The Sixth International Symposium on Operations Research and Its Applications (ISORA'06)
Xinjiang, China, August 8–12, 2006

A Robust Collaborative Optimization Method Under Multidisciplinary Uncertainty*

Huihui Liu† Wenhui Fan

CIMS ERC Tsinghua University, Beijing 100084, China

Abstract Multidisciplinary design optimization (MDO) is a useful technique on complex product design in recent years. Collaborative optimization (CO) is an effective MDO methods based decomposition which is for deterministic optimization. However, many uncertainties exist in product design such as model error and design variables error. And the propagation of uncertainties in multidisciplinary is more complicated than in a single disciplinary because of the coupled systems. Therefore, robust design has become more important in engineering systems, and its research and applications have extended to multidisciplinary design environment from primary single disciplinary. To make reliable decisions, some researchers have studied several useful methods on multidisciplinary design optimization under uncertainty environment.

In this paper, a new robust collaborative optimization (RCO) method is proposed based on system uncertainty analysis (SUA) method. First, given the probabilistic distribution of model error and design variables, the mean and variance of system output is calculated by the SUA method. Then using implicit uncertainty propagation (IUP) method, we get the uncertain estimation of auxiliary design variables that is introduced in CO method. In the following, we embed both SUA and IUP methods into CO method framework, and put the estimation of variables which is output from SUA and IUP methods into CO calculation flow, then the optimization calculation process will not stop until system become convergent. Finally, we realize a new robust collaborative optimization method. Compared to the existing RCO method, our method's advantage is the probabilistic presentation of uncertainty objective functions and constraints instead of the variation presentation in the exiting RCO method, so we can know more information about the system performance that is influenced by uncertainty design parameters and variables. Then we can make more reliable decisions in designing the engineering systems according to the probability distributed of system objective function.

Keywords uncertainty; multidisciplinary; RCO; SUA; IUP

---

*National natural science foundation No. 60474059.
†Email:liuhh04@mails.tsinghua.edu.cn. Tel: 010-62789635-1002

Nomenclature

SUA system uncertainty analysis	RCO robust collaborative optimization
IUP implicit uncertainty propagation	ΔT uncertainty associate with simulation tool
μ mean	$\frac{\partial T}{\partial x}$ disciplinary sensitivity matrix
σ variance	x_{sh} shared design variable F objective function
x_i design variable of disciplinary i	F^r robust objective function
x_{aux} auxiliary design variable	g constraints
y linking variable	g^r robust constraint

1 Introduction

Multidisciplinary design optimization (MDO) is a useful technique on complex product design in recent years. In order to get the optimization solution of the whole system, MDO take effective strategy to deal with the relationship of every subsystem. To design different complex systems, Researchers have so far developed many MDO methods [4,5] including individual disciplinary feasible(IDF), multidisciplinary feasible (MDF), collaborative optimization(CO),concurrent subsystem optimization (CSSO).Of those methods, CO is an effective method which decomposes the whole system into a double-level nonlinear optimization problem. To solve subsystem coupling, CO introduces auxiliary design variables and consistency constraints in the system optimization level; disciplinary optimization processes in the subsystem optimization level. At present, CO method has been becoming more and more popular to solve MDO problem because it entitles subsystem disciplinary personnel to have decision-making and it suits for great multidisciplinary problem. More detailed information on CO method can be found in reference [4]. Generally speaking, uncertainty exists in a multidisciplinary design system which includes uncertainty of design variables and uncertainty associated with simulation tools [2,6]. The propagation of uncertainty in a multidisciplinary system is more complicated than in a single-disciplinary as a result of coupled system which can be seen in the following figure.

Fig. 1 shows how uncertainty propagates in a three disciplinary system.

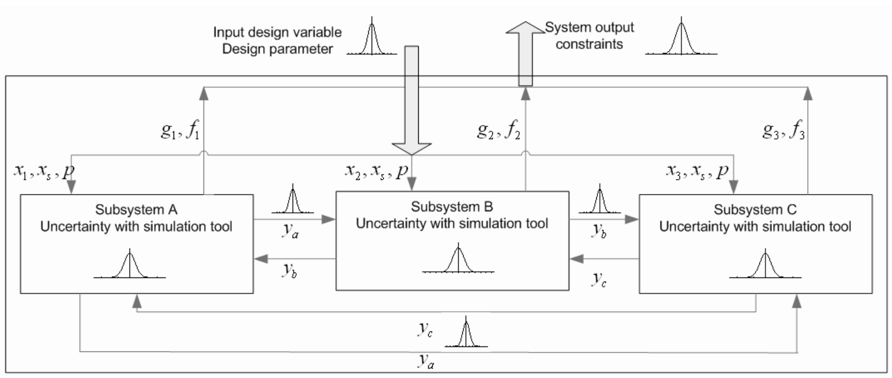

Figure 1: The uncertainty propagation in coupled-disciplinary system

As is clearly seen in figure 1, these uncertainties will more or less influence the system performance, even lead to convergence failure. As a result, robust design has been gaining so much attention that its applications have been extended to making reliable decisions when designing complex engineering systems in a multidisciplinary design environment. Though the usefulness of robust design is widely acknowledged for multidisciplinary design systems, its implementation is rare. One of the reasons is due to the complexity and computational burden associated with the evaluation of performance variations caused by the randomness (uncertainty) of a system. Therefore, how to effectively manage the propagation of uncertainty (that is to say uncertainty analysis) is the foundation of robust design in a multidisciplinary system. In recent years since 1997, some researchers have done plenty of work on robust design in the context of multidisciplinary design. Of those research, Renaud and Su [8] in 1997 proposed two robust optimization methods in which they utilized sensitivity analysis and design of experiment to make multidisciplinary system analysis, then got the evaluation of system objective function and realized robust optimization; Gu [9] put forward a worst case estimation of propagated uncertainty method, with their approach, model uncertainty is denoted by a range of system output ; the worst case concept and the first-order sensitivity analysis are used to evaluate the interval of the end performance of a multidisciplinary system. And then by integrating the uncertainty analysis method into a robust MDO framework, a robust collaborative optimization (RCO) method is formed [1]. However, the uncertainty of the method is represented as variation instead of as probability, so it could not accommodate generic probabilistic representations of uncertain variable and model error estimations. To solve the problem, another approach proposer Du and Chen [3,7] developed three techniques including system uncertainty analysis method (SUA), concurrent subsystem uncertainty analysis method (CSSUA), modified concurrent subsystem uncertainty analysis method (MCSSUA). Given the probabilistic representations of the uncertainty, the above three methods quickly evaluated the mean and the variance of a system output so as to improve the computational efficiency. But what a pity is that although the three methods for uncertainty analysis facilitate the integration of the robust design with MDO in all-in-one type, they did not show a new robust MDO method which can be applied in a multi-level optimization system.

In this paper, a new robust collaborative optimization (RCO) method based on SUA method is proposed. Compared to original RCO method proposed by Gu, this method has the advantage in that it accommodates generic probabilistic representations of uncertain variable and model error estimations as well as its improvement of the computational efficiency.

2 The flow chart of the new RCO method

Considering a three-disciplinary coupled system, the flow chart of the method is shown in Fig.2. The basic approach can be described in detail as follows:

1. Specify targets of design variables: The system level optimizer specifies the values of shared design variables and auxiliary design variables, which are sent to the

disciplines as targets to be matched.

2. Process subsystem level optimization: According to the targets, the subspace optimizers process subsystem level optimization. Each subsystem has autonomy and can solve the robust optimization problem in parallel. Then they report the minimized discrepancy value back to the system level. Also using system uncertainty analysis method, each of subspace optimizer sends the disciplinary sensitivity information as well as the mean and variances value of linking variables obtained at the end of the subspace optimization to the Implicit Uncertainty Propagation module.

3. Implicit uncertainty propagation: based on the IUP methodology in reference [1], the IUP module in the RCO method produces estimates of propagated uncertainty in the auxiliary design variables using the disciplinary sensitivities. These estimates are then sent back to the subspace optimizer and the system optimizer, where they are used to calculate the robust constraint and robust objective function respectively.

4. Process system level optimization: According to the information of robust objective function and the variances of auxiliary design variables calculated in step 3, we can perform system level optimization to get the robust optimization solutions.

5. Convergency check: repeat the above steps until no further improvement in the system level objective value is obtained. A convergency check performs during iteration and if the convergency criterion is not satisfied, then we must go to step 1 and repeat the whole robust optimization process again.

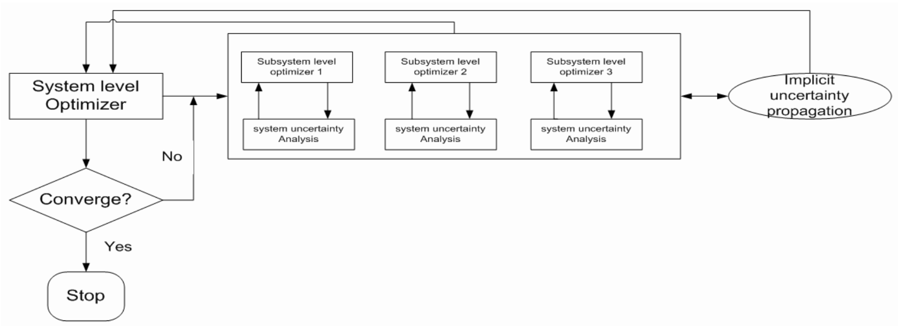

Figure 2: The flow chart of the new RCO method

3 Formulation of the new RCO method

The essence of robust design is to improve the quality of a product by minimizing the adverse effects of variation without necessarily eliminating the causes of variation. The general formulation for a deterministic optimization problem is stated as follows:

$$\text{Minimize:} \quad f(x,u(x)) \tag{1}$$

$$\text{Subject to:} \quad \begin{cases} h(x,u(x)) = 0 \\ g(x,u(x)) \leq 0 \\ (x)_{\min} \leq x \leq (x)_{\max} \end{cases} \tag{2}$$

Considering random distribution variables, when we process robust design, the above statement can be described as follows, where α, β, γ are weight factors for functional variations.

$$\text{Minimize:} \quad F^r = \alpha\mu_F + (1-\alpha)\sigma_F \tag{3}$$

$$\text{Subject to:} \quad \begin{cases} g^r = \mu_g + \beta\sigma_g \leq 0 \\ h^r = |\mu_h| + \gamma\sigma_h \leq \varepsilon \\ (\mu_x)_{\min} \leq \mu_x \leq (\mu_x)_{\max} \end{cases} \tag{4}$$

In the new RCO method, mathematically, the robust optimization can be described as follows

$$\text{Minimize:} \quad F^r(x^0_{sys}) = \alpha\mu_F + (1-\alpha)\sigma_F \tag{5}$$

$$\text{Subject to:} \quad \begin{cases} \mu_F = F(\mu_{x^0_{sys}}), \\ \sigma_F = \left|\frac{\partial}{\partial x_{sh}} F(x^0_{sys})\sigma_{x^0_{sh}}\right| + \left|\frac{\partial}{\partial x_{aux}} F(x^0_{sys})\sigma_{x_{aux}}\right|, \\ x^0_{sys} = (x^0_{sh}, x^0_{aux}) \\ g^r_i(x_{ssi}) = \mu_{gi} + \beta\sigma_{gi} \leq 0 \\ \mu_{gi} = g_i(\mu_{xssi}), \\ x_{ssi} = ((x_{sh})_i, (x_{aux})_{ji}, (x_{local})_i) \\ \sigma_{gi} = \left|\frac{\partial}{\partial (x_{sh})_i} g_i(x_{ssi})\sigma_{(x_{sh})_i}\right| + \left|\frac{\partial}{\partial (x_{aux})_{ji}} g_i(x_{ssi})\sigma_{(x_{aux})_{ji}}\right| \\ \qquad + \left|\frac{\partial}{\partial (x_{local})_i} g_i(x_{ssi})\sigma_{(x_{local})_i}\right| \end{cases} \tag{6}$$

Fig.3 shows the framework of the new robust collaborative optimization method in a three discipline system, the detailed data flow can be clearly seen in the framework.

In order to calculate the influence of design variables to system level objective function and subsystem constraint function, as is seen in Eq 4, we must get the variance values of auxiliary design variables σ_{xaux}. σ_{xaux} can be got by IUP module, that is:

$$\begin{Bmatrix} \sigma(x_{aux})_a \\ \sigma(x_{aux})_b \\ \sigma(x_{aux})_c \end{Bmatrix} = \begin{Bmatrix} \frac{dy_a}{dx} \\ \frac{dy_b}{dx} \\ \frac{dy_c}{dx} \end{Bmatrix} \sigma x + \begin{Bmatrix} I_a & -\frac{\partial T_a}{\partial (x_{aux})_{ba}} & -\frac{\partial T_a}{\partial (x_{aux})_{ca}} \\ -\frac{\partial T_b}{\partial (x_{aux})_{ab}} & I_b & -\frac{\partial T_b}{\partial (x_{aux})_{cb}} \\ -\frac{\partial T_c}{\partial (x_{aux})_{ac}} & -\frac{\partial T_c}{\partial (x_{aux})_{bc}} & I_c \end{Bmatrix}^{-1} \sigma y \tag{7}$$

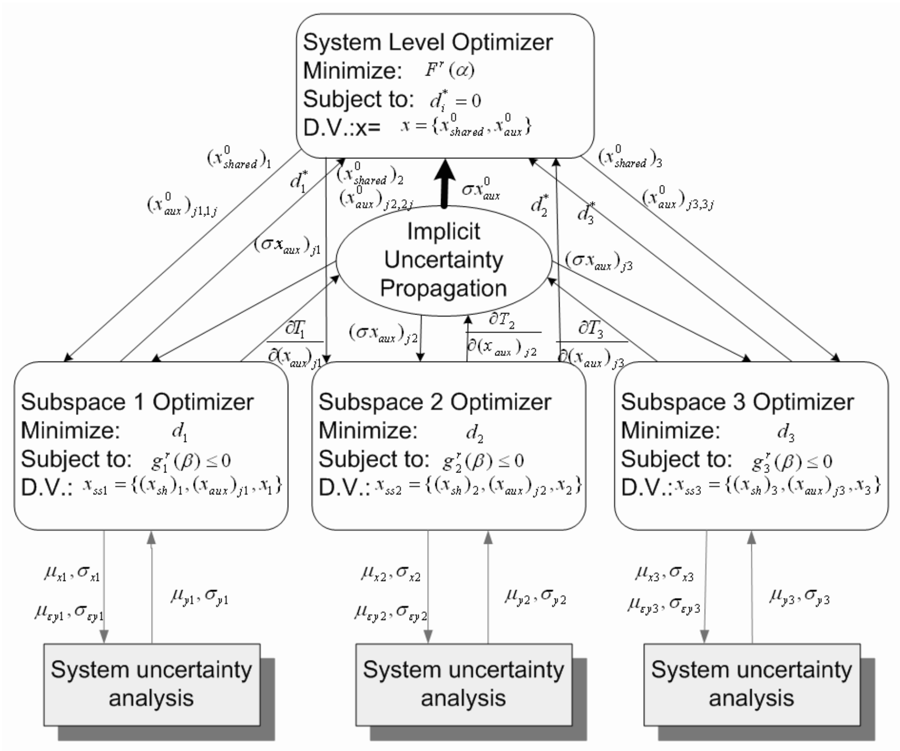

Figure 3: The framework of the new RCO method

In Eq 7, σx is the variance of design variables which is given, $\sigma y$is the variance of linking variables which can be calculated by Eq 8. In the below equation, $\sigma_x, \sigma_{\varepsilon y}$is the variance of design variable and model uncertainty which are given in probabilistic representation. Eq 8 can be got from SUA method.

$$\begin{pmatrix} \sigma_{y1}^2 \\ \sigma_{y2}^2 \\ \sigma_{y3}^2 \end{pmatrix} = A \begin{pmatrix} \frac{\partial T_1}{\partial x_s} \\ \frac{\partial T_2}{\partial x_s} \\ \frac{\partial T_3}{\partial x_s} \end{pmatrix} \sigma_{xsh}^2 + A \begin{pmatrix} \frac{\partial T_1}{\partial x_1} & 0 & 0 \\ 0 & \frac{\partial T_2}{\partial x_2} & 0 \\ 0 & 0 & \frac{\partial T_3}{\partial x_3} \end{pmatrix} \begin{pmatrix} \sigma_{x1}^2 \\ \sigma_{x2}^2 \\ \sigma_{x3}^2 \end{pmatrix} + A \begin{pmatrix} \sigma_{\varepsilon y1}^2 \\ \sigma_{\varepsilon y2}^2 \\ \sigma_{\varepsilon y3}^2 \end{pmatrix} \tag{8}$$

Where

$$A = \begin{pmatrix} I_1 & -\frac{\partial T_1}{\partial y_2} & -\frac{\partial T_1}{\partial y_3} \\ -\frac{\partial T_2}{\partial y_1} & I_2 & -\frac{\partial T_2}{\partial y_3} \\ -\frac{\partial T_3}{\partial y_1} & -\frac{\partial T_3}{\partial y_2} & I_3 \end{pmatrix}^{-1}$$

When we get the estimates of auxiliary design variables by IUP method, these estimates are then sent back to discipline optimizer as well as system level optimizer. Then the robust optimization performs in system level optimizer until we get a robust optimization solution.

4 Conclusion and future work

Based on the system uncertainty analysis method and implicit uncertainty propagation, a new robust collaborative optimization is proposed in the paper. In order to accommodate the probabilistic representation of system uncertainty associated with design variables and simulation models, the method introduces SUA method to calculate the linking variables and system output. Then the IUP module is utilized to get the variances values of auxiliary design variables which are used for system decoupling. Compared to the method presented by Gu, the efficiency of robust optimization apparently gets improved because of the advantage of SUA method. Therefore, more reliable design can be performed using the new robust collaborative optimization method.

Apparently, the paper just proposed the framework and flow chart of the new method, we need a complex engineering application to verify the correctness of the method. Furthermore, we have much work to realize the method in the integration software, and put the uncertainty algorithm module into MDO through second-develop of software.

References

[1] Xiaoyu Gu. Uncertainty in simulation based multidisciplinary design optimization. *Ph.D. Dissertation*, Graduate School of the University of Notre Dame, April 2003.

[2] Xiaoping Du and Wei Chen. Propagation and management of uncertainties in simulation-based collaborative system design. `http://www.uic.edu/labs/idel/pdf/WCSMO3.pdf`

[3] Xiaoping Du and Wei Chen. Efficient uncertainty analysis methods for multidisciplinary robust design. American Institute of Aeronautics and Astronautics, Inc., 2001.

[4] Natalia Alexandrov and Srinivas Kodiyalam. Initial results of MDO method evaluation study. *AIAA-98-4884*.

[5] Srinivas Kodiyalam. Evaluation of methods for multidisciplinary design optimization (MDO). *NASA/CR-1998-208716*.

[6] Harish Agarwal and John E. Renaud. Uncertainty quantification using evidence theory in multidisciplinary design optimization. *Reliability Engineering and System Safety*, 85, 281–194, 2004.

[7] Xiaoping Du and Wei Chen. An efficient approach to probabilistic uncertainty analysis in simulation-based multidisciplinary design. *AIAA 2000-0423*.

[8] Su and Renaud. Automation differentiation in robust optimization. *AIAA 1072-1079*.

[9] X. Gu and J. E. Renaud. Worst case propagated uncertainty of multidisciplinary systems in robust design optimization. *Struct. Multidisc. Optim.*, 20, 190–213, Springer-Verlag, 2000.