we find that the controlled network converges on a stable state after a few steps. The chaotic neural network changes from non–periodic chaotic dynamics to a stable orbit with some periodicity p which depends on both the ζ^* and t_c. Here, we show the sequences of the outputs for the controlled network in Fig. 2 and Fig. 3 with $\zeta^* = 5$ and $t_c = 5,6$ for initial pattern of Fig. 2(a). The output pattern is controlled and the initial pattern is obtained with a periodicity $p = 20$ and 12 for $t_c = 5$ and 6 respectively. Similar results are found for other initial patterns and patterns with noise.

We investigate the relation with the initial state and the stable output pattern of the controlled network by calculating the Hamming distance. The Hamming distance is defined as following:

$$H^p = \sum_{i=1}^{100} |x_i - x_i^p|, \tag{6}$$

where x_i is a initial state, x_i^p is the ith component of the pth pattern. For the pth stored pattern, the Hamming distance will be 0 or 100 when initial state is the *p*th stored pattern or its reverse pattern exactly, respectively. In Fig. 3, we have shown the hamming distance between the initial pattern of Fig. 2(a) and the output pattern under control with control parameters $\zeta^* = 5$ and $t_c = 5$.

As compared to the previous phase space constraint method [5], where we get a desired output pattern with periodicity $p = 1$, the present method provides a another way of targeting a controlled output pattern with temporal period $p > 1$. Here we have applied a control by limiting the refractoriness ζ at varying time interval t_c to the neurons which provides a controlled output patterns from any initial pattern with temporal period p, where the period p depends on the control parameters ζ^* and t_c. This method has a advantage over the continuous time control method [5] as the controlled chaotic neural network maintains its internal dynamics.

4 Conclusions

We have extended the chaos control method proposed by He *et al*. to target a controlled output pattern. We have applied the chaos control method by limiting the refractoriness ζ of chaotic neural network at varying time interval t_c. In this method, the divergence or spreads of an orbit is limited by applying the control at varying time interval. The numerical simulation has proved that by employing the phase space constraint method at varying time interval the chaotic motion of the chaotic neural network can be controlled and the network converges on one of its stored patterns or their reverses with temporal period $p > 1$. The period p of the controlled network depends on the control parameters ζ^* and t_c. The continuous time control method provided a controlled output pattern with period one where the dynamics of the network is lost while the present method of controlling the chaos at varying time interval provides a controlled output pattern with higher periods without any loss of network dynamics. Such chaotic neural network under control with desired temporal periods of different patterns may have advantage over the controlled fixed temporal period one patterns in the applications such as pattern recognition or memory search etc. The controlled chaotic neural networks need to be explored further for their rich spatio–temporal behavioral output patterns for possible applications to different fields of computational, physical, and biological sciences.

References

[1] K. Aihara, T. Takebe, and M. Toyoda, “Chaotic neural networks,” *Phys. Lett. A*, vol. 144, pp. 333–340, Mar. 1990.

[2] M. Adachi, and K. Aihara, “Associative dynamics in a chaotic neural network," *Neural Networks*, vol. 10, pp. 83–98, 1997.

[3] J. Kuroiwa, S. Nara, and K. Aihara, “Response properties of a single chaotic neuron to stochastic inputs,” *Int. J Bifurcat. Chaos*, vol. 5, pp. 1447–1460, 2001.

[4] G. He, Z. Cao, P. Zhu, and H. Ogura, “Controlling chaos in a chaotic neural network,” *Neural Networks*, vol. 16, pp. 1195–1200, 2003.

[5] G. He, Z. Cao, H. Chen, and P. Zhu, "Controlling chaos in a chaotic neural network based on phase space constraint," *Int. J Mod. Phys. B*, vol. 17, pp. 4209–4214, 2003.

[6] G. He, J. Kuroiwa, H. Ogura, P. Zhu, Z. Cao, and P. Chen, "A type of delay feedback control of chaotic dynamics in a chaotic neural network," *IEICE trans. Fundamentals*, vol. E87–A, pp. 1765–1771, 2004.

[7] E. Ott, C. Grebogi, and J. A. Yorke, "Controlling chaos," *Phys. Rev. Lett.*, vol. 64, pp. 1196–1199, 1990.

[8] K. Pyragas, "Continuous control of chaos by self-controlling feedback," *Phys. Lett. A*, vol. 170, pp. 421–428, 1992.

[9] M. L. Pecora and T. L. Carroll, "Synchronization in chaotic systems," *Phys. Rev. Lett.*, vol. 64, pp. 821–824, 1990.