Analysis of the Road Traffic Based on SPCP

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1 Introduction

The transportation of people and goods has been necessary in any society for thousands of years. In modern times, transportation has become a foundation component of human activity. A great deal of problems related to traffic and transportation have been ingeniously resolved in the past and perhaps an even greater number of problems will have to be overcome in the future. Now it has been realized that more and more problems connected to traffic congestion, especially road traffic congestion in relation to the type of road networks, need to be understood and controlled.

There exist many types of road networks in urban cities, and among these networks some road segments are crowded with cars while others are not. To evaluate the phenomenon quantitatively, Oyama and Taguchi [7][8] devised the SPCP (shortest path counting problem) and gave an approach that reflects the crowdedness of each road segment with respect to some special type networks. This approach has been examined to be effective by applying them to several real cities in Japan (Oyama and Taguchi [9]), and recently, some related studies such as Li [4], Oyama [6], Oyama and Morohosi [10], have also been done on the topic. However, the existing studies do not give further discussions on the fundamental properties of SPCP such as the maximum value, the expectation and the variance *etc.* accompanied with the shape of a road traffic network, and they do not consider the direction of a road segment, which may not be neglected in a real road traffic network. Here, in this paper, by considering two idealized typical networks — grid type networks, which are common in North American cities and radial - circular type networks, which usually appear in traditional European cities, we discuss some useful properties of the above two road networks from the viewpoint of SPCP. Moreover, using the obtained results, we also compare and examine their

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effectiveness. Finally, by considering the concrete direction of a road segment in the above two road networks, we give some proposals on the design of car tracks.

The remainder of this paper is organized as follows. In the next section, we describe the concept of SPCP, and then give the results of SPCP in grid type networks and radial - circular type networks (Oyama and Taguchi [7][8]). Then in the third section, we investigate their properties such as maximum values, expectations, variances, *etc.*, and compare their effectiveness based on SPCP. In the fourth section, we extend SPCP to DSPCP (Directed Shortest Path Counting Problem) by considering the concrete directions in the two road networks. In the fifth section, we give some observations on the number of car tracks of a road segment with different directions based on the results of the previous section. Lastly, in the sixth section, we summarize the results of the study.

2 SPCP

2.1 The concept of SPCP

In a network $N = (B, E)$ with the vertex set B and the edge set E, the shortest path problem is to find a path with the shortest length from a specified origin vertex to another specified destination vertex (O-D).

Generally, when a special type network is given, there may exist more than one shortest path between two vertices. Here, in determining a unique shortest path between any two vertices in case there exist two or more shortest paths having the same lengths, the following rules are applied.

1. The number of turns is minimized.
2. The number of left turns is maximized when there exist the shortest paths having an equal number of turns.

As a result, assuming the above two rules are sufficient to determine a unique path, there are $c(c-1)$ shortest paths in a network with $|B| = c$, so among all these $c(c-1)$ shortest paths, SPCP requires us to count the number of the shortest paths passing each edge. Here, we use the symbol $\omega(\epsilon)$, which is called the weight of the edge ($\epsilon \in E$), to denote the number of the shortest paths involving the edge ϵ for a given network.

2.2 The result of SPCP in a grid type network

We consider SPCP of a grid type network $G(m, n)$ as shown in Figure 1, consisting of $(m + 1)(n + 1)$ grid points.

Here, the set of grid points of $G(m, n)$ is expressed as $\{x_{yz} | 1 \leq y \leq m + 1, 1 \leq z \leq n + 1\}$. In this grid type network $G(m, n)$, Z_{kl} and H_{kl} indicate a vertical edge element connecting x_{kl} with $x_{k+1,l}$ and a horizontal edge element connecting x_{kl} with $x_{k,l+1}$, respectively. Regarding the weight $\omega(\epsilon)$ of the edge ϵ with respect to the shortest paths in a grid type network $G(m, n)$, we obtain the following theorem.

Theorem 1 *For a given grid type network $G(m,n)$, the weights of the edge elements Z_{kl} and H_{kl} with respect to the shortest paths can be expressed as*

$$\begin{cases} \omega(Z_{kl}) = 2k(m+1-k)(n+1) & 1 \le k \le m, \quad 1 \le l \le n+1 \\ \omega(H_{kl}) = 2l(n+1-l)(m+1) & 1 \le k \le m+1, \quad 1 \le l \le n \end{cases} \quad (1)$$

For more details, refer to Oyama and Taguchi [7][8].

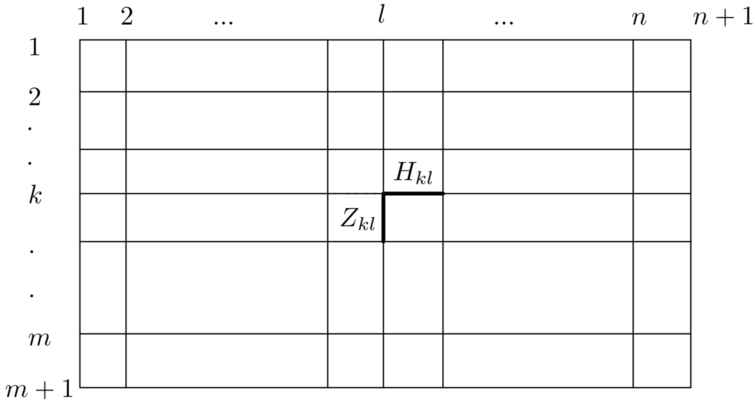

Figure 1 Grid-type network $G(m,n)$

2.3 The result of SPCP in a radial-circular type network

Now we consider SPCP of a circular type network $T(m,n)$, with m circular roads and n radial roads as shown in Figure 2. Here, we suppose that a radial-circular type network is divided by n radial roads with the same angle between two neighboring roads.

Then similarly to the above subsection, the set of points of $T(m,n)$ is expressed as $\{x_{yz} | 1 \le y \le m, 1 \le z \le n\}$ and O expresses the center. In a radial-circular type network $T(m,n)$, R_{kl} and C_{kl} indicate a radial edge element connecting x_{kl} with $x_{k+1,l}$ and a circular edge element connecting x_{kl} with $x_{k,l+1}$, respectively. Regarding the weight $\omega(\epsilon)$ of the edge ϵ with respect to the shortest paths in a radial-circular type network $T(m,n)$, we obtain the following theorem.

Theorem 2 *For a given radial-circle type network $T(m,n)$, the weights of the edge elements R_{kl} and C_{kl} with respect to the shortest paths can be expressed as*

$$\begin{cases} \omega(R_{kl}) = 2kmn - 2k^2(2p_0+1) + 2k & 1 \le k \le m, \quad 1 \le l \le n \\ \omega(C_{kl}) = (2k-1)p_0(p_0+1) & 1 \le k \le m, \quad 1 \le l \le n \end{cases} \quad (2)$$

Here, radian 2 plays an important role in determining the shortest path between any two vertices within $T(m,n)$, and when the angle of two vertices formed with the center is greater than radian 2, then only radial roads are selected, while it is smaller than radian 2, a circular road and a radial road (when necessary), are selected. Using this property, in our calculation, we define a parameter p_0 which is denoted as $p_0 = [\frac{n}{\pi}]$, where $[x]$ is the greatest integer not greater than x. Obviously, $p_0 + 1$ shows the number of the radial roads within radian 2.

For more details, also refer to Oyama and Taguchi [7][8].

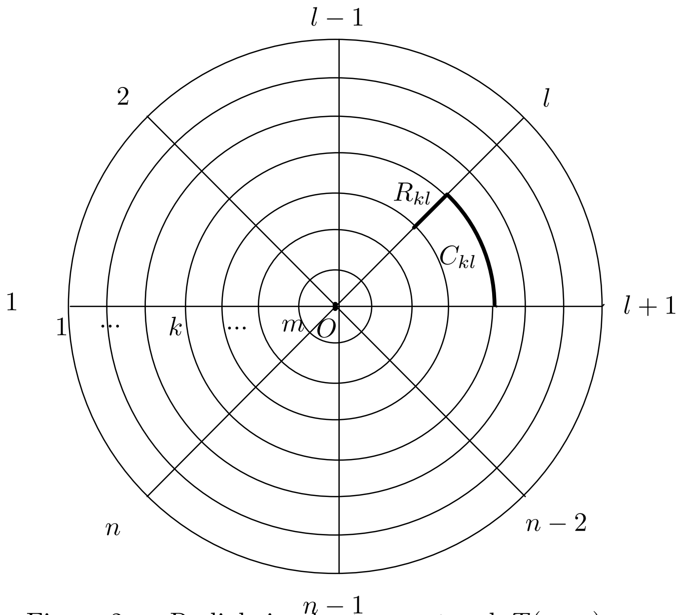

Figure 2 Radial-circular type network $T(m, n)$

3 Some properties of SPCP in two road networks

Analyzing the results of Theorem 1 from the aspects of maximum values, expectations and variances, they could be given as below. Here, to make the analysis convenient, we suppose fraction expressions are permitted for $k_{\max}$ (k which maximizes $\omega(Z_{kl})$ in $G(m, n)$, or maximizes $\omega(R_{kl})$), $\omega(C_{kl})$) in $T(m, n)$), $l_{\max}$ (l which maximizes $\omega(H_{kl})$ in $G(m, n)$) in the following.

$$\begin{cases} \omega_{\max}(Z_{kl}) = \frac{(n+1)(m+1)^2}{2} & \text{when } k_{\max} = \frac{(m+1)}{2} \\ \omega_{\max}(H_{kl}) = \frac{(m+1)(n+1)^2}{2} & \text{when } l_{\max} = \frac{(n+1)}{2} \end{cases} \tag{3}$$

$$\begin{cases} E[\omega(Z_{kl})] = \frac{(n+1)(m+1)(m+2)}{3} \\ E[\omega(H_{kl})] = \frac{(m+1)(n+1)(n+2)}{3} \end{cases} \tag{4}$$

$$\begin{cases} V[\omega(Z_{kl})] = \frac{(m+1)(m+2)(m-1)(m-2)(n+1)^2}{45} \\ V[\omega(H_{kl})] = \frac{(n+1)(n+2)(n-1)(n-2)(m+1)^2}{45} \end{cases} \tag{5}$$

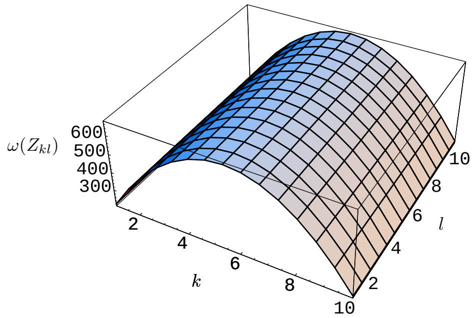

Figure 3 The distribution of $\omega(Z_{kl})$ as a function of k,l

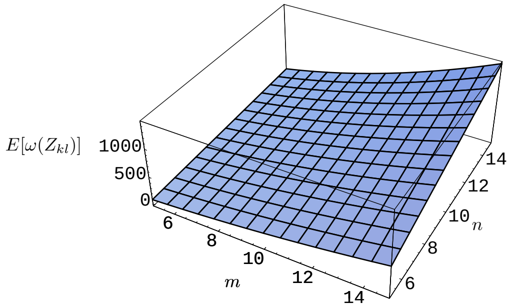

Figure 4 The distribution of $E[\omega(Z_{kl})]$ as a function of m,n

The most typical characteristic shown in (3) $\sim$ (5) is the symmetry for the obtained results of $\omega(Z_{kl})$, $\omega(H_{kl})$ with respect to m, n. From (3) $\sim$ (5), we can also see that the maximum weights usually happen at the center of a grid type network (Figure 3), and the expectations and the variances of vertical edges are more easily affected by the number of horizontal roads m (Figure 4 $\sim$ 5), while the values of horizontal edges are more easily affected by the number of vertical roads n.

We next calculate the expectation and the variance of $\omega(\epsilon)$ in $G(m, n)$.

$$\begin{cases} E[\omega(\epsilon)] = \frac{(n+1)(m+1)[m(n+1)(m+2)+n(m+1)(n+2)]}{3(2mn+m+n)} \\ V[\omega(\epsilon)] = \frac{m(m+1)(m+2)(m-1)(m-2)(n+1)^3}{45(2mn+m+n)} + \frac{n(n+1)(n+2)(n-1)(n-2)(m+1)^3}{45(2mn+m+n)} \end{cases} \quad (6)$$

Fixing the number of the vertices in $G(m, n)$ as $|B| = (m+1)(n+1) = c$, *i.e.*, fixing the total number of the shortest paths in $G(m, n)$ as $c(c-1)$, we theoretically find that the minimum values of $E[\omega(\epsilon)](E_{\min}), V[\omega(\epsilon)](V_{\min})$ are obtained when $m = n$, meaning that the same number of vertical roads and horizontal roads are desirable when designing a grid type network over an urban area by considering SPCP. We omit the proof of the result here since it is easy to be derived.

On the other hand, analyzing the results of Theorem 2 also from the aspects of maximum values, expectations and variances, they could be given as below.

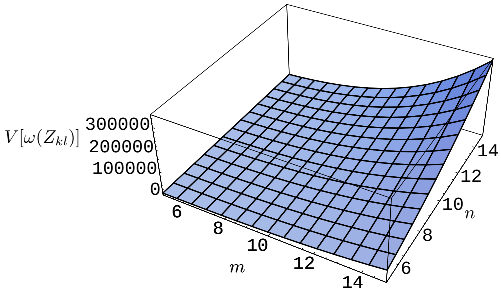

Figure 5 The distribution of $V[\omega(Z_{kl})]$ as a function of m,n

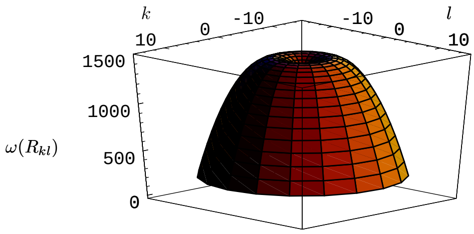

Figure 6 The distribution of $\omega(R_{kl})$ as a function of k,l

$$\begin{cases} \omega_{\max}(R_{kl}) = \frac{(mn+1)^2}{2(2p_0+1)} & \text{when } k_{\max} = \frac{(mn+1)}{2(2p_0+1)} \\ \omega_{\max}(C_{kl}) = (2m-1)p_0(p_0+1) & \text{when } k_{\max} = m \end{cases} \tag{7}$$

$$\begin{cases} E[\omega(R_{kl})] = \frac{1}{3}(m+1)(2-2m+3mn-2p_0-4mp_0) \\ E[\omega(C_{kl})] = mp_0(p_0+1) \end{cases} \tag{8}$$

$$\begin{cases} V[\omega(R_{kl})] = \frac{1}{45}(m+1)(m-1)(16m^2-30m^2n+15m^2n^2-16p_0+60mp_0 \\ \qquad +64m^2p_0-60mnp_0-60m^2np_0+44p_0^2+120mp_0^2+64m^2p_0^2-4) \\ V[\omega(C_{kl})] = \frac{1}{3}(m+1)(m-1)p_0^2(1+p_0^2) \end{cases} \tag{9}$$

From (7) $\sim$ (9), we can understand that the maximum weights of radial roads happen at the inner side and they move to inside with the increasing of the number of radial roads n (Figure 6 $\sim$ 7), while those of circular roads happen at the inmost side in a radial-circular type network. We can also understand that the expectations and the variances of radial edges are much more affected by the number of circular roads m, while the values of circular edges are much more affected by the number of radial roads n (Figure 8 $\sim$ 9).

Similarly to the above, we calculate the expectation and the variance of $\omega(\epsilon)$ in $T(m,n)$.

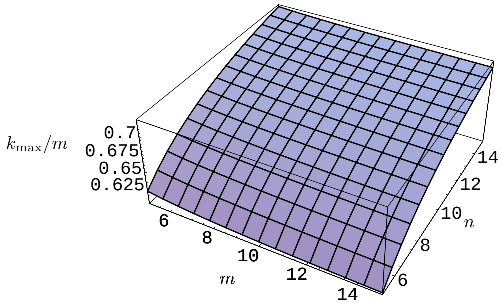

Figure 7 The distribution of $k_{\max}/m$ as a function of m,n

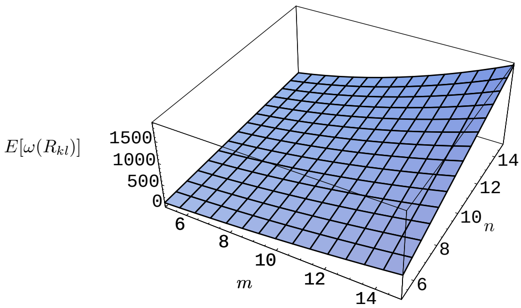

Figure 8 The distribution of $E[\omega(R_{kl})]$ as a function of m,n

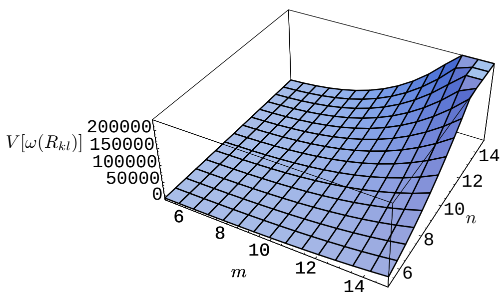

Figure 9 The distribution of $V[\omega(R_{kl})]$ as a function of m,n

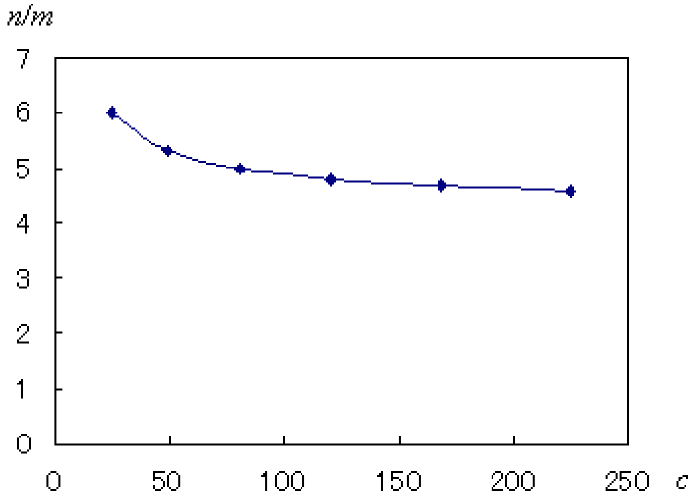

Figure 10 n/m as a function of c for $E_{\min}, V_{\min}$

$$\left\{\begin{array}{l} E[\omega(\epsilon)] = \frac{1}{2}mp_0(p_0+1) + \frac{1}{6}(m+1)(2-2m+3mn-2p_0-4mp_0) \\ V[\omega(\epsilon)] = \frac{1}{180}(28-80m^2+52m^4+60mn+120m^2n-60m^3n-120m^4n+15m^2n^2 \\ \qquad +90m^3n^2+75m^4n^2-8p_0-300mp_0-200m^2p_0+300m^3p_0+208m^4p_0 \\ \qquad +60mnp_0-210m^2np_0-510m^3np_0-240m^4np_0-98p_0^2-120mp_0^2 \\ \qquad +475m^2p_0^2+660m^3p_0^2+208m^4p_0^2-90m^2np_0^2-90m^3np_0^2-60p_0^3 \\ \qquad +60mp_0^3+330m^2p_0^3+120m^3p_0^3-30p_0^4+75m^2p_0^4) \end{array}\right. \tag{10}$$

Fixing the number of the vertices in $T(m,n)$ as $|B| = mn+1 = c$, *i.e.*, fixing the total number of the shortest paths in $T(m,n)$ as $c(c-1)$, we numerically find that the minimum values of $E[\omega(\epsilon)](E_{\min}), V[\omega(\epsilon)](V_{\min})$ are obtained when n/m holds between $4.5 \sim 6$ for some realistic values of c such as $25 \le c \le 225$, and it decreases with the increasing of the number of the vertices c (Figure 10). The result also offers valuable information when designing a radial-circular type network over an urban area by considering SPCP.

Lastly, we compare $E_{\min}$, $V_{\min}$ of $G(m,n)$ and $T(m,n)$ through Figure 11 $\sim$ 12 to examine their efficiencies from the viewpoint of urban planning. The results show in most cases, a radial-circular type network $T(m,n)$ is relatively more effective than a grid type network $G(m,n)$ when considering their expectations, variances of SPCP, and the tendency becomes much more clear as the number of the vertices c increases.

4 DSPCP

4.1 The concept of DSPCP

Based on SPCP, we now define DSPCP (Directed Shortest Path Counting Problem) by differentiating the concrete direction in each edge, i.e., DSPCP requires us to count the number of the shortest paths passing each directed edge.

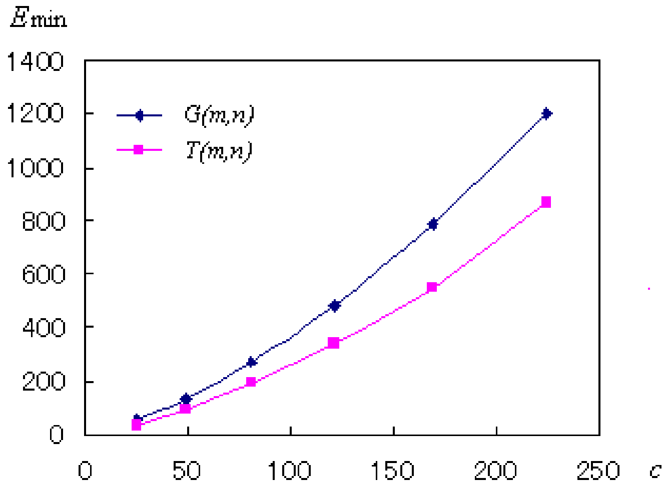

Figure 11 $E_{\min}$ as a function of c

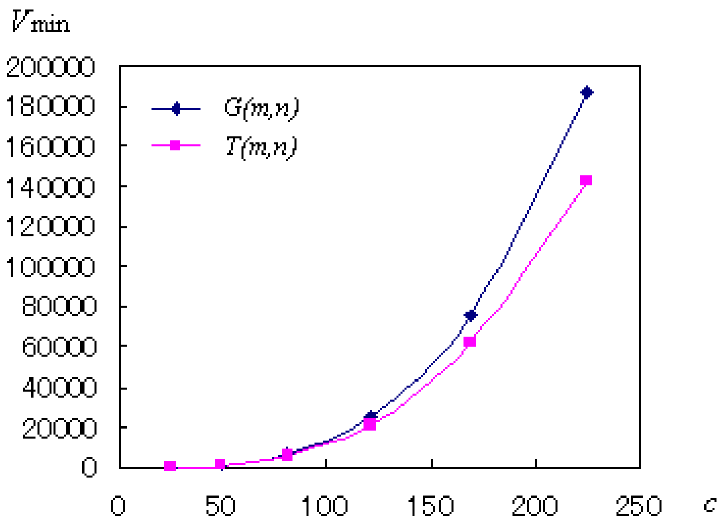

Figure 12 $V_{\min}$ as a function of c

To facilitate this, in an edge ϵ, we define ϵ^+ as a directed one from left to right or for downwards, and ϵ^- as a directed one from right to left or for upwards in a grid type network. Moreover, we define ϵ^+ as a directed one from the center to the circumference or for anti-clockwise, and ϵ^- as a directed one from the circumference to the center or for clockwise in a radial-circular type network.

4.2 The result of DSPCP in a grid type network

Theorem 3 *For a given grid type network $G(m,n)$, the weights of the vertical edge elements Z_{kl}^+ and Z_{kl}^- with respect to the shortest paths can be expressed as*

$$\begin{cases} \omega(Z_{kl}^+) = k(m+1-k)(2n+3-2l) & 1 \le k \le m, \quad 1 \le l \le n+1 \\ \omega(Z_{kl}^-) = k(m+1-k)(2l-1) & 1 \le k \le m, \quad 1 \le l \le n+1 \end{cases} \tag{11}$$

On the other hand, the weights of the horizontal edge elements H_{kl}^+ and H_{kl}^- with respect to the shortest paths can be expressed as

$$\begin{cases} \omega(H_{kl}^+) = l(n+1-l)(2k-1) & 1 \le k \le m+1, \quad 1 \le l \le n \\ \omega(H_{kl}^-) = l(n+1-l)(2m+3-2k) & 1 \le k \le m+1, \quad 1 \le l \le n \end{cases} \tag{12}$$

Proof. We calculate the number of the points in the following 6 blocks (Figure 13). Then,

$$\begin{cases} \mid s_1 \mid = k(l-1) \\ \mid s_2 \mid = k \\ \mid s_3 \mid = k(n+1-l) \\ \mid s_4 \mid = (m+1-k)(n+1-l) \\ \mid s_5 \mid = m+1-k \\ \mid s_6 \mid = (m+1-k)(l-1) \end{cases}$$

Since the shortest paths from a point in block s_2 to a point in block s_5, a point in block s_2 to a point in block s_4, and a point in block s_3 to a point in block s_5, pass through Z_{kl}^+, we can calculate $\omega(Z_{kl}^+)$ with respect to the shortest paths as the following.

$$\begin{aligned} \omega(Z_{kl}^+) &= |s_2| \cdot |s_5| + |s_2| \cdot |s_4| + |s_3| \cdot |s_5| \\ &= k(m+1-k) + k(m+1-k)(n+1-l) + k(n+1-l)(m+1-k) \\ &= k(m+1-k)(2n+3-2l) \end{aligned}$$

Similarly to the above, we also calculate $\omega(Z_{kl}^-)$ as below, respectively.

$$\begin{aligned} \omega(Z_{kl}^-) &= |s_5| \cdot |s_2| + |s_5| \cdot |s_1| + |s_6| \cdot |s_2| \\ &= k(m+1-k) + k(m+1-k)(l-1) + k(l-1)(m+1-k) \\ &= k(m+1-k)(2l-1) \end{aligned}$$

On the other hand, let us calculate the number of the points in the following 6 blocks (Figure 14). Then,

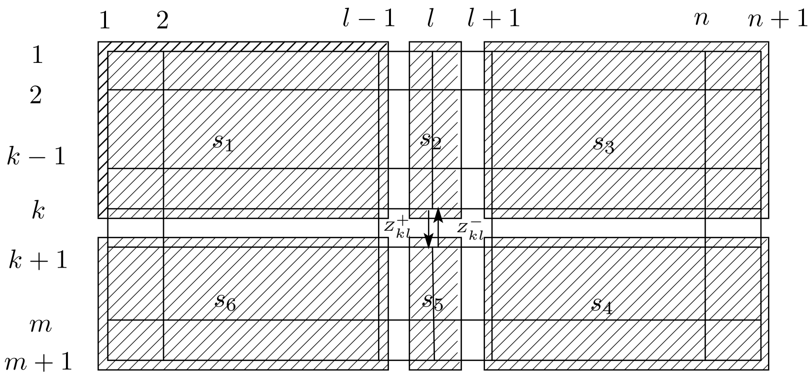

Figure 13 The number of the points in s_1– s_6

$$\begin{cases} \mid t_1 \mid = (k-1)l \\ \mid t_2 \mid = (k-1)(n+1-l) \\ \mid t_3 \mid = n+1-l \\ \mid t_4 \mid = (m+1-k)(n+1-l) \\ \mid t_5 \mid = (m+1-k)l \\ \mid t_6 \mid = l \end{cases}$$

Using the same method introduced above, we can calculate $\omega(H^+_{kl})$ and $\omega(H^-_{kl})$.

$$\omega(H^+_{kl}) \quad = \quad |t_6| \cdot |t_3| + |t_6| \cdot |t_2| + |t_1| \cdot |t_3|$$

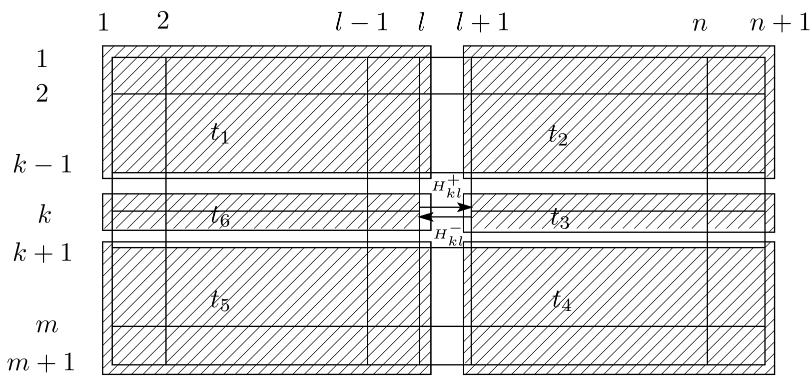

Figure 14 The number of the points in t_1– t_6

$$
\begin{aligned}
&= l(n+1-l)+l(k-1)(n+1-l)+(k-1)l(n+1-l) \\
&= l(n+1-l)(2k-1) \\
\omega(H_{kl}^{-}) &= |t_3|\cdot|t_6|+|t_4|\cdot|t_6|+|t_3|\cdot|t_5| \\
&= l(n+1-l)+l(m+1-k)(n+1-l)+(m+1-k)l(n+1-l) \\
&= l(n+1-l)(2m+3-2k)
\end{aligned}
$$

■

4.3 The result of DSPCP in a radial-circular type network

Theorem 4 *For a given radical-circular type network $T(m,n)$, the weights of the radial edge elements R_{kl}^{+} and R_{kl}^{-} with respect to the shortest paths can be expressed as*

$$\omega(R_{kl}^{+}) = \omega(R_{kl}^{-}) = k(mn+1) - k^2(2p_0+1) \quad 1 \le k \le m, \quad 1 \le l \le n \tag{13}$$

On the other hand, the weights of the circular edge elements C_{kl}^{+} and C_{kl}^{-} with respect to the shortest paths can be expressed as

$$\omega(C_{kl}^{+}) = \omega(C_{kl}^{-}) = \tfrac{1}{2}(2k-1)p_0(p_0+1) \quad 1 \le k \le m, \quad 1 \le l \le n \tag{14}$$

Proof. We calculate the number of the points in the following 4 blocks (Figure 15). Then,

$$
\begin{cases}
|\ s_0\ | = 1 \\
|\ s_1\ | = k \\
|\ s_2\ | = (2p_0+1)(m-k) \\
|\ s_3\ | = m[n-(2p_0+1)]
\end{cases}
$$

Since the shortest paths from the points in block s_0, s_2, s_3 to the points in block s_1, all pass through R_{kl}^{+}, we calculate $\omega(R_{kl}^{+})$ with respect to the shortest paths as the following.

$$
\begin{aligned}
\omega(R_{kl}^{+}) &= |s_0|\cdot|s_1|+|s_2|\cdot|s_1|+|s_3|\cdot|s_1| \\
&= k+k(2p_0+1)(m-k)+km[n-(2p_0+1)] \\
&= k(mn+1)-k^2(2p_0+1)
\end{aligned}
$$

Similarly to the above, we also calculate $\omega(R_{kl}^{-})$ as below, respectively.

$$
\begin{aligned}
\omega(R_{kl}^{-}) &= |s_1|\cdot|s_0|+|s_1|\cdot|s_2|+|s_1|\cdot|s_3| \\
&= k+k(2p_0+1)(m-k)+km[n-(2p_0+1)] \\
&= k(mn+1)-k^2(2p_0+1)
\end{aligned}
$$

On the other hand, let us calculate the number of the points in the following 4 blocks (Figure 16). Here,

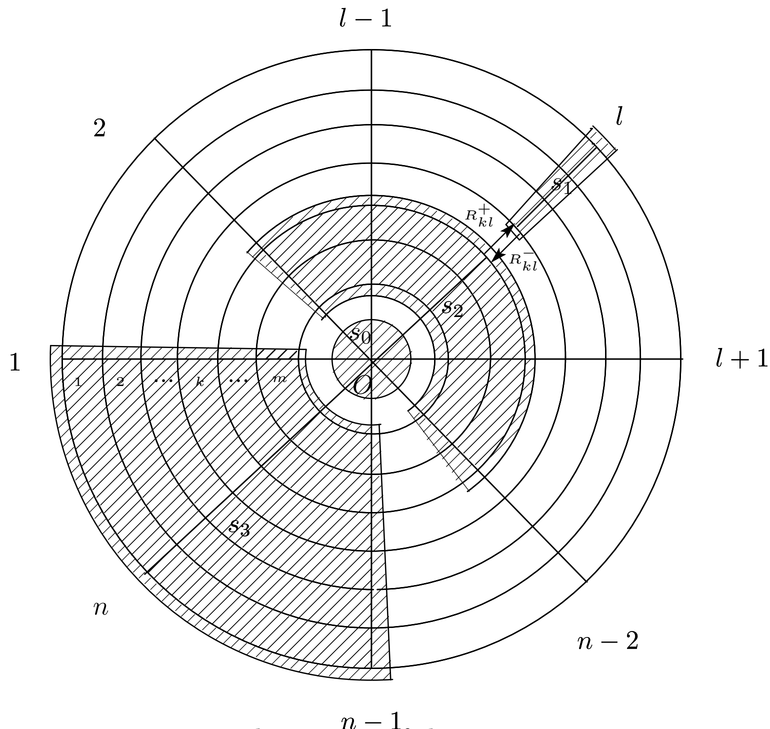

Figure 15 The number of the points in s_1– s_4

$$
\begin{cases}
\mid s'_1 \mid = (k-1)p_0 \\
\mid s'_2 \mid = p_0 \\
\mid s'_3 \mid = p_0 \\
\mid s'_4 \mid = (k-1)p_0
\end{cases}
$$

Then since the shortest paths from a point in block s'_3 to a point in block s'_2, whose angle formed with the center O is smaller than radian 2, a point in block s'_3 to a point in block s'_1, whose angle formed with the center O is smaller than radian 2, and a point in block s'_4 to a point in block s'_2, whose angle formed with the center O is also smaller than radian 2, all pass through C^+_{kl}, we calculate $\omega(C^+_{kl})$ with respect to the shortest paths as the following.

$$
\begin{aligned}
\omega(C^+_{kl}) &= |s'_3 \to s'_2| + |s'_3 \to s'_1| + |s'_4 \to s'_2| \\
&= \frac{1}{2}p_0(p_0+1) + (k-1)\frac{1}{2}p_0(p_0+1) + (k-1)\frac{1}{2}p_0(p_0+1) \\
&= \frac{1}{2}(2k-1)p_0(p_0+1)
\end{aligned}
$$

Similarly to the above, we calculate $\omega(C^-_{kl})$ as below, respectively.

$$\begin{aligned}\omega(C^-_{kl}) &= |s'_2 \to s'_3| + |s'_1 \to s'_3| + |s'_2 \to s'_4| \\ &= \frac{1}{2}p_0(p_0+1) + (k-1)\frac{1}{2}p_0(p_0+1) + (k-1)\frac{1}{2}p_0(p_0+1) \\ &= \frac{1}{2}(2k-1)p_0(p_0+1)\end{aligned}$$

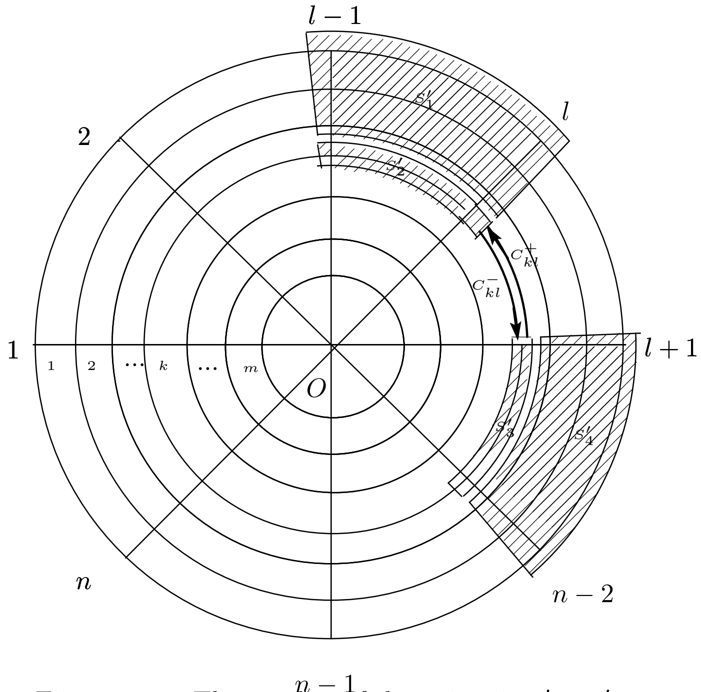

Figure 16 The number of the points in $s'_1 \sim s'_4$

■

5 The proportion of car tracks with different directions in two road networks

Analyzing the results of Theorem 3, we can give the distribution of the proportion of car tracks with different directions in a grid type network.

$$\begin{cases} \omega(Z^+_{kl})/\omega(Z_{kl}) = 1 - \frac{2l-1}{2(n+1)} & 1 \le l \le n+1 \\ \omega(Z^-_{kl})/\omega(Z_{kl}) = \frac{2l-1}{2(n+1)} & 1 \le l \le n+1 \end{cases} \tag{15}$$

$$\begin{cases} \omega(H^+_{kl})/\omega(H_{kl}) = \frac{2k-1}{2(m+1)} & 1 \le k \le m+1 \\ \omega(H^-_{kl})/\omega(H_{kl}) = 1 - \frac{2k-1}{2(m+1)} & 1 \le k \le m+1 \end{cases} \tag{16}$$

From (15), we can see that $\omega(Z_{kl}^{+})/\omega(Z_{kl})$ decreases, $\omega(Z_{kl}^{-})/\omega(Z_{kl})$ increases with the increasing of l, meaning when considering DSPCP in a grid type network, it is reasonable to design relatively more car tracks for downwards of vertical roads in left side, while relatively more car tracks for upwards of vertical roads in right side, and almost the same number of car tracks for both sides near the center (Figure 17).

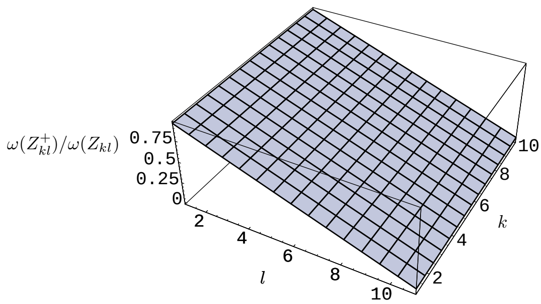

Figure 17 The proportion of $\omega(Z_{kl}^{+})/\omega(Z_{kl})$

Similarly to the above, from (16), we can also see that the relatively more car tracks for left directions of horizontal roads are needed in upper place, while relatively more car tracks for right directions of horizontal roads are needed in lower place, and almost the same number of car tracks for both sides are needed near the center.

Figure 18 shows the imagination on the design of the number of car tracks in a grid type network based on our investigation.

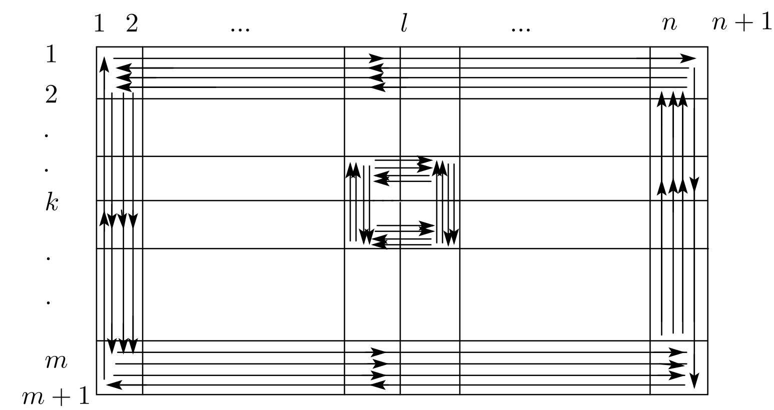

Figure 18 The number of car tracks in a grid type network

On the other hand, analyzing the results of Theorem 4, we can easily find that the same number of car tracks for both sides are needed in each edge (Figure

19). This shows an interesting property of the radial-circular type network when considering DSPCP.

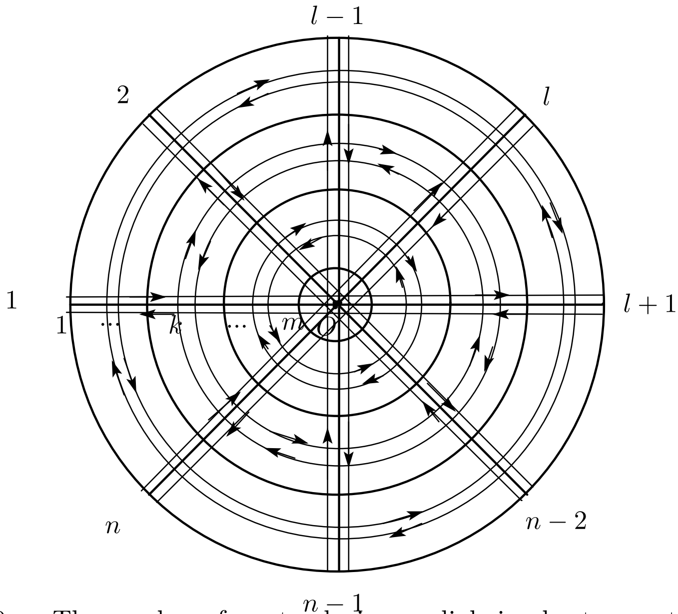

Figure 19 The number of car tracks in a radial-circular type network

6 Conclusions

Analyzing and comparing the properties of the road traffic by considering the shapes of various networks are important so that we can see much work on this field such as Koshizuka[3], and Li and Fushimi[5]. In this paper, using SPCP, a quantitative method proposed by Oyama and Taguchi [7][8], we investigate and compare the properties of two typical road networks–grid type and radial-circular type from the viewpoint of maximum weights, expectations, variances, and also from the viewpoint of car tracks. The results of the study are concluded as follows:

1. The maximum road traffic usually reaches peak at the center except the radial roads of a radial-circular network, where the peak takes place near the center (inner side) but not at the center.

2. When considering the expectations and the variances of road traffic, vertical edges are more easily affected by the number of horizontal roads m, horizontal edges are more easily affected by the number of vertical roads n in a grid type network, while radial edges are much more affected by the number of circular roads m, circular edges are much more affected by the number of radial roads n in a radial-circular type network. On the other hand, to minimize the

above-mentioned values, the relation $n/m = 1$ is usually required for a grid type network, while the relation $4.5 \leq n/m \leq 6$ generally holds for some realistic values of c in a radial-circular type network. Moreover, when fixing the total number of the shortest paths and comparing two road networks with minimum values, the structure of a radial-circular type network seems relatively effective in holding down road traffic.

3. When considering SPCP by differentiating the directions of each road segment, *i.e.*, DSPCP, in a grid type network, the number of car tracks is expected to be designed with the same proportion at the center for both sides, and then gradually decreasing the proportion for clockwise direction, increasing the proportion for anti-clockwise direction as the edge gets farther from the center toward the circumference. However, in a radial-circular type network, car tracks are always desired to be designed with the same proportion for both sides, no matter for radial roads, or for circular roads.

In real urban planning, the estimation of road traffic flows and the design of a road traffic network in association with its shape, the number of car tracks for different directions *etc.*, should be considered and evaluated from many aspects. That is, the results given here could not be used or applied as they are. However, in the initial stage of decision making, this kind of study is considered to be valuable and meaningful by offering important and useful information before designing a road network.

As we described in the first section of the paper, although the feasibility of SPCP has already been examined through its applications in real societies (Oyama and Taguchi [9]), and its O-D distribution has been theoretically proved to be a good approach of the simple power model (Li [4], Horwood [2], Smeed [12]), we are now also considering to develop some related models which reflect other O-D distributions such as those based on Clark's model (Clark [1]) and Sherratt's model (Sherratt [11]) *etc.* as the remaining topics, which seem also interesting from the aspect of traffic engineering.

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