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Newsvendor Bounds and Heuristics for Optimal Policy of Serial Supply Chains with and without Expedited Shippings*

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Abstract We consider N-stage serial production/distribution systems with stationary demand at the most downstream stage. First, we study classical Clark-Scarf model with both average cost and discounted cost criteria. The optimal echelon base-stock levels are obtained in terms of only probabilistic distributions of leadtime demand. This analysis yields a novel approach for developing bounds and simple heuristics for optimal echelon base-stock polices. In addition to deriving known bounds, we develop several new upper bounds for both average cost and discounted cost models. Second and more important, we extend this idea to a more general model with two transportation modes between stages: the regular and expedited shippings (Lawson and Porteus (2000)). The optimal inventory policy for this system is known to be echelon base-stock policy, which can be computed through minimizing $2N$ nested convex functions recursively. We again develop simple newsvendor type of lower and upper bounds for the optimal control parameters, as well as a simple near optimal heuristic. Extensive numerical results show that the heuristics for both models perform well. The bounds and heuristic enhance the accessibility and implementability of the optimal policies in supply chains with single and dual transportation modes.

1 Introduction

We consider serial periodic-review production/distribution systems with N stages. Stochastic customer demand arises at stage 1, stage 1 replenishes from stage 2, stage 2 replenishes from stage 3, etc., and stage N replenishes its inventory from an outside supplier with ample stock. The random demands in different periods are independent and identically distributed. When demand in a period exceeds the stock level at stage 1, the excess is backlogged.

In the classcial Clark and Scarf (1960) serial system, there is only one transportation mode between each stage. Lawson and Porteus (2000) (see also Muharremoglu and Tsitsilkis (2003)) consider a system with two modes of transportation

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between any two adjacent stages, referred to as regular shipping and expedited shipping, with transportation leadtimes being 1 and 0 respectively. The expedited shipping cost is higher than the regular shipping cost. The 0 leadtime allows one to ship a product from any stage to stage 1 in no time, if needed, by using expedited shipping between stages. There is a linear holding cost at each stage, and a linear shortage cost at stage 1 when shortage occurs, and the problem is to minimize the total discounted cost over the planning horizon. They shown that echelon base-stock policies are optimal.

In this chapter, we start from the classical Clark-Scarf system by further studying the recursive computation of the echelon base-stock levels, for both average cost and discounted total cost criteria. The optimal echelon base-stock levels are obtained in terms of only probability distributions of leadtime demand. This analysis yields a novel approach for developing bounds and simple heuristics for optimal echelon base-stock polices. To illustrate, we show how the bounds of Zipkin (2000) and Shang and Song (2003) can be obtained from these equations. The explicit equations allow us to obtain new bounds for the optimal echelon base-stock levels for serial inventory systems with both average cost and discounted cost criteria in a unified manner.

Next, we study the infinite horizon problem of the model that includes two transportation modes with the objective of minimizing the total discounted cost. It is known that the optimal stationary policies of this system can be obtained by solving $2N$ nested convex optimization problems recursively. Despite its simple form, however, it is not easy to see the key determinants of the optimal policy and minimum cost from the recursion. Therefore, in this chapter we aim at developing simple newsvendor bounds and heuristics for the optimal inventory control policies that can shed light on the effect of system parameters.

Multi-echelon inventory system is a fundamental model for supply chain management and has been studied extensively since the seminal work of Clark and Scarf (1960), who show that an echelon base-stock policy is optimal for a finite horizon problem. Federgruen and Zipkin (1984) extend this result to infinite horizon and prove the optimality of a stationary order-up-to level policy. Chen and Zheng (1994) present a simple lower-bound approach to prove the optimality of echelon base-stock policy for the average cost criterion, and Chen (2000) further extends the result to the batch ordering case. See Gallego and Zipkin (1999) for a detailed summary and a review of the relevant literature.

There has been a number of papers in the literature on constructing simple bounds of cost or optimal policies for inventory systems. Zheng (1992) studies stochastic (Q,R) model and presents bounds for optimal ordering quantity for single stage problem, and Gallego (1998) develops closed form distribution-free bounds for (Q,R) policies. Glasserman (1999) establishes bounds and asymptotics for performance measures for single as well as serial capacitated system. Hopp et al. (1997) suggest an easily implementable heuristic for one-warehouse multiple-retailer systems, which is a simple closed form solution of the system control parameters. Gallego and Zipkin (1999) discuss the issue of stock positioning and construct three heuristics to calculate the system average cost. Zipkin (2000) introduces lower bound for a two-stage system by restricting the possibility of holding inventory at the upper stream stage. Dong and Lee (2003) develop lower bounds for optimal policies of serial system with discounted cost criterion, while Shang and Song (2003) obtain simple newsvendor type of bounds and develop simple heuristics for serial systems with average cost criterion, using a different approach than that of Dong and Lee. Chao and Zhou (2004) present another approach for constructing bounds and heuristics for serial systems, and they obtain a series of upper and lower bounds for optimal base-stock levels. Another related work on bounds and heuristics for serial systems is Gallego and Ozalp (2004). In all these references only one transportation mode is considered between stages.

Another stream of research that closely relates to ours is on optimality of control policies for single- and multi-stage inventory models with single and multiple supply modes.

The earliest work on inventory models with two delivery modes can be traced back to Barankin (1961), who studies a single period problem. Daniel (1963) is regarded as the first work on a multi-period single-stage model with one regular supplier and one emergency supplier, with leadtimes being 1 and 0 respectively. Fukuda (1964) extends the work of Daniel to the case where the leadtimes of the two supply modes are L and $L+1$ respectively. Whittmore and Saunders (1977) consider the dual-supplier problem with arbitrary length of leadtimes and demonstrate that the optimal control policy is very complicated and state-dependent if the difference in leadtimes is greater than 1. Because of the complexity of the optimal policy, Scheller-Wolf et al. (2003) and Veeraraghavan and Scheller-Wolf (2004) focus on evaluation and optimization of two classes of policies, i.e., “single index" and “dual index" policies. These studies only focus on single stage inventory system. Other related work on single-stage inventory systems with multiple transportation modes includes Feng et al. (2003) and Feng et al. (2004).

For multi-echelon serial systems with the option of expedited shipping between stages, Lawson and Porteus (2000) consider serial systems with dual transportation modes. Under the assumptions that the leadtime difference of regular and expedited supply is one period for each stage and additive linear shipping cost, they obtain the form of optimal inventory control policy. The control parameters of each echelon consist of two numbers, one for regular shipping and the other for expedited shipping. Muharremoglu and Tsitsiklis (2003) extend Lawson and Porteus’ second assumption by introducing “supermodular" shipping cost structure and characterize the optimal policy as extended echelon base-stock type.

Supply chain models with multiple transportation modes have gained momentum in recent years due to the increasing popularity of outsourcing, see for example McMillan (1990), Venkatesan (1992), and van Mieghem (1999). Cost and leadtime are two important measures of the suppliers for outsourcing. A supplier who provides shorter leadtime usually has higher price. To balance this tradeoff, companies often adopt multiple sourcing strategies by sharing its business with multiple suppliers. Consequently, companies need to strategically determine the ordering quantity from each supplier based on its inventory status and demand forecast in order to minimize cost. The strategic importance of utilizing multiple suppliers with long and short lead time was first recognized by the US fashion industry. Many firms in this sector have moved their major manufacturing facilities offshore to take advantage of the lower production cost. However, some still prefer to maintain costly domestic facilities so that they can better respond to changes in market demand. The combination of 'quick-response, or short leadtime' suppliers with 'low cost, long leadtime' suppliers has been viewed by many as an appropriate strategy to meet fickle customer demand. Some references in this area are Fisher, et al. (1994), Fisher and Raman (1996), Eppen and Iyer (1997), Donohue (2000), and Haksoz and Seshadri (2004).

There are two main technical differences between the systems with regular and expedited shippings and the classical serial inventory system with one transportation mode. First, the optimal inventory replenishment policy is no longer myopic but instead it is one period ahead, which increases not only the complexity of computation of the optimal base-stock levels but also the difficulty of developing bounds for the optimal echelon base-stock levels. Second, there are two types of induced penalty cost functions, one is between stages and the other is within stage. In this chapter we develop several lower and upper newsvendor simple bounds for the optimal echelon base-stock levels for the multi-echelon system. Based on these bounds we develop a simple heuristic. Numerical studies show that the heuristic performs very well. To simplify the exposition and without loss of generality, we assume leadtime for regular and expedited supply is 1 and 0, respectively. If leadtime between stages is not 1, say L, then it can be covered by inserting $L-1$ stages each representing work in progress (WIP). In the resulting model there will be L options to reduce the shipping time, to $\ell = 0, 1, \ldots, L-1$ time units respectively, and the costs for these different options satisfy additivity structure. See Lawson and Porteus (2000) and Muharremoglu and Tsitsilkis (2003) for more discussion.

The rest of the chapter is organized as follows. In Section 2, we present the probabilistic solution, bounds and heuristic for the optimal policy of Clark-Scarf model with average and discounted cost criteria. In Section 3, we develop the bounds and heuristic for the serial system with regular and expedited shippings. We conclude the chapter with some discussion in Section 4. All of the technical proofs are refered to Chao and Zhou (2004) and Zhou and Chao (2006).

Throughout the chapter, we use “expedited order" and “expedited supply" interchangeably. Furthermore, we use “increasing" and “decreasing" in a non-strict sense, i.e., they represent “non-decreasing" and “non-increasing" respectively. For any real numbers a and b, $a \wedge b = \min\{a,b\}$, $a \vee b = \max\{a,b\}$, $a^+ = \max\{a,0\}$ and $a^- = \max\{-a,0\}$.

2 Classical Serial System

In this section, we study the classical serial system with both average cost and discounted cost.

2.1 Probabilistic Solution

Consider a continuous-review single item serial inventory system with N stages. The demand process is compound Poisson with continuous demand sizes. Demand only occurs at stage 1. Stage 1 places order from stage 2, stage 2 orders from stage 3, etc., and stage N orders from the outside with ample supply. There are constant transportation time between stages, and unsatisfied demand is fully backlogged at stage 1.

The following notation will be used for stage $i = 1,2,\ldots,N$:

$L_i =$ the leadtime between stage i to stage $i+1$,

$D_i =$ the leadtime demand during L_i units of leadtime,

$F_i(.) =$ the distribution function of D_i,

$y_i =$ echelon inventory position at stage i after ordering,

$h_i =$ echelon i inventory holding cost rate,

$H_i =$ installation i inventory holding cost rate, i.e., $H_i = \sum_{j=i}^{N} h_j$,

$IP_i =$ echelon inventory position of stage i,

$b =$ backorder cost rate at stage 1.

For convenience we let $L_{i,j}$ represent the leadtime between stage i and stage $j+1$, i.e., $L_{i,j} = \sum_{k=i}^{j} L_k$, let $D_{i,j}$ represent the demand during leadtime $L_{i,j}$, i.e., $D_{i,j} = \sum_{k=i}^{j} D_k$, let $F_{i,j}$ be the distribution function of $D_{i,j}$, and $\bar{F}_{i,j} = 1 - F_{i,j}$. Clearly, $L_{i,i} = L_i, D_{i,i} = D_i, F_{i,i} = F_i$ and they will be used interchangeably. Let $L_{i,j} = 0, D_{i,j} = 0$ for $j < i$.

Echelon base-stock policy is known to be optimal for this system with both average cost and discounted total cost criteria (see Federgruen and Zipkin (1984) and Chen and Zheng (1994)).

We first consider the case of minimizing total discounted cost with discount factor α, the algorithm for computing the optimal base-stock levels is: Let $G_0^1(x) = (H_1 + b)x^-$. For $j = 1,2,\ldots,N$, compute

$$G_i(x) = \alpha^{L_i} h_i E(x - D_i) + \alpha^{L_i} E[G_{i-1}^{i}(x - D_i)], \tag{1}$$

$$s_i^* = \arg\min G_i(x), \tag{2}$$

$$G_i^{i+1}(x) = G_i(x \wedge s_i^*). \tag{3}$$

To the best of our knowledge, no recursive equations have ever been reported in the literature for serial inventory model with discounted cost criterion. The following result gives the optimal base-stock levels for discounted cost criterion in terms of the leadtime demand distributions.

Proposition 1. *Assuming $s_1^*, \ldots, s_{i-1}^*$, the optimal echelon base-stock level for stage i, s_i^*, for $i = 1, 2, \ldots, N$, is the solution of*

$$h_i + \sum_{j=1}^{i-1} \alpha^{L_{j,i-1}} h_j P(D_{j+1,i} \geq y - s_j^*, D_{j+2,i} \geq y - s_{j+1}^*, \ldots, D_{i,i} \geq y - s_{i-1}^*)$$
$$- \alpha^{L_{1,i-1}} (H_1 + b) P(D_{1,i} \geq y, D_{2,i} \geq y - s_1^*, \ldots, D_{i,i} \geq y - s_{i-1}^*) = 0. \quad (4)$$

The left hand side of the equation is increasing in y.

Proposition 1 presents explicit form dependency of the optimal inventory control strategies on its determinants. In the next section we will see how these results can be applied to develop simple upper and lower bounds for the optimal control parameters.

For the average cost case, it turns out that the probabilistic solution for optimal echelon base-stock levels is also given by (4) except that we need to set $\alpha = 1$. We remark that the probabilistic solution for the average cost case is related to van Houtum, et al. (1996) and van Houtum and Zijm (1991), who also study serial inventory model with average cost. However, equation (4) with $\alpha = 1$ is more succinct that is amenable for developing bounds and heuristics as we will demonstrate in the next section.

2.2 Bounds

The explicit results of optimal echelon base-stock levels given in the last section can be used to derive simple bounds for optimal control parameters. The idea is simple: If we approximate the left hand side of (4) by another function which yields a simple solution, then the solution serves as an approximation for s_i^*. Furthermore, since the left hand side of (4) is increasing in y, if we approximate the left hand side by a smaller increasing function, say $\bar{g}(y)$, then $\bar{g}(y) = 0$ will give an upper bound for s_i^*; while if we approximate the left hand side of (4) by a larger, increasing function $\underline{g}(y)$, then $\underline{g}(y) = 0$ will give a lower bound for s_i^*.

We will first present the lower and upper bounds for the serial system with discounted cost criterion.

Proposition 2. *An upper bound for the optimal echelon base-stock level of serial inventory model with total discounted cost criterion is, if*

$$\alpha^{L_{1,i-1}} (H_1 + b) - \sum_{j=1}^{i-1} \alpha^{L_{j,i-1}} h_j > h_i, \quad (5)$$

$$s_i^u = \bar{F}_{1,i}^{-1} \left(\frac{h_i}{\alpha^{L_{1,i-1}} (H_1 + b) - \sum_{j=1}^{i-1} \alpha^{L_{j,i-1}} h_j} \right), \quad i = 1, \ldots, N, \quad (6)$$

and otherwise

$$s_i^u = 0, \qquad i = 1, \ldots, N.$$

Another upper bound for the discounted cost case is, if (5) is satisfied then

$$\hat{s}_i^u = s_i^* + \bar{F}_i^{-1}\left(\frac{h_i}{\alpha^{L_{1,i-1}}(H_1+b) - \sum_{j=1}^{i-1}\alpha^{L_{j,i-1}}h_j}\right), \quad i = 1,\ldots,N, \tag{7}$$

and (5) is not satisfied then $\hat{s}_i^u = 0$. *Inductively we obtain upper bound,*

$$\tilde{s}_i^u = \sum_{j=1}^{i}\bar{F}_j^{-1}\left(\frac{h_j}{\alpha^{L_{1,j-1}}(H_1+b) - \sum_{k=1}^{j-1}\alpha^{L_{k,j-1}}h_k}\right), \quad i = 1,\ldots,N,$$

where $\bar{F}_j^{-1}(x)$, $j = 1,\ldots,i$, *is understood as 0 if either* $x \geq 1$ *or* $x \leq 0$. *And a lower bound for the optimal echelon base-stock level is*

$$s_i^l = \bar{F}_{1,i}^{-1}\left(\frac{\sum_{j=1}^{i}\alpha^{-L_{1,j-1}}h_j}{H_1+b}\right), \qquad i = 1,\ldots,N. \tag{8}$$

We note that the lower bound for the discounted cost case has been obtained by Dong and Lee (2003). The upper bound is, to the best of our knowledge, new.

Remark 1. It can be easily demonstrated by some examples that the two upper bounds, i.e. (6) and (7), do not have a dominating relationship, hence anyone can be a better bound, depending on the instance.

Remark 2. It can be seen from the proof of Proposition 2 (see Chao and Zhou (2004)) that, for any $1 \leq k \leq i$, we have

$$\begin{aligned} h_i + \sum_{j=1}^{i-1}\alpha^{L_{j,i-1}}h_j P(D_{j+1,i} \geq y - s_j^*, D_{j+2,i} \geq y - s_{j+1}^*, \ldots, D_i \geq y - s_{i-1}^*) \\ - \alpha^{L_{1,i-1}}(H_1+b)P(D_{1,i} \geq y, D_{2,i} \geq y - s_1^*, \ldots, D_i \geq y - s_{i-1}^*) \\ \geq h_i - \Big(\alpha^{L_{1,i-1}}(H_1+b) - \sum_{j=1}^{i-1}\alpha^{L_{j,i-1}}h_j\Big)P(D_{1,k} \geq y - s_{k-1}^*). \end{aligned}$$

Therefore we have a sequence of upper bounds for s_i^*: For $k = 1,\ldots,i$,

$$\hat{s}_i^u = s_{k-1}^* + \bar{F}_{1,k}^{-1}\left(\frac{h_i}{\alpha^{L_{1,i-1}}(H_1+b) - \sum_{j=1}^{i-1}\alpha^{L_{j,i-1}}h_j}\right), \qquad i = 1,\ldots,N,$$

where s_0^* is understood as 0.

For the average cost case, we can obtain similar results by letting $\alpha = 1$ in the proceeding analysis and so we only summarize the results and omit the proof. Proposition 3 has been obtained by Shang and Song (2003) using a completely different method. The second result, Proposition 4, is new.

Proposition 3. *An upper bound for the optimal echelon base-stock level of serial inventory model with average cost criterion is*

$$s_i^u = \bar{F}_{1,i}^{-1}\left(\frac{h_i}{\sum_{j=i}^N h_j + b}\right), \qquad i = 1, \dots, N, \tag{9}$$

and a lower bound for the optimal echelon base-stock level is

$$s_i^l = \bar{F}_{1,i}^{-1}\left(\frac{\sum_{j=1}^i h_j}{\sum_{j=1}^N h_j + b}\right), \qquad i = 1, \dots, N. \tag{10}$$

Proposition 4. *An upper bound for s_i^* is*

$$\hat{s}_i^u = s_{i-1}^* + \bar{F}_i^{-1}\left(\frac{h_i}{\sum_{j=i}^N h_j + b}\right), \qquad i = 1, \dots, N. \tag{11}$$

Inductively we obtain another simple upper bound for s_i^*

$$\tilde{s}_i^u = \sum_{j=1}^{i} \bar{F}_j^{-1}\left(\frac{h_j}{\sum_{k=j}^N h_k + b}\right), \qquad i = 1, \dots, N. \tag{12}$$

Remark 3. One might also wish to obtain a lower bound for s_i^* in the form of s_{i-1}^* plus a nonnegative number. We argue that this is not possible. It is known that the solution obtained from the computational algorithm (1), (2) and (3) may not satisfy relationship $s_1^* \le s_2^* \le \cdots \le s_N^*$, see for example Gallego and Ozer (2004). Hence if $s_i^* < s_{i-1}^*$ then s_{i-1}^* is already an upper bound for s_i^*. Thus it is not possible to give a lower bound of s_i^* in the form s_{i-1}^* plus a nonnegative number. It is also well-known that, in that case, we can define $\bar{s}_i^* = \min\{s_i^*, s_{i+1}^*, \dots, s_N^*\}$ to give an optimal policy that satisfies $\bar{s}_1^* \le \bar{s}_2^* \le \cdots \le \bar{s}_N^*$.

Based on these bounds, several heuristics can be constructed for the optimal base-stock levels, using the approaches of Shang and Song (2003) and Gallego and Ozer (2004). For example, we can just simply use average of s_i^l and s_i^u,

$$s_i^h = \frac{s_i^u + s_i^l}{2}$$

to approximate s_i^*. And s_i^u is the smaller one of the two upper bounds, i.e., $\min\{s_i^u, \tilde{s}_i^u\}$. If the average is not an integer, we can either round up or round down the value to obtain the nearest integer value for s_i^h. Our numerical studies show that the heuristic works very well. Extensive numerical results for bounds and heuristics are reported in Zhou (2006).

3 Serial System with Regular and Expedited Shppings

In this section, we first derive the probabilistic solution for the optimal base-stock leverls , then develop lower and upper bounds as well as a simple heuristic for the serial system with regular and expedited shippings.

3.1 Probabilistic Solution

Consider an infinite-horizon periodic-review serial inventory system with dual transportation modes. There are N stages, denoted by $1,2,\ldots,N$, stage i orders from stage $i+1$ ($i=1,\ldots,N-1$), and stage N orders from an external supplier with unlimited stock. There are two ordering decisions at each stage: Expedited order and regular order. Demand occurs only at stage 1, and excess demand is fully backlogged at stage 1. The regular order has leadtime 1, and the expedited order has leadtime 0. The demands in different periods are i.i.d. random variables. At the beginning of each period, the firm decides the ordering quantities for two supply options at each stage. The objective is to minimize the total discounted cost over an infinite planning horizon.

The events sequence is as follows: First at the beginning of the period, each stage receives the regular order placed in the previous period; second, expedited order is placed from its upstream stage which is delivered immediately; third, regular order is placed from the upstream which will be delivered at the beginning of next period; finally, demand is realized at stage 1 and all costs are calculated.

For $i=1,2,\ldots,N$, define:

$x_i =$ initial echelon inventory level at stage i;

$y_i^E =$ echelon inventory level at stage i after placing the expedited order;

$y_i^R =$ echelon inventory position at stage i after placing the regular order;

$\bar{c}_i^E =$ unit expedited shipping cost from stage $i+1$ to stage i;

$\bar{c}_i^R =$ unit regular shipping cost from stage $i+1$ to stage i, i.e., $\bar{c}_i^R < \bar{c}_i^E$;

$h_i =$ unit echelon i inventory holding cost per period;

$H_i =$ unit installation i inventory holding cost per period, s.t., $H_i = \sum_{j=i}^{N} h_j$.

$b =$ unit demand backlog cost per period;

$D_j =$ demand in period j, $j=1,2,\ldots$;

$D =$ generic one-period demand;

$F(.) =$ cumulative distribution function of D;

$\bar{F}(.) = 1-F(.)$;

$D(j) =$ j-period demand, $j=1,2,\ldots$;

$F_j(.) =$ cumulative distribution function of $D(j)$, $j=1,2,\ldots$;

$\bar{F}_j(.) = 1-F_j(.)$, $j=1,2,\ldots$;

$\alpha =$ the discount factor, i.e., $0<\alpha\leq 1$.

As explained in Lawson and Porteus (2000), x_i denotes the sum of on-hand stock from stages 1 to i, less the backlog at stage 1; y_i^E is the echelon stock level of stage i after all expediting at stage i and upstream stages, but before expediting into stage stage $i-1$, has taken place. Similarly, y_i^R is the echelon inventory position of stage i after both expedited order and regular order are placed from stage $i+1$. Clearly, $y_i^R - y_i^E \geq 0$ represents the number of regular units placed into the regular flow from stage $i+1$, while $y_i^E - x_i \geq 0$ represents the number of units expedited to stage i from stage $i+1$. Since a product can be expedited to stage i in no time through expedition between stages, $y_i^E - x_i$ has no upper limit. Also, note that $D(1) = D$ and $F_1 = F$. The state of the system at the beginning of a period, before any decision is made, is $\mathbf{x} = (x_1, \ldots, x_N)$.

The following result is easily verified, and it is originally due to Karush (1958).

Lemma 5. *Let $g(x)$ be a convex function with a minimizer s, then for any $x \le y$,*

$$\min_{x \le z \le y} g(z) = g(x \vee s) + g(y \wedge s) - g(s).$$

Note that $g(x \vee s)$ is an increasing convex function of x while $g(y \wedge s)$ is a decreasing convex function of y.

Let $f(\mathbf{x})$ be the minimum expected total discounted cost given initial echelon inventory level $\mathbf{x} = (x_1, x_2, \ldots, x_N)$. Let $L_1(x) = h_1 E[x-D] + (H_1 + b)E[(x-D)^-]$ and $L_i(x) = h_i E[x-D]$ for $i > 1$. The optimality equation is

$$f(\mathbf{x}) = \min_{x_i \le y_i^E \le y_i^R \le y_{i+1}^E} \left\{ \sum_{i=1}^{N} (\bar{c}_i^E (y_i^E - x_i) + \bar{c}_i^R (y_i^R - y_i^E) + L_i(y_i^E)) \right. \\ \left. + \alpha E[f(\mathbf{y}^R - D)] \right\}, \quad (13)$$

where $\mathbf{y}^R = (y_1^R, \ldots, y_N^R)$ and $\mathbf{y}^R - D = (y_1^R - D, \ldots, y_N^R - D)$. For ease of exposition, we shift the cost $-\bar{c}_i^E x$ to the previous period and after some simple algebra, we obtain

$$f(\mathbf{x}) = \min_{x_i \le y_i^E \le y_i^R \le y_{i+1}^E} \left\{ \sum_{i=1}^{N} ((\bar{c}_i^E - \bar{c}_i^R) y_i^E + \alpha c_i^E E[D] + (\bar{c}_i^R - \alpha \bar{c}_i^E) y_i^R \right. \\ \left. + h_i E[(y_i^E - D)]) + L_1(y_1^E) + \alpha E[f(\mathbf{y}^R - D)] \right\}.$$

Let $c_i^E = \bar{c}_i^E - \bar{c}_i^R + h_i > 0$ and $c_i^R = \alpha \bar{c}_i^E - \bar{c}_i^R > 0$, the reason for $c_i^R > 0$ is that otherwise the regular order will never be used and the model collapses to the one with single supply mode which is not the interest of this section. We call c_i^E the relative unit expedited ordering cost and c_i^R the relative unit regular ordering cost (in the rest of the section, we may occasionally skip the "relative" for simplicity).

By suppressing the terms that do not affect the optimization, we can finally rewrite the optimality equation as,

$$f(\mathbf{x}) = \min_{x_i \le y_i^E \le y_i^R \le y_{i+1}^E} \left\{ \sum_{i=1}^{N} (c_i^E y_i^E - c_i^R y_i^R) + (b + H_1) E[(y_1^E - D)^-] \right. \\ \left. + \alpha E[f(\mathbf{y}^R - D)] \right\} \quad (14)$$

It can be shown that $f(\mathbf{x})$ is additively convex,

$$f(\mathbf{x}) = \sum_{i=1}^{N} f_i(x_i), \tag{15}$$

where each $f_i(x_i)$ is a convex function (see Lawson and Porteus (2000)). Let $G_1^E(y) = c_1^E y + (H_1 + b)E[(y-D)^-]$, which is a convex function with minimizer s_1^E, a finite number. Applying Lemma 1 to (14) yields $f_1(x_1) = G_1^E(x_1 \vee s_1^E)$. Let

$$G_{1,1}(y) = G_1^E(y \wedge s_1^E) - G_1^E(s_1^E) + \alpha E[G_1^E((y-D) \vee s_1^E)],$$

referred to as the induced penalty cost within the stage, and

$$G_1^R(y) = G_{1,1}(y) - c_1^R y.$$

Let s_1^R be the minimizer of convex function $G_1^R(.)$. Substituting (15) into (14) and applying Lemma 1 yields,

$$\begin{aligned}
&\min_{x_1 \le y_1^E \le y_1^R \le y_2^E} \{G_1^E(y_1^E) - c_1^R y_1^R + \alpha E[f_1(y_1^R - D)]\} \\
=&G_1^E(x_1 \vee s_1^E) + \min_{y_1^R \le y_2^E} \{-c_1^R y_1^R + G_{1,1}^E(y_1^R)\} \\
=&G_1^E(x_1 \vee s_1^E) + \min_{y_1^R \le y_2^E} G_1^R(y_1^R) \\
=&G_1^E(x_1 \vee s_1^E) + G_1^R(y_2^E \wedge s_1^R).
\end{aligned}$$

Let $G_{1,2}(y) = G_1^R(y \wedge s_1^R)$, also called induced penalty cost but which is between stages, and

$$G_2^E(y) = c_2^E y + G_{1,2}(y).$$

Let s_2^E be the minimizer of convex function $G_2^E(.)$. This process can be continued and we obtain, in general for $i \ge 1$, after G_i^E is defined with minimizer s_i^E, that

$$\begin{aligned}
G_{i,i}(y) =&G_i^E(y \wedge s_i^E) - G_i^E(s_i^E) + \alpha E[G_i^E((y-D) \vee s_i^E)], && (16)\\
G_i^R(y) =&G_{i,i}(y) - c_i^R y, && (17)\\
G_{i,i+1}(y) =&G_i^R(y \wedge s_i^R), && (18)\\
G_{i+1}^E(y) =&c_{i+1}^E y + G_{i,i+1}(y). && (19)
\end{aligned}$$

And that, for all $i \ge 1$,

$$f_i(x_i) = G_i^E(x_i \vee s_i^E).$$

Note that all these functions are convex, and in particular, $G_{i,i+1}$ is decreasing convex and f_i is increasing convex.

The optimal policy for this system is top-down echelon base-stock policies (Lawson and Porteus (2000)). The top-down base-stock policy works as follows. Starting from stage N, each stage tries to raise its echelon inventory position to the expedited order-up-to level s_i^E and regular order-up-to level s_i^R, taking upstream decisions as fixed and ignoring downstream decisions. More formally, a policy with $2N$ base-stock levels is a top-down base-stock policy if the actual decisions can be constructed from the base-stock levels as follows.

$$\begin{aligned} y_N^R &= s_N^R \vee x_N, \\ y_i^E &= s_i^E \vee x_i \wedge y_i^R \qquad i = 1,2,\ldots,N, \\ y_i^R &= s_i^R \vee x_i \wedge y_{i+1}^E \qquad i = 1,2,\ldots,N-1. \end{aligned}$$

Lemma 6. *(1) $s_i^E \le s_{i-1}^R$, for $i = 2,\ldots,N$.*
(2) $s_i^E \le s_i^R$, for $i = 1,\ldots,N$.

We can develop probabilistic solutions for the optimal base-stock levels s_i^E and s_i^R. From equations (16)-(19), the optimal control parameters s_i^E and s_i^R are, respectively, the solution of $(G_i^E(y))' = 0$ and $(G_i^R(y))' = 0$. Let $D_1, D_2, \ldots$ be demands in periods $1,2,\ldots$. For stage 1, taking derivative of $G_1^E(y)$ with respect to y yields

$$c_1^E - (H_1 + b)P(D_1 > y) = 0,$$

hence the optimal expedited base-stock level for stage 1 is

$$s_1^E = \bar{F}^{-1}\left(\frac{c_1^E}{H_1 + b}\right). \tag{20}$$

Note that if $c_1^E \ge H_1 + b$, then $s_1^E = -\infty$ and expedited shipping is never used at stage 1. To solve for s_1^R, it follows from Lemma 6 that we only need to consider the solution of $(G_1^R(y))' = 0$ on $y \ge s_1^E$. It follows from (16) that s_1^R is the solution of

$$-c_1^R + \alpha c_1^E P(D_2 \le y - s_1^E) - \alpha(H_1 + b)P(D_2 \le y - s_1^E, D_1 + D_2 > y) = 0. \tag{21}$$

Some further algebraic derivations yield that s_2^E is the solution of

$$\begin{aligned} c_2^E - c_1^R + c_1^E \mathbf{1}[y < s_1^E] + \alpha c_1^E P(D \le y - s_1^E) - (H_1 + b)P(D > y)\mathbf{1}[y < s_1^E] \\ - \alpha(H_1 + b)P(D \le y - s_1^E, D(2) > y) = 0, \end{aligned}$$

that s_2^R is the solution of

$$\begin{aligned} - c_2^R + \alpha c_2^E P(D \le y - s_2^E) - \alpha c_1^R(D \le y - s_2^E, D > y - s_1^R) \\ + \alpha c_1^E P(D \le y - s_2^E, D > y - s_1^R, D > y - s_1^E) \\ + \alpha^2 c_1^E P(D \le y - s_2^E, D > y - s_1^R, D(2) \le y - s_1^E) \\ - \alpha(H_1 + b)P(D \le y - s_2^E, D > y - s_1^E, D > y - s_1^R, D(2) > y)) \\ - \alpha^2(H_1 + b)P(D \le y - s_2^E, D > y - s_1^R, D(2) \le y - s_1^E, D(3) > y) = 0, \end{aligned}$$

and that s_3^E is the solution of

$$\begin{aligned}
c_3^E - c_2^R + (c_2^E - c_1^R)\mathbf{1}[y < s_2^E] + c_1^E\mathbf{1}[y < s_1^E, y < s_2^E] \\
+ \alpha c_1^E P(D \le y - s_1^E)\mathbf{1}[y < s_2^E] \\
+ \alpha c_2^E P(D \le y - s_2^E) - \alpha c_1^R (D \le y - s_2^E, D > y - s_1^R) \\
+ \alpha c_1^E P(D \le y - s_2^E, D > y - s_1^R, D > y - s_1^E) \\
+ \alpha^2 c_1^E P(D \le y - s_2^E, D > y - s_1^R, D(2) \le y - s_1^E) \\
- (H_1 + b)P(D > y)\mathbf{1}[y < s_1^E, y < s_2^E] \\
- \alpha(H_1 + b)P(D \le y - s_1^E, D(2) > y)\mathbf{1}[y < s_2^E] \\
- \alpha(H_1 + b)P(D \le y - s_2^E, D > y - s_1^R, D > y - s_1^E, D(2) > y)) \\
- \alpha^2(H_1 + b)P(D \le y - s_2^E, D > y - s_1^R, D(2) \le y - s_1^E, D(3) > y) = 0.
\end{aligned}$$

This process can be continued to give probability solutions for s_i^E and s_i^R for any i. As can be expected, the expression becomes extremely complicated as i increases. The detailed derivation of these equations see Zhou and Chao (2006).

The following lemma presents the dependency of optimal policy on system parameters.

Lemma 7. *(1) s_i^E is decreasing in c_j^E for $j \le i$, independent of c_j^E for $j > i$, increasing in c_j^R for $j < i$, independent of c_j^R for $j \ge i$ and increasing in b.*

(2) s_i^R is decreasing in c_j^E for $j \le i$,independent of c_j^E for $j > i$, increasing in c_j^R for $j \le i$, independent of c_j^R for $j > i$ and increasing b.

Thus, for each stage, if the expedition cost gets higher, then less expedition will be used and the base-stock level for expedited order becomes lower. If the expedition cost of any downstream stage gets higher, then less expedition will be used in that stage and so the echelon expedited base-stock level of current stage becomes lower as well. But if the regular shipping cost gets higher, then the regular echelon base stock level becomes higher. The explanation for the latter is the same as that of a single-stage infinite horizon inventory model with periodic review and one ordering opportunity in each period, for which it is well-known that the base-stock level is increasing in purchasing cost. As a result, if downstream stage's regular shipping cost becomes higher, then the downstream stage tends to order more so both the echelon expedited and regular order-up-to levels of current stage become higher. That s_i^R is decreasing in c_j^E can be explained as follows: As we can consider the expedited manager as the downstream of the regular order manager, when the expedition cost is higher, less expedited orders are placed and consequently, the echelon base-stock level for regular shipping will also become lower. Finally, that both s_i^E and s_i^R are increasing in b is intuitively clear: with higher shortage cost, then each stage should keep higher (echelon) inventory to reduce shortage cost.

Lemma 8. *For $i = 2, \ldots, N$, $s_i^E \ge s_{i-1}^E$ if and only if $c_{i-1}^R > c_i^E$.*

The previous results we have obtained not only show some structural properties of the optimal base-stock levels but more importantly, they will be used in the derivation of bounds for the optimal base-stock levels and computational heuristics in the remainder of the section.

3.2 Bounds

In this subsection, we develop several sets of newsvendor lower bounds and upper bounds for the optimal echelon base-stock levels.

The basic ideas used in developing upper and lower bounds are as follows: The optimal base-stock level for emergency shipping s_i^E is determined by $(G_i^E(y))' = 0$. Since $(G_i^E(y))'$ is an increasing function of y, if we can find a function g such that $G_i^E(y))' \leq g(y)$, then the solution of $g(y) = 0$ is a lower bound for s_i^E. Similarly, if we can find a function g such that $G_i^E(y))' \geq g(y)$, then the solution of $g(y) = 0$ is an upper bound for s_i^E. The same argument applies to s_i^R which is determined by $(G_i^R(y))' = 0$. Moreover, the simpler and tighter the $g(y)$ function is, the simpler and better the resulting bound.

Theorem 9. *For $i = 1, \ldots, N$, the lower bounds for s_i^E and s_i^R are, respectively,*

$$\underline{s}_i^{E1} = \max\left\{\bar{F}^{-1}\left(\frac{\sum_{j=1}^{i}(c_j^E - c_{j-1}^R)}{H_1 + b}\right), \bar{F}^{-1}\left(\frac{\sum_{j=1}^{i}\alpha^{i-j}(c_j^E - c_{j-1}^R)}{\alpha^{i-1}(H_1 + b)}\right)\right\}, \tag{22}$$

and

$$\underline{s}_i^{R1} = \max\left\{\bar{F}^{-1}\left(\frac{-c_i^R + \sum_{j=1}^{i}(c_j^E - c_{j-1}^R)}{H_1 + b}\right), \bar{F}^{-1}\left(\frac{-c_i^R + \sum_{j=1}^{i}\alpha^{i-j+1}(c_j^E - c_{j-1}^R)}{\alpha^{i}(H_1 + b)}\right)\right\}. \tag{23}$$

For notational convenience, for $i = 1, \ldots, N,\ j = 1, \ldots, i$, let

$$\begin{aligned} A_{i,i} &= 0 \\ B_{i,j} &= c_i^R + \alpha A_{i,j}^+, \\ A_{i,j} &= -c_i^E + B_{i-1,j}. \end{aligned}$$

In the next two theorems, we give other sets of lower bounds for s_i^E and s_i^R.

Theorem 10. *For $i = 1, 2, \ldots, N$, if $\sum_{j=1}^{i}\alpha^{i-j}(c_j^E - c_{j-1}^R)) \leq \alpha^{i-1}(H_1 + b)$, then the lower bounds for s_i^E and s_i^R are,*

$$\underline{s}_i^{E2} = \max\left\{F_k^{-1}\left(\frac{A_{i,i-k+1}}{\sum_{l=1}^{i-k+1}\alpha^{i-l}(c_l^E - c_{l-1}^R)}\right), k = 2, \ldots, i\right\}, \tag{24}$$

$$\underline{s}_i^{R2} = \max\left\{ F_{k+1}^{-1}\left(\frac{B_{i,i-k+1}}{\sum_{l=1}^{i-k+1}\alpha^{i-l+1}(c_l^E - c_{l-1}^R)}\right), k = 1,\dots,i \right\}. \tag{25}$$

Theorem 11. *For $i = 2,\dots,N$*

$$\underline{s}_i^{E3} = s_{i-1}^E + \max\left\{ F^{-1}\left(\frac{c_{i-1}^R - c_i^E}{\sum_{j=1}^{i-1}\alpha^{i-j}(c_j^E - c_{j-1}^R)}\right), F^{-1}\left(\frac{c_{i-1}^R - c_i^E}{\alpha c_{i-1}^E}\right)\right\}, \tag{26}$$

if $c_{i-1}^R - c_i^E >= 0$. And $i = 1,\dots,N$

$$\underline{s}_i^{R3} = s_i^E + \min\left\{ F^{-1}\left(\frac{c_i^R}{\sum_{j=1}^{i}\alpha^{i-j+1}(c_j^E - c_{j-1}^R)}\right), F^{-1}\left(\frac{c_i^R}{\alpha c_i^E}\right)\right\}. \tag{27}$$

It follows that, if the relative unit regular order cost of downstream stage is greater than the relative unit expedited cost of the current stage, i.e., $c_{i-1}^R > c_i^E$, then the expedited order-up-to level of the current stage is higher than that of its downstream stage. Although this bounds depend on the optimal s_{i-1}^E and s_i^E, in computation, we can use the largest available lower bounds of s_{i-1}^E and s_i^E to replace s_{i-1}^E and s_i^E in (26) and (27), respectively.

Note that the last set of lower bounds for the optimal expedited order-up-to level is equal to the lower bound of the optimal expedited order-up-to level of its downstream plus a number, which can be either positive or negative infinity; the last lower bounds for optimal regular order-up-to level is equal to the lower bound of the optimal expedited order-up-to level of its own stage plus a nonnegative number. Furthermore, we remark that the lower bounds derived above do not have a dominating relationship. That is, any lower bound can be a better one, depending on the problem instance.

We next develop three sets of upper bounds for the optimal echelon base-stock levels of each stage.

Theorem 12. *For $i = 1,\dots,N$, the upper bounds for s_i^E and s_i^R are, respectively,*

$$\bar{s}_i^{E1} = \bar{F}_i^{-1}\left(\frac{c_i^E - c_{i-1}^R + \alpha c_{i-1}^E}{H_1 + b - \sum_{j=1}^{i-2}(\alpha c_j^E - c_j^R)}\right), \tag{28}$$

if $c_i^E + \sum_{j=1}^{i-1}(\alpha c_j^E - c_j^R) \le H_1 + b$; otherwise $\bar{s}_i^{E1} = -\infty$; and

$$\bar{s}_i^{R1} = \bar{F}_{i+1}^{-1}\left(\frac{\alpha c_i^E - c_i^R}{\alpha\left(H_1 + b - \sum_{j=1}^{i-1}(\alpha c_j^E - c_j^R)\right)}\right), \tag{29}$$

if $\sum_{j=1}^{i}(\alpha c_j^E - c_j^R) \le H_1 + b$; otherwise, $\bar{s}_i^{R1} = -\infty$.

Proposition 13. *(1) If $c_i^E + \sum_{j=1}^{i-1}(\alpha c_j^E - c_j^R) > H_1 + b$, then $s_j^E = -\infty$ for $j \geq i$. (2) If $\sum_{j=1}^{i}(\alpha c_j^E - c_j^R) > H_1 + b$, then $s_j^R = -\infty$ for $j \geq i$.*

The second set of upper bounds is

Theorem 14. *The upper bounds for s_i^E and s_i^R, $i = 2, \dots, N$ are*

$$\bar{s}_i^{E2} = \bar{s}_{i-1}^{R2}, \tag{30}$$

and

$$\bar{s}_i^{R2} = s_{i-1}^R + \min\left\{ \bar{F}^{-1}\left(\frac{\alpha c_i^E - c_i^R}{\alpha\left(H_1 + b - \sum_{j=1}^{i-1}(\alpha c_j^E - c_j^R)\right)} \right), F^{-1}\left(\frac{c_i^R}{\alpha c_i^E}\right) \right\}. \tag{31}$$

Again, for computation of (31), the available smallest upper bound of s_{i-1}^R is used instead of the optimal one.

In the following we develop another set of newsvendor upper bounds for the optimal base-stock levels. Let

$$C_0 = 0$$
$$C_i = c_i^E - c_{i-1}^R - C_{i-1}^-, i = 1, \dots, N.$$

Theorem 15. *The third set of upper bounds is*

$$\bar{s}_i^{E3} = \min\left\{ \bar{F}^{-1}\left(\frac{C_i}{H_1 + b}\right), \bar{F}_2^{-1}\left(\frac{C_i + \alpha C_{i-1}^+}{H_1 + b}\right) \right\}, \quad i = 1, \dots, N, \tag{32}$$

and

$$\bar{s}_i^{R3} = \bar{F}_2^{-1}\left(\frac{-c_i^R + \alpha C_i}{\alpha(H_1 + b)}\right), \quad i = 1, \dots, N. \tag{33}$$

Similar to the case with lower bounds, none of these upper bounds dominate the other. That is, any of these upper bounds can be sharper, depending on the problem instance.

3.3 Heuristics and Numerical Results

In this subsection, we develop a simple heuristic based on the lower and upper bounds we derived for the optimal echelon base-stock levels of each stage. We also present numerical studies to demonstrate the effectiveness of the heuristic method.

For $i = 1, 2, \dots, N$, let

$$\underset{\sim}{s_i}^E = \max\{\underline{s}_i^{Ej}, j = 1, 2, 3\}, \qquad \underset{\sim}{s_i}^R = \max\{\underline{s}_i^{Rj}, j = 1, 2, 3\};$$
$$\tilde{s}_i^E = \min\{\bar{s}_i^{Ej}, j = 1, 2, 3\}, \qquad \tilde{s}_i^R = \min\{\bar{s}_i^{Rj}, j = 1, 2, 3\}.$$

It is clear that $\underset{\sim}{s_i}^E \le s_i^E \le \tilde{s}_i^E$ and $\underset{\sim}{s_i}^R \le s_i^R \le \tilde{s}_i^R$. Moreover, note that from Lemma 8, if $c_{i-1}^R > c_i^E$ and $\tilde{s}_i^E > \tilde{s}_{i-1}^E$, then we set $\tilde{s}_i^E = \tilde{s}_{i-1}^E$.

For $i = 1, 2 \ldots, N$ and $0 \le \beta \le 1$, set

$$s_i^{Eh} = \left[\beta \underset{\sim}{s_i}^E + (1-\beta)\tilde{s}_i^E\right], \quad s_i^{Rh} = \left[\beta \underset{\sim}{s_i}^R + (1-\beta)\tilde{s}_i^R\right], \tag{34}$$

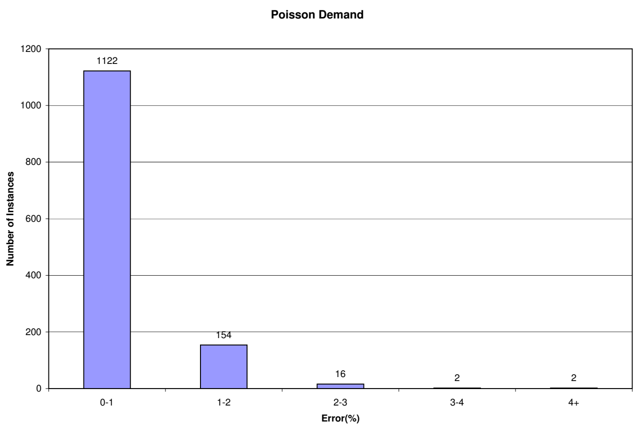
in which [ ] is the round off operator. We choose $\beta = 0.5$ as the heuristic policy. The heuristic policy works in exactly the same manner as the original top-down echelon base-stock policy but using s_i^{Eh} and s_i^{Rh} as the echelon base-stock levels for stage i.

In the following we present two groups of numerical examples classified by the demand distributions to illustrate the effectiveness of this heuristic.

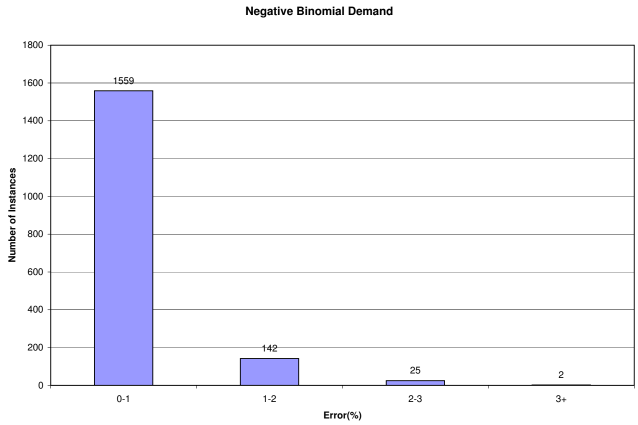
We use the relative error on the optimal system cost as the measure for effectiveness of the heuristic. Let $\widehat{f}(\mathbf{x})$ denote the cost of the heuristic policy, the relative error of the heuristic is defined as

$$Error\% = \frac{\widehat{f}(\mathbf{x}) - f(\mathbf{x})}{f(\mathbf{x})} \times 100\%.$$

In Group 1, we use Poisson demand with arrival rate $\lambda = 5, 10, 50$. We compare the optimal and heuristic policies for a three-stage system. The parameters for the examples are $b = 30, 60$, $h_i = 0.1, 1$, $\bar{c}_i^E = 4, 10$, $\bar{c}_i^R = 2, 6$, for $i = 1, 2, 3$ and $\alpha = 0.95, T = 100$. By restricting $\bar{c}_i^E > \bar{c}_i^R$, we generate 432 instances for each demand arrival rate. The average relative error among 432 instances for $\lambda = 5$ is 0.57% with the maximum 3.06%, for $\lambda = 10$ is 0.52% with the maximum 4.28% and for $\lambda = 50$ is 0.33% with the maximum 1.70%. The average relative error for all 1296 instances is 0.47%.



To show that the heuristic is robust under larger demand variance comparing to mean, in Group 2, we use Negative Binomial demand with four sets of mean and variance (30, 120), (30, 40), (6, 24), and (6, 8) while keep other parameters the same as Group 1. These demand parameters generate 4 set of numerical examples and each set includes 432 instances. The average relative error among 432 instances for the first set is 0.42% with the maximum 3.62%, for the second is 0.37% with the maximum 2.65% and for the third is 0.49% with the maximum 2.64% and for the fourth is 0.48% with the maximum 2.88%. The average relative error for all 1728 instances is 0.44%. The numerical results indeed validate that the effectiveness of our heuristic under larger demand variance.



From our numerical studies, we find that it is more cost efficient to reduce the holding cost at upstream stages and the expedited ordering cost at downstream stages. In addition, the downstream's optimal echelon base-stock levels are independent of upstream's cost parameters and upstream's optimal echelon base stock levels are increasing as downstream's ordering costs increase and decreasing when downstream's holding cost increases. Moreover, the increase of backlog cost rate has larger impact on the order-up-to level of expedited shipment (regular shipment) when the unit expedited shipping cost is relatively small (large).

4 Conclusion

In this chapter, we first present an explicit form solution for the optimal echelon base-stock policy of Clark-Scarf model for both average cost and discounted cost criteria. These simple expressions clearly identify the key determinants of the optimal optimal policy, and they provide a novel approach to construct simple bounds and approximations of the optimal solutions. We illustrate the lower bound of Dong and Lee (2003) for the discounted cost and the lower and upper bounds of Shang and Song (2003) for the average cost. We also present new upper bounds for both average cost and discounted cost criteria. By further extending the idea, we derive newsvendor-type lower bounds and upper bounds for infinite horizon, periodic review, serial inventory system with expedited and regular supply, and based on which we develop a simple and effective heuristic. We use numerical examples to demonstrate the effectiveness of the heuristics for both models.

In Section 3, the leadtimes for regular and expedited ordering are assumed to be 1 and 0 respectively. By inserting stages to stand for leadtime, we can obtain models where leadtimes between stages $i+1$ and i is l_i, and the firm is allowed by expedite shipping between any two stages. Through expedition, the firm can ship the product from stage $i+1$ to i in ℓ units of time for any $\ell = 0, 1, \ldots, l_i - 1$. The cost for such expedition will have to satisfy the relationship entailed by the model. See Muharremoglu and Tsitsilkis (2003). In particular, the result for the case where the leadtimes for stage $i+1$ and i are $l_i + 1$ and l_i can be presented in similar fashion to those in the chapter. This is a natural extension of the Fukuda model (1964) to serial supply chains. In that case the recursive algorithms for computing the optimal echelon base-stock levels are as follows. For convenience, let $D(l)$ be the leadtime demand over l periods. Let $G_1^E(y) = c_1^E y + (H_1 + b)E[(y - D(l_1))^-]$, and let s_1^E be the minimizer of G_1^E, for $i \geq 1$, compute:

$$
\begin{aligned}
G_{i,i}(y) =& G_i^E((y - D(l_i) \wedge s_i^E) - G_i^E(s_i^E) + \alpha E[G_i^E((y - D(l_i + 1)) \vee s_i^E)], \\
G_i^R(y) =& G_{i,i}(y) - c_i^R y, \quad s_i^R = \arg\min G_i^R(y), \\
G_{i,i+1}(y) =& G_i^R(y \wedge s_i^R), \\
G_{i+1}^E(y) =& c_{i+1}^E y + G_{i,i+1}(y), \quad s_{i+1}^E = \arg\min G_{i+1}^E(y).
\end{aligned}
$$

We should accordingly revised the bounds developed to reflect the arbitrary leadtimes. For instance, the F_i and F_{i+1} in Theorems 2 and 4 should represent the leadtime demand distributions over $\sum_{j=1}^{i} l_j$ and $\sum_{j=1}^{i+1} l_j$ periods, respectively.

The results reported in this chapter can also be extended to the case where, for some stages, there is only one transportation mode, while the others have two transportation modes. In that case there is one echelon base-stock level for those stages with only one transportation mode, and two echelon base-stock levels for those stages with two transportation modes. Discussions on these extensions can be found in Zhou (2006).

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