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Two Interval Parameter Fuzzy Programming Models for Petroleum Solid Waste Management

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Abstract This paper presents two interval parameter fuzzy programming models for the planning of petroleum solid waste management systems under uncertainty. In these models, fixed violation limits are introduced to relax the critical constraints so that the results produced by these models give more useful information to decision makers. These two models are also applied to a hypothetical planning problem of waste flow allocation and treatment/disposal facilities expansion. The results provide a number of decision alternatives under various system conditions. It is helpful for decision makers to make tradeoffs between system benefit and reliability.

Keywords interval parameter; fuzzy programming; uncertainty; petroleum solid waste

1 Introduction

The high demand for oil as fuel and feedstock has led to intensive crude oil exploration and production activities. At the same time, increased activities create more oil waste. Since the pollution from petroleum solid waste such as residual oil, disused catalysts, and rotten tanks and pipes may pose a variety of impacts, risks, and liabilities, petroleum solid waste management (PSWM) has been of much concern in recent years.

In PSWM, many system parameters and their interactions generally show high degrees of intrinsic variability and uncertainty. For dealing with uncertainties, Huang et al. (1993) proposed interval parameter fuzzy programming. This model can be solved by an interactive two-step solution algorithm (Huang et al., 1993). However, this solutions may not be satisfying to decision makers. Recently, Huang et al. (2002) extended this model to an interval fuzzy violation analysis model that provides more tradeoff information between system benefits and reliability. But in this method,

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the values of violation variables can only be controlled by the total violation. This method has two drawbacks: (1) the sum of violation variables may satisfy the total violation, but a single violation variable may exceed its own violation limit; (2) since each constraint reflects different aspects in PSWM, it is difficult to explain the summation of all violation variables. For example, in the application of the hypothetical problem in section 3, the constraints denote system cost, system capacity, and the daily waste generation rate respectively. It would be difficult to understand the sum of the violations for these constraints.

In this paper, two new interval fuzzy violation analysis models are developed to improve the existing models. In both models, a set of fixed violation limits with parameters are introduced to substitute violation variables in constraints. The solutions to these models under different violation levels of system constraints can help decision makers to identify and evaluate alternative system designs and to determine which design can most efficiently achieve the desired system objectives. These two models will be employed in a hypothetical PSWM system to demonstrate its potential usefulness.

This paper is organized as follows. In Section 2, we describe the two interval parameter fuzzy programming models. In Section 3, we apply these two models to a hypothetical planning problem of waste flow allocation and treatment/disposal facilities expansion. In Section 4, we analyze the results obtained from Section 3. Finally, we conclude the paper in Section 5.

2 Methodology

We first consider an interval-parameter linear program (ILP) as follows:

$$\begin{aligned} \text{minimize} \quad & f^{\pm} = C^{\pm}X^{\pm} \\ \text{subject to} \quad & A^{\pm}X^{\pm} \leq B^{\pm} \\ & X^{\pm} \geq 0 \end{aligned}$$

where $A^{\pm} \in \{\mathbb{R}^{\pm}\}^{m\times n}$, $B^{\pm} \in \{\mathbb{R}^{\pm}\}^{m\times 1}$, $C^{\pm} \in \{\mathbb{R}^{\pm}\}^{1\times n}$ and $X^{\pm} \in \{\mathbb{R}^{\pm}\}^{n\times 1}$. We use $\mathbb{R}^{\pm}$ to denote a set of interval numbers. In this model, the interval numbers are introduced to express uncertainties. However, when the model stipulations are very uncertain, the outputs will lead to little use or no use for decision makers (Huang et al. 1993).

When we incorporate the flexible fuzzy linear programming and interval programming into the one framework, we could formulate this interval-parameter fuzzy linear programming as follows (Huang et.al.,1993):

$$\begin{aligned} \text{maximize} \quad & \lambda^{\pm} \\ \text{subject to} \quad & C^{\pm}X^{\pm} \leq f^{+}_{opt1} - \lambda^{\pm}[f^{+}_{opt1} - f^{-}_{opt1}] \\ & A^{\pm}X^{\pm} \leq B^{+} - \lambda^{\pm}[B^{+} - B^{-}] \\ & 0 \leq \lambda^{\pm} \leq 1 \\ & X^{\pm} \geq 0 \end{aligned}$$

where f_{opt1}^- and f_{opt1}^+ denote the least and most desirable system objectives respectively, and $\lambda^{\pm}$ is a control variable corresponding to the degree to which the model's solution fulfills the fuzzy goal or constraints. According to the solution algorithm provided by Huang et al. (1993), this model could be solved by dividing it into two submodels. Let the interval solution be denoted as $X_{opt}^{\pm} = (x_{1opt}^{\pm}, x_{2opt}^{\pm}, \ldots, x_{mopt}^{\pm})^T$, $x_{jopt}^{\pm} = [x_{jopt}^-, x_{jopt}^+]$, $j = 1, 2, \ldots, m$, $f_{opt}^{\pm} = [f_{opt}^-, f_{opt}^+]$ and $\lambda_{opt}^{\pm} = [\lambda_{opt}^-, \lambda_{opt}^+]$. However, decision makers may not be satisfied with these simply solutions. Instead, they may desire tradeoff information between system reliability (with low risk) and system benefit (with low net cost). Thus, information could be presented by different risk levels of violating system constraints (Huang et al. 2002). To deal with this issue, an interval parameter fuzzy programming with violation variables model was proposed. In this model, a set of violation variables were introduced to relax the constraints and to facilitate violation analysis. This related model can be rewritten as follows (Huang et al. 2002):

$$\begin{aligned} \text{maximize} \quad & \lambda^- \\ \text{subject to} \quad & C^+X^+ - V_f \leq f_{opt1}^+ - \lambda^-[f_{opt1}^+ - f_{opt1}^-] \\ & A^-X^+ - V \leq B^+ - \lambda^-[B^+ - B^-] \\ & V_f + \textstyle\sum_{i=1}^m V_i \leq \lambda^- TV \\ & 0 \leq \lambda^- \leq 1 \\ & X^+ \geq 0 \end{aligned}$$

where V_f is a violation variable for the objective function; $V = (V_1, V_2, \ldots, V_m)^T$ is the vector of violation variables for each constraint; TV is the total tolerable violation limit; and m is the number of constraints. With varied TV levels, different λ^- values could be generated. These solutions are useful for analyzing tradeoffs between the system satisfaction levels and the associated risks (of violating the fuzzy goal and constraints) (Huang et al. 2002). Unfortunately, there are two drawbacks in the this method. First, some violation variables value may exceed their own violation limit, although the sum may satisfy the total violation. And secondly, since each constraint reflects different aspects, it is difficult to explain the meaning of adding all violation variables together. In order to deal with this concern, we propose the following model:

$$\begin{aligned} \text{maximize} \quad & \lambda^- \\ \text{subject to} \quad & C^+X^+ - (1-\lambda^-)U_0 \leq f_{opt1}^+ - \lambda^-[f_{opt1}^+ - f_{opt1}^-] \\ & A^-X^+ - (1-\lambda^-)U \leq B^+ - \lambda^-[B^+ - B^-] \\ & 0 \leq \lambda^- \leq 1 \\ & X^+ \geq 0 \end{aligned}$$

In this model, U_0 is a fixed violation limit for the objective function; $U = (U_1, U_2, \ldots, U_m)^T$ is the vector of fixed violation limit for each constraint; and $(1-\lambda^-)$ represents the percentage of violation limit (i.e. violation amount for each constraint). From this model, we will see, when λ^- equals 1, the model will be reduced to an original interval fuzzy linear programming model. The corresponding solution can be interpreted the lowest system cost, the lowest reliability in fulfilling system requirements, and the highest risk. Conversely, when λ^- equals 0, the violation values will reach their limits. By solving this model, the solutions will provide useful alternatives for decision makers. This model is easy to implement, however, since all constraints use the same λ, this may make some constraints over-satisfied and leave others not well satisfied. We propose another model that introduces a set of parameters α_l $(0 \le \alpha_l \le 1)$ to the constraints. This model can be described as follows:

$$\begin{aligned}
\text{maximize} \quad & \lambda^- \\
\text{subject to} \quad & C^+X^+ - (1-\alpha_0)U_0 \le f^+_{opt1} - \lambda^-[f^+_{opt1} - f^-_{opt1}] \\
& A^-X^+ - \begin{pmatrix} (1-\alpha_1)U_1 \\ \vdots \\ (1-\alpha_m)U_m \end{pmatrix} \le B^+ - \lambda^-[B^+ - B^-] \\
& \textstyle\sum_{l=0}^{m} \alpha_l/(m+1) \ge \lambda^- \\
& 0 \le \lambda^- \le 1 \\
& 0 \le \alpha_l \le 1, l = 0,1,2,\ldots,m \\
& X^+ \ge 0
\end{aligned}$$

where m is the numbers of original constraints.

3 Application

We apply the two new models of the previous section to a hypothetical study area. The hypothetical region includes two oil refinery plants, one oil production site and three waste management facilities (one landfill and two incinerators), as show in Figure 1. All kinds of liquid and solid waste from industries such as residual oil,

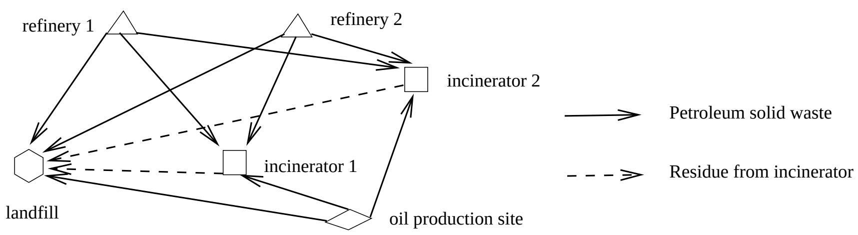

Figure 1: A hypothetical case of PSWM system

dismissed catalysts, rotten tanks and pipes are transported to the incinerators or the landfill. The waste generation rates for two oil refinery plants and one oil production site in different periods are showed in Table 1.

Table 1: Waste generation rates

	time period		
	k=1	k=2	k=3
Refinery 1 (WG_{1k})	[200,250]	[225,275]	[250,300]
Refinery 2 (WG_{2k})	[350,400]	[375,425]	[400,450]
Oil production site (WG_{3k})	[275,325]	[300,350]	[325,375]

Table 2: Data for the costs of transportation and operation

	time period		
	$k=1$	$k=2$	$k=3$
Cost of transportation to landfill($/ton)			
Refinery 1 ($c_{11k}^{\pm}$)	[12.1,16.1]	[13.3,17.7]	[14.6,19.5]
Refinery 2 ($c_{21k}^{\pm}$)	[10.5,14.0]	[11.6,15.4]	[12.8,16.9]
Oil production site ($c_{31k}^{\pm}$)	[12.7,17.0]	[14.0,18.7]	[15.4,20.6]
Cost of transportation to incinerator 1 ($/ton)			
Refinery 1 ($c_{12k}^{\pm}$)	[12.8,19.6]	[10.6,14.1]	[11.7,15.5]
Refinery 2 ($c_{22k}^{\pm}$)	[10.1,13.4]	[11.1,14.7]	[12.2,16.2]
Oil production site ($c_{32k}^{\pm}$)	[8.80,11.7]	[9.7,12.8]	[10.6,14.0]
Cost of transportation to incinerator 2 ($/ton)			
Refinery 1 ($c_{13k}^{\pm}$)	[12.1,16.1]	[13.3,17.7]	[14.6,19.5]
Refinery 2 ($c_{23k}^{\pm}$)	[12.8,17.2]	[14.1,18.8]	[15.5,20.7]
Oil production site ($c_{33k}^{\pm}$)	[4.2,5.6]	[4.6,6.2]	[5.1,6.8]
Cost of residue transportation to landfill ($/ton)			
Incinerator 1 ($d_{2k}^{\pm}$)	[4.7,6.3]	[5.2,6.9]	[5.7,7.6]
Incinerator 2 ($d_{3k}^{\pm}$)	[13.4,17.9]	[14.7,19.7]	[16.2,21.7]
Revenue from incinerators ($/ton)			
$RE_k^{\pm}$	[15,25]	[15,25]	[15,25]
Facility operating costs ($/ton)			
$TE_{1k}^{\pm}$ (landfill)	[30,45]	[40,60]	[50,80]
$TE_{2k}^{\pm}$ (incinerator 1)	[55,75]	[60,85]	[65,95]
$TE_{3k}^{\pm}$ (incinerator 2)	[50,70]	[60,80]	[65,85]

The assumed planning horizon is 15 years that is considered as three equal periods (5 years for each). The landfill has a capacity of $[0.625, 0.775] * 10^6$ tons, and the two waste-to-energy facilities have treatment capacities of $[100, 125]$ and $[200, 225]$ tons/day, respectively. The incinerators generate residues of 20% - 30% of the total incoming wastestream, and the residues will be transported to the landfill and disposed immediately. The revenues from the incinerators are $15-25$$ per ton combusted. Table 2 lists transportation costs and relevant operation costs for the three resources in the three time periods.

According to the region's environmental policy, during the planning horizon, the landfill can be incremented by $[1.55, 2.5] * 10^6$ tons. And the incinerators can be expanded once by any of three options in each time period. Table 3 gives the detailed expansion options and the relevant costs for the expansion.

Table 3: Capacity expansion options for PSWM facilities

	time period		
	$k=1$	$k=2$	$k=3$
Capacity expansion option for two incinerators (ton/day)			
Option 1 (ΔTC_1)	100	100	100
Option 2 (ΔTC_2)	150	150	150
Option 3 (ΔTC_3)	200	200	200
Capacity expansion option for landfill (10^6ton)			
ΔTC	[1.55,2.50]	[1.55,2.50]	[1.55,2.50]
Capital cost for landfill expansion (\$$10^6$ present value)			
FLC_k	[13,15]	[13,15]	[13,15]
Capacity cost for incinerator expansion, $j=2,3$ (10^6 ton)			
Option 1 (FTC_{j1k})	10.5	8.3	6.5
Option 2 (FTC_{j2k})	15.2	11.9	9.3
Option 3 (FTC_{j3k})	19.8	15.5	12.2

Therefore, the problem under consideration is how to effectively distribute the waste flows from these three resources and select appropriate capacity expansion schemes to minimize net system cost. Since uncertainties exist in the system components, the above problem can be formulated as an interval fuzzy mixed integer linear programming (IFMILP) as follows.

maximize $\lambda^{\pm}$
subject to

$$1825\sum_{i=1}^{3}\sum_{k=1}^{3}[(c_{i1k}^{\pm}+TE_{1k}^{\pm})\cdot x_{i1k}^{\pm}+\sum_{j=2}^{3}((c_{ijk}^{\pm}+TE_{jk}^{\pm})\cdot x_{ijk}^{\pm}-RE_{k}^{\pm}\cdot x_{ijk}^{\pm}$$
$$+RF^{\pm}\cdot(d_{jk}^{\pm}+TE_{1k}^{\pm})\cdot x_{ijk}^{\pm})]+\sum_{j=2}^{3}\sum_{m=1}^{3}\sum_{k=1}^{3}(FTC_{jmk}^{\pm}\cdot Z_{jmk}^{\pm}+\sum_{k=1}^{3}(FLC_{k}^{\pm}\cdot Y_{k}^{\pm})$$
$$\leq f_{opt1}^{+}-\lambda^{\pm}\cdot(f_{opt1}^{+}-f_{opt1}^{-}),\quad \text{(system cost constraint)};$$

$$1825\sum_{i=1}^{3}\sum_{k=1}^{k'}(x_{i1k}^{\pm}+RF^{\pm}\cdot\sum_{j=2}^{3}x_{ijk}^{\pm})\leq TL^{-}-\lambda^{\pm}(TL^{+}-TL^{-})+\triangle TC\cdot\sum_{k=1}^{k'}Y_{k}^{\pm},$$
$$k'=1,2,3,\quad \text{(landfill capacity constraints)};$$

$$\sum_{i=1}^{3}x_{ijk'}^{\pm}\leq WTE_{j}^{+}-\lambda^{\pm}\cdot(WTE_{j}^{+}-WTE_{j}^{-})+\sum_{m=1}^{3}\sum_{k=1}^{k'}(Z_{jmk}^{\pm}\cdot\triangle TC_{m}),$$
$$j=2,3;k'=1,2,3,\quad \text{(incinerator capacity constraints)};$$

$$\sum_{j=1}^{3} x_{ijk}^{\pm} \geq WG_{ik}^{+} - \lambda^{\pm} \cdot (WG_{ik}^{+} - WG_{ik}^{-}),$$

$$i = 1,2,3; k = 1,2,3, \quad \text{(waste disposal constraints)};$$

$$\sum_{k=1}^{3} Y_k^{\pm} \leq 1, \quad \text{(landfill expansion constraints)};$$

$$\sum_{m=1}^{3} Z_{jmk} \leq 1, j = 2,3; k = 1,2,3, \quad \text{(incinerator expansion constraints)};$$

$$Y_k = 0 \text{ or } 1, \text{ and } Z_{jmk} = 0 \text{ or } 1, j = 2,3; k = 1,2,3; m = 1,2,3;$$

$$0 \leq \lambda^{\pm} \leq 1, \quad \text{(membership degree)};$$

$$x_{ijk}^{\pm} \geq 0, i = 1,2,3, j = 2,3; k = 1,2,3.$$

In the above formula, the coefficient $1825 = 365 \times 5$, which means the total days in a period of five years; $\lambda^{\pm}$ is a control variable corresponding to the degree (membership grade) of satisfaction for the constraints and objective function; f_{opt1}^{-} and f_{opt1}^{+} is the most and least desirable system objective function value; i is the index for three resources, $i = 1$ (refinery 1), $i = 2$ (refinery 2) and $i = 3$ (oil production site); j is the index for PSWM facilities, $j = 1$ (landfill), $j = 2$ (incinerator 1) and $j = 3$ (incinerator 2); k is the index for periods, $k = 1$ ($0 \sim 5$ years), $k = 2$ ($6 \sim 10$ years) and $k = 3$ ($11 \sim 15$ years); m is the index for facility capacity expansion type; $RF^{\pm}$ is the residue flow rate from incinerators to landfill (ton/day); $x_{ijk}^{\pm}$ is the solid waste stream from resource i to facility j in period k (ton/day); $\Delta TC^{\pm}$ is the total amount of landfill expansion capacity for landfill (ton); $Y_k^{\pm}$ is the binary variable for expansion of landfill in period k; $TL^{\pm}$ is the capacity of landfill (ton); $WTE_j^{\pm}$ is the capacity of incinerator j ($j = 2,3$) (ton/day); $Z_{jmk}^{\pm}$ is the binary variable for expansion option m of incinerator j in period k; ΔTC_m is the amount of capacity expansion option m for incinerators (ton/day); $C_{ijk}^{\pm}$ is the waste transportation cost from resources i to facility j in period k (\$/ton); $d_{jk}^{\pm}$ is the residue transportation cost from incinerator j to landfill in period k (\$/ton); $TE_{jk}^{\pm}$ is the operating cost of PSWM facility j in period k (\$/ton); $RE_k^{\pm}$ is the revenue of unit PSW incinerated in period k (\$/ton); $FLC_k^{\pm}$ is unit capacity expansion cost for landfill in period k(\$/ton); $FTC_{jmk}^{\pm}$ is the unit cost of capacity expansion option m for incinerator j in period k (\$/ton); $WG_{ik}^{\pm}$ is the daily generation rate of solid waste for resource i in period k (ton/day).

From Section 2, our first constraint-relaxed IFMILP model, denoted by *model A*, can be formulated as follows.

maximize λ^{-}
subject to

$$1825 \sum_{i=1}^{3} \sum_{k=1}^{3} [(c_{i1k}^{+} + TE_{1k}^{+}) \cdot x_{i1k}^{+} + \sum_{j=2}^{3} ((c_{ijk}^{+} + TE_{jk}^{+}) \cdot x_{ijk}^{+} - RE_k^{-} \cdot x_{ijk}^{+}$$

$$+ RF^{+} \cdot (d_{jk}^{+} + TE_{1k}^{+} \cdot x_{ijk}^{+})] + \sum_{j=2}^{3} \sum_{m=1}^{3} \sum_{k=1}^{3} (FTC_{jmk}^{+} \cdot Z_{jmk}) + \sum_{k=1}^{3} (FLC_k^{+} \cdot Y_k^{+})$$

$$-(1 - \lambda^{-})U_0 \leq f_{opt1}^{+} - \lambda^{-} \cdot (f_{opt1}^{+} - f_{opt1}^{-}), \quad \text{(system cost constraint with violation)};$$

$$1825\sum_{i=1}^{3}\sum_{k=1}^{k'}[x_{i1k}^{+}+RF^{+}\cdot\sum_{j=2}^{3}x_{ijk}^{+}]-(1-\lambda^{-})U_{k'}\leq TL^{-}-\lambda^{-}(TL^{+}-TL^{-})+\triangle TC\cdot\sum_{k=1}^{k'}Y_{k}^{\pm},$$
$$k'=1,2,3,\quad \text{(landfill capacity constraints with violation);}$$

$$\sum_{i=1}^{3}x_{ijk'}^{+}-(1-\lambda^{-})U_{k'+3(j-1)}\leq WTE_{j}^{+}-\lambda^{-}(WTE_{j}^{+}-WTE_{j}^{-})+\sum_{m=1}^{3}\sum_{k=1}^{k'}(Z_{jmk}\cdot\triangle TC_{m}),$$
$$j=2,3;k'=1,2,3,\quad \text{(incinerator capacity constraints with violation);}$$

$$\sum_{j=1}^{3}x_{ijk}^{+}+(1-\lambda^{-})\cdot U_{k+3(i-1)+9}\geq WG_{ik}^{+}-\lambda^{-}\cdot(WG_{ik}^{+}-WG_{ik}^{-}),$$
$$i=1,2,3;k=1,2,3\quad \text{(waste disposal constraints with violation);}$$

$$\sum_{k=1}^{3}Y_{k}\leq 1,\quad \text{(landfill expansion constraints);}$$

$$\sum_{m=1}^{3}Z_{jmk}\leq 1,j=2,3;k=1,2,3,\quad \text{(incinerator expansion constraints);}$$

$$Y_{k}=0\text{ or }1,\text{ and }Z_{jmk}=0\text{ or }1,j=2,3;k=1,2,3;m=1,2,3;$$
$$0\leq\lambda^{-}\leq 1,\quad \text{(membership degree);}$$
$$x_{ijk}^{+}\geq x_{ijk_{opt}}^{-},i=1,2,3,j=2,3;k=1,2,3.$$

where U_0 is a fixed violation limit for the original objective function, and U'_k, $U_{k'+3(j-1)}$, and $U_{k+3(i-1)+9}$ $(i=1,2,3,\ j=2,3,\ k=1,2,3,\ k'=1,2,3)$ are violation limits for constraints.

From Section 2, our second constraint-relaxed IFMILP model, denoted by *model B*, can be formulated as follows.

maximize λ^{-}
subject to

$$1825\sum_{i=1}^{3}\sum_{k=1}^{3}[(c_{i1k}^{+}+TE_{1k}^{+})\cdot x_{i1k}^{+}+\sum_{j=2}^{3}((c_{ijk}^{+}+TE_{jk}^{+})\cdot x_{ijk}^{+}-RE_{k}^{-}\cdot x_{ijk}^{+}$$
$$+RF^{+}(d_{jk}^{+}+TE_{1k}^{+}\cdot x_{ijk}^{+})]+\sum_{j=2}^{3}\sum_{m=1}^{3}\sum_{k=1}^{3}(FTC_{jmk}^{+}\cdot Z_{jmk})+\sum_{k=1}^{3}(FLC_{k}^{+}\cdot Y_{k}^{+})$$
$$-(1-\alpha_{0})U_{0}\leq f_{opt1}^{+}-\lambda^{-}\cdot(f_{opt1}^{+}-f_{opt1}^{-}),\quad \text{(system cost constraint with violation);}$$

$$1825\sum_{i=1}^{3}\sum_{k=1}^{k'}[x_{i1k}^{+}+RF^{+}\cdot\sum_{j=2}^{3}x_{ijk}^{+}]-(1-\alpha_{k'})U_{k'}\leq TL^{-}-\lambda^{-}(TL^{+}-TL^{-})$$
$$+\triangle TC\cdot\sum_{k=1}^{k'}Y_{k}^{\pm},k'=1,2,3,\quad \text{(landfill capacity constraints with violation);}$$

$$\sum_{i=1}^{3} x_{ijk'}^{+} - (1-\alpha_{k'+3(j-1)})U_{k'+3(j-1)}$$
$$\leq WTE_j^{+} - \lambda^{-} \cdot (WTE_j^{+} - WTE_j^{-}) + \sum_{m=1}^{3}\sum_{k=1}^{k'} (Z_{jmk} \cdot \triangle TC_m),$$
$$j = 2,3; k' = 1,2,3, \quad \text{(incinerator capacity constraints with violation);}$$

$$\sum_{j=1}^{3} x_{ijk}^{+} + (1-\alpha_{k+3(i-1)+9})U_{k+3(i-1)+9} \geq WG_{ik}^{+} - \lambda^{-} \cdot (WG_{ik}^{+} - WG_{ik}^{-}),$$
$$i = 1,2,3; k = 1,2,3, \quad \text{(waste disposal constraints with violation);}$$
$$\sum_{l=0}^{m} \alpha_l/(m+1) \geq \lambda^{-}, \quad \text{(average } \alpha_l \text{ constraint);}$$
$$\sum_{k=1}^{3} Y_k \leq 1, \quad \text{(landfill expansion constraints);}$$
$$\sum_{m=1}^{3} Z_{jmk} \leq 1, j = 2,3; k = 1,2,3, \quad \text{(incinerator expansion constraints);}$$
$$Y_k = 0 \text{ or } 1, \text{ and } Z_{jmk} = 0 \text{ or } 1, j = 2,3; k = 1,2,3; m = 1,2,3;$$
$$0 \leq \lambda^{-} \leq 1, \quad \text{(membership degree);}$$
$$0 \leq \alpha_l \leq 1, l = 0,1,\ldots,m;$$
$$x_{ijk}^{+} \geq x_{ijk_{opt}}^{-}, i = 1,2,3, j = 2,3; k = 1,2,3.$$

where U_0 is a fixed violation limit for the original objective function, U_k', $U_{k'+3(j-1)}$, and $U_{k+3(i-1)+9}$, $(i = 1,2,3,\ j = 2,3,\ k = 1,2,3,\ k' = 1,2,3)$, are violation limits for each constraints, and $(1-\alpha_l)$ $(l = 0,1,\ldots,m)$ are parameters for the percentage of the fixed violation limits.

4 Result analysis

4.1 IFMILP model

Table 4 shows the solutions to the IFMIP model. The net system cost is [300.96, 576.534], and corresponding λ^{-} value ranges [0.035, 0.959]. Since λ^{-} is a control variable corresponding to the membership degree, a low λ^{-} value $(= 0.035)$ implies a relatively low possibility for satisfying the objective function and constraints.

The solutions for integer variables are that, when λ^{-} has a higher value, the expansion of the landfill should be taken in period 2; in comparison, when λ^{-} has a lower value, the landfill will be expanded in period 1 corresponding to a high flow rate; both incinerators 1 and 2 should be expanded in period 1, with an incremental capacity of 200 tons/day and 150 tons/day respectively.

Table 4: Solutions to IFMIP model

Resource	Facility	Period	Variable	Solution
Oil refinery plant 1	flow to landfill (ton/day)	1	$x_{111}^{\pm}$	[128,174]
		2	$x_{112}^{\pm}$	[0,46]
		3	$x_{113}^{\pm}$	[0,46]
	flow to incinerator 1 (ton/day)	1	$x_{121}^{\pm}$	[0,0]
		2	$x_{122}^{\pm}$	[227,227]
		3	$x_{123}^{\pm}$	[228,228]
	flow to incinerator 2 (ton/day)	1	$x_{131}^{\pm}$	[74,74]
		2	$x_{132}^{\pm}$	[0,0]
		3	$x_{133}^{\pm}$	[24,24]
Oil refinery plant 2	flow to landfill (ton/day)	1	$x_{211}^{\pm}$	[54,100]
		2	$x_{212}^{\pm}$	[303,349]
		3	$x_{213}^{\pm}$	[329,375]
	flow to incinerator 1 (ton/day)	1	$x_{221}^{\pm}$	[298,298]
		2	$x_{222}^{\pm}$	[74,74]
		3	$x_{223}^{\pm}$	[73,73]
	flow to incinerator 2 (ton/day)	1	$x_{231}^{\pm}$	[0,0]
		2	$x_{232}^{\pm}$	[0,0]
		3	$x_{233}^{\pm}$	[0,0]
Oil production site	flow to landfill (ton/day)	1	$x_{311}^{\pm}$	[0,46]
		2	$x_{312}^{\pm}$	[0,46]
		3	$x_{313}^{\pm}$	[0,32]
	flow to incinerator 1 (ton/day)	1	$x_{321}^{\pm}$	[0,0]
		2	$x_{322}^{\pm}$	[0,0]
		3	$x_{323}^{\pm}$	[0,0]
	flow to incinerator 2 (ton/day)	1	$x_{331}^{\pm}$	[277,277]
		2	$x_{332}^{\pm}$	[302,302]
		3	$x_{333}^{\pm}$	[327,341]
$\lambda^{\pm}$				[0.035,0.959]
System cost(10^6)				[300.96, 576.534]

4.2 Model A

Since the system capacities and system cost are interval numbers, the violation limit used in this paper is a percentage of the mean value of the right hand side for each constraint and original objective function. In this paper, the largest violation limit is 30% of the whole system capacity or system cost because it will be impossible in practice if violation limits exceed 30%.

Solving the new constraint-relaxed IFMIP model A, the relationship between λ^- and the system cost can be obtained, under the various violation limits. These results, showed in table 5, indicate that with an increasing λ^-, the system cost decreases, but the risk of violating the system constraints increases; in comparison, with a lower λ^- level, the system cost is high, but the reliability in fulfilling system requirements increases.

Table 5 also shows how the type of expansion capacity changes with the different

Table 5: System cost, violation amount and λ^- levels

Violation limits (% system capacity)	2	3	5	8	10	15	20	30
λ^- levels	0.1841	0.2236	0.3095	0.3919	0.4441	0.5382	0.5989	0.6864
System costs ($million)	538.66	532.51	512.62	495.04	483.00	461.83	449.11	429.90
Violation amount								
Landfill (10^6 ton)	0.0114	0.0163	0.0242	0.0340	0.0389	0.0485	0.0562	0.0658
Incinerator 1 (ton/day)	1.83	2.62	3.88	5.47	6.25	7.79	9.03	10.58
Incinerator 2 (ton/day)	3.47	4.95	7.34	10.33	11.81	14.72	17.05	19.99
Binary variables with value 1 for:								
Landfill	Y_1	Y_1	Y_1	Y_1	Y_1	Y_1	Y_1	Y_1
Incinerator 1	Z_{211}							
Incinerator 2	Z_{321}	Z_{311},Z_{332}	Z_{331}	Z_{311},Z_{312}	Z_{321}	Z_{311}	Z_{311}	Z_{313}

Table 6: Reduced system cost under different λ^- levels

Violation limits (% system capacity)	1	3	4	5	8	10	15	20	30
λ^- levels	λ_0^-	λ_1^-	λ_2^-	λ_3^-	λ_4^-	λ_5^-	λ_6^-	λ_7^-	λ_8^-
	0.134	0.224	0.267	0.310	0.392	0.444	0.538	0.599	0.686
System costs	c_0	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
($million)	551.98	532.51	522.64	512.62	495.04	483.00	461.82	449.11	429.90
System costs		Δc_1	Δc_2	Δc_3	Δc_4	Δc_5	Δc_6	Δc_7	Δc_8
reduction ($million)		19.47	29.34	39.36	56.94	68.98	90.16	102.87	122.08
$\Delta\lambda^-$		$\Delta\lambda_1^-$	$\Delta\lambda_2^-$	$\Delta\lambda_3^-$	$\Delta\lambda_4^-$	$\Delta\lambda_5^-$	$\Delta\lambda_6^-$	$\Delta\lambda_7^-$	$\Delta\lambda_8^-$
		0.09	0.133	0.176	0.258	0.31	0.404	0.465	0.552
$\Delta c/\Delta\lambda^-$ ($million per unit of λ^-)		216.33	220.6	223.64	220.7	222.51	223.16	221.22	221.16

λ^-. With a increased λ^- value, the capacity expansion will generally decrease and expansion time will be delayed. The reason is that due to a increased λ^-, the right hand sides of constraints decrease, which would force a low flow rate, so that the capacity expansion would decrease or lag behind. At the same time the system cost will be low.

We use 9 scenarios to analyze the variations of system cost reduction under different λ^- levels. We use λ_0^- =0.134 (the violation limit is 1%) as the reference scenario. The data for the ratio of system costs reduction (Δc_m) to λ^- value variation ($\Delta\lambda_m^-$) is listed in Table 6. In this table, $\Delta c_m = c_0 - c_m$ and $\Delta\lambda_m^- = \lambda_m^- - \lambda_0^-$, $m = 1,2,\ldots,8$. These results indicate that the ratio ($\Delta c/\Delta\lambda^-$) generally increases as violation limits increase. When the λ^- value is low (less than or equal to 0.31), $\Delta c/\Delta\lambda^-$ increases faster than the ratio with a high λ^- value. As a result, a conclusion can be drawn that $\lambda^- \leq 0.31$ is not a good choice for decision makers. However, with a high λ^- value, the risk of violating system constraints increases. Thus, decision makers should be very careful to make a good compromise between system benefit and environmental objectives.

Table 7: λ^- levels, system costs and violation amount under mixed violation limit (violation limits vary only for waste generation constraints)

Violation limits (%capacity)	3	5	10	15	20
λ^- levels	0.3388	0.3686	0.4441	0.4951	0.5476
System costs($million)	518.126	508.184	483.00	465.978	448.471
Violation amount for waste generation rate	6.4467	10.26	18.07	24.61	29.41

Table 8: System costs and λ^- levels (violation limit is 5% of capacity/system cost)

Average α_l	$10\lambda^-$	$8\lambda^-$	$5\lambda^-$	$3\lambda^-$	λ^-	$0.5\lambda^-$	$0.25\lambda^-$
λ^- levels	λ_0^-	λ_1^-	λ_2^-	λ_3^-	λ_4^-	λ_5^-	λ_6^-
	0.0989	0.1222	0.1886	0.2678	0.3981	0.3993	0.4131
System costs	c_0	c_1	c_2	c_3	c_4	c_5	c_6
($million)	562.9501	560.8542	554.4566	531.54	493.8291	493.4884	489.4913
Δc		2.0958	8.4935	31.41	69.1209	69.4616	73.4587
$\Delta\lambda^-$		0.0233	0.0897	0.1689	0.2992	0.3004	0.3142
$\Delta c/\Delta\lambda^-$		89.833	94.688	185.968	231.02	231.23	233.796

In this method, decision makers can also choose violation limits as the different percentage of system capacity or system cost according to the actual desire. We analyze how system cost, violation amount and λ^- will change when only violation limits for waste generation constraints are different. These results are presented in Table 7. These provide useful information for decision makers: when they select the waste allocation patterns, what economy and the risk they should face.

4.3 Model B

We will continue to use the percentage of the mean value of the right hand side as violation limit for each constraint and original objective function. In this paper, the largest violation limit is 20% of the whole system capacity or system cost. We present Tables 8, 9 and 10 to analyze the relationship between system cost and λ^- levels. In these three tables, the violation limits are 5%, 10% and 15% of system cost or system capacity respectively. The three tables indicate that system cost will decrease with an increased λ^- level. We present the ratio of system cost reduction (Δc_i) to λ^- value variation ($\Delta\lambda_i^-$) under three different violation limit (i.e., 5%, 10% and 15%). In these table, $\Delta c_i = c_0 - c_i$ and $\Delta\lambda_i^- = \lambda_i^- - \lambda_0^-$ $(i = 1, 2, \ldots, 6)$. These results indicate that the ratio ($\Delta c/\Delta\lambda^-$) will increase when λ^- increases. When λ^- is less than 0.2, different violation limits have little influence on it. After that, the ratio shows that, with an increased violation limit, the ratio will decrease. The reason is that large violation amounts will lead to a high λ^- value and a low system cost. Therefore, decision makers should be more careful: if they want to choose a low system cost, that means they must face a high risk of violating system constraints.

The violation amount for each constraint at a certain λ^- level can be quantified

Table 9: System costs and λ^- levels (violation limit is 10% of capacity/system cost)

Average α_l	$10\lambda^-$	$8\lambda^-$	$5\lambda^-$	$3\lambda^-$	λ^-	$0.5\lambda^-$	$0.25\lambda^-$
λ^- levels	λ_0^-	λ_1^-	λ_2^-	λ_3^-	λ_4^-	λ_5^-	λ_6^-
	0.0994	0.1236	0.1944	0.3137	0.6093	0.7135	0.7184
System costs	c_0	c_1	c_2	c_3	c_4	c_5	c_6
($million)	562.902	560.702	554.4	540.373	454.842	424.674	423.263
Δc		2.2	8.502	22.529	108.082	138.228	139.639
$\Delta\lambda^-$		0.0241	0.095	0.2143	0.5099	0.6141	0.619
$\Delta c/\Delta\lambda^-$		91.13	89.523	105.128	211.98	225	225.588

Table 10: System costs and λ^- levels (violation limit is 15% of capacity/system cost)

Average α_l	$10\lambda^-$	$8\lambda^-$	$5\lambda^-$	$3\lambda^-$	λ^-	$0.5\lambda^-$	$0.25\lambda^-$
λ^- levels	λ_0^-	λ_1^-	λ_2^-	λ_3^-	λ_4^-	λ_5^-	λ_6^-
	0.0994	0.124	0.1962	0.3209	0.7413	0.9766	1
System costs	c_0	c_1	c_2	c_3	c_4	c_5	c_6
($million)	562.885	560.69	554.24	541.05	439.15	370.68	363.90
Δc		2.194	8.645	21.835	123.735	192.20	198.99
$\Delta\lambda^-$		0.0246	0.0968	0.2215	0.6419	0.8772	0.9
$\Delta c/\Delta\lambda^-$		89.16	89.34	98.59	192.77	219.13	221.1

by solving this model. Table 11 lists the relationship among λ^- levels, system costs and violation amount when the violation limit is 15% of the system cost or system capacity (in this table we only show the value of α_l that is not equal to 1, because, if $\alpha_l = 1$, then there is no violation amount on that constraint). This table shows that λ^- will increase when the violation amount is increased. The violation amount will be used in the following order: first, the violation amount for the system cost constraint will be used; when it reaches to its limit, the violation amount for the daily waste generation constraints begin to join it; then the violation amount for the capacity of landfill constraint in period 3 will be added; and finally, the violation amount for some incinerator constraints will be added.

Table 11 also lists the change of the capacity expansion type with the different λ^- levels and the corresponding violation amount. With the greater violation amount needed by constraints, λ^- level will increase and the right hand side will decrease. This will force the left hand side (flow rate) to decrease too. Therefore, the expansion of capacity will decrease or lag behind. This conclusion is the same as the one obtained by model A.

In model B, we also could find that λ^- value has a certain upper bound depend on violation limits. Here, we use the violation limit of 5% of system cost/system capacity for all constraints as an example. In Table 12, since almost all violation amounts, which reach their limits, are used, λ^- obtains an upper bound of value 0.4131 at this situation.

Since the original objective function is to minimize the system cost, the best

Table 11: System costs, α_l values and λ^- levels (violation limit is 15% of capacity/system cost. The capacity expansion for landfill will occur in period 1)

Average α_l	$10\lambda^-$	$8\lambda-$	$5\lambda^-$	$3\lambda^-$	λ^-	$0.5\lambda^-$	$0.25\lambda^-$
λ^- levels	0.0994	0.124	0.1962	0.3209	0.7413	0.9766	1
System costs ($million)	562.885	560.69	554.24	541.05	439.15	370.68	363.90
Binary variables with value 1	Z_{232},Z_{321}	Z_{232},Z_{321}	Z_{232},Z_{321}	Z_{212},Z_{331}	Z_{311}		
α_l	α_1 =0.892	α_1 =0.85	α_1 =0.64	α_1 =0.29	α_1 =0 α_{13} =0.26 α_{14} =0.96 α_{15},α_{16} =0 α_{18} =0.87 α_{19} =0	α_1 =0 α_{11} =0.28 α_{12},α_{13} =0 α_{14},α_{15} =0 α_{16},α_{17} =0 α_{18},α_{19} =0	α_1,α_4 =0 α_8 =0.8 α_7,α_9 =0 α_{10},α_{11} =0 α_{12},α_{13} =0 α_{14},α_{15} =0 α_{16},α_{17} =0 α_{18},α_{19} =0

Table 12: Relationship between λ^- and α_l (violation limit is 5% of the capacity/system cost)

Average λ_l	$10\lambda^-$	$5\lambda^-$	λ^-	$0.5\lambda^-$	$0.25\lambda^-$	$0.2\lambda^-$
λ^- levels	0.0989	0.1886	0.3981	0.3993	0.4131	0.4131
α_l	α_1 =0.79	α_1 =0 α_{16} =0.92	α_1,α_4 =0 α_{10} =0.56 α_{11},α_{12} =0 α_{13},α_{14} =0 α_{15},α_{16} =0 α_{17},α_{18} =0 α_{19}	α_1,α_4 =0 α_9 =0.79 α_6,α_7 =0 $\alpha_8\alpha_{10}$ =0 α_{11},α_{12} =0 α_{13},α_{14} =0 α_{15},α_{16} =0 α_{17},α_{18} =0 α_{19} =0	$\alpha_1,\alpha_4,\alpha_5$ =0 α_2 =0.96 α_6,α_7 =0 α_8,α_9 =0 α_{10},α_{11} =0 α_{12},α_{13} =0 α_{14},α_{15} =0 α_{16},α_{17} =0 α_{18},α_{19} =0	$\alpha_1,\alpha_4,\alpha_5$ =0 α_2 =0.96 α_6,α_7 =0 α_8,α_9 =0 α_{10},α_{11} =0 α_{12},α_{13} =0 α_{14},α_{15} =0 α_{16},α_{17} =0 α_{18},α_{19} =0

choice for decision makers is to reduce the violation limit for the system cost constraint. Using model B, we can solve different violation limits for each constraints to achieve the best results. Table 13 shows the solutions based on a violation limit of 15% of system capacity, 5% of system cost, and 10% of daily generation rate for waste generation constraints.

Tables 10 and 13 indicate that λ^- will decrease and the corresponding system cost will increase, since violation limit are cut down. It means that if decision makers want a low risk of violating system constraints, they may face a high system cost. The three tables also present that, since the violation limits decrease, more constraints will need the violation amount.

4.4 Comparison of the two new IFMIP models

Generally speaking, the results from these two new models are consistent. When the violations amount increase, λ^- will increase, the system cost will decrease and the expansion for capacity will decrease or defer. However, model B allows more

Table 13: System costs and λ^- levels (violation limits are 15% of capacity, 5% of system cost and 10% of waste generation rate. The capacity expansion for landfill will occur in period 1)

Average α_l	$10\lambda^-$	$8\lambda^-$	$5\lambda^-$	$2\lambda^-$	λ^-	$0.5\lambda^-$	$0.2\lambda^-$
λ^- levels	0.0989	0.1222	0.1890	0.3829	0.5473	0.6162	0.6198
System costs ($million)	562.950	560.854	554.349	498.224	450.657	430.723	429.691
Binary variables with value 1	Z_{232},Z_{321}	Z_{232},Z_{321}	Z_{232},Z_{321}	Z_{311}, Z_{332}	Z_{311}	Z_{313}	Z_{313}
α_l	α_1 =0.79	α_1 =0.58	α_1 =0	α_1 =0	α_1 =0	α_1, α_4 =0	α_1, α_4 =0
			α_{16} =0.95	α_{13} =0.55	α_4 =0.69	α_8 =0.85	α_5 =0.36
				α_{15}, α_{16} =0	α_{12}, α_{13} =0	α_9, α_{10} =0	α_6, α_7 =0
				α_{19} =0	α_{14}, α_{15} =0	α_{11}, α_{12} =0	α_8, α_9 =0
					α_{16} =0	α_{13}, α_{14} =0	α_{10}, α_{11} =0
					α_{17} =0.71	α_{15}, α_{16} =0	α_{12}, α_{13} =0
					α_{18}, α_{19} =0	α_{17}, α_{18} =0	α_{14}, α_{15} =0
						α_{19} =0	α_{16}, α_{17} =0
							α_{18}, α_{19} =0

depth in which to analyze the information for violations than model A. Since model A only uses one λ^-, it would make every constraint have violation amount, even if some of them did not need violation amount at all. For example, in table 13, when the average α_l is greater than or equal to λ^-, since the landfill has a capacity expansion at period 1, there is no violation amount for the landfill constraint in period 1 and 2 (i.e. $\alpha_2 = 1$ and $\alpha_3 = 1$), and there is only violation amount ($0.0328 * 10^6$ ton) at period 3. At the same situation, since incinerator 2 has a expansion of 100 tons/day in period 1, three constraints for incinerator 2 have no violation amount.

5 Conclusions

In this study, we reviewed some interval parameter programming models. We then proposed two new interval parameter fuzzy programming models based on the interval parameter violation analysis approach. In these models, the fixed violation limit is introduced for each constraint. Then we applied these models to the hypothetical case of PSWM system. After solving this problem, a number of decision alternatives under various system conditions were generated. These results provided useful information for decision makers to help them to identify desirable waste flow allocation patterns. They also provided the relationship between system benefit and reliability as well as the risk they may have to face. And also, we compared the two models, the conclusion is that these two models have the same trend for analyzing the generated alternatives. However model B allows decision makers to get more in-depth information.

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