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Application of Interval-PROMETHEE Method for Decision Making in Investing

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Abstract This paper introduces the interval-PROMETHEE method to the investment decision making of value investing. Five performance criteria are used for measuring the growth potential of firms and are represented by interval numbers based on the real financial data. The interval-PROMETHEE method is applied to rank the 20 randomly selected stocks in Shanghai Stock Exchange. The portfolio with the top 5 stocks was proven to have higher return than the top 10 stocks within the 17-month investment period. The empirical study showed the effectiveness of the interval-PROMETHEE method in the decision making process of value investing.

Keywords Value investing; Multi-criteria decision making; Interval-PROMETHEE

1 Introduction

Value investing initially proposed by Graham and well developed later by Buffet has proven to be a successful investment strategy and has been paid more and more attention in the last three decades. As suggested, the main process of BVI consists of two phases. The first is to find the equities with extraordinary overall performance based on several financial ratios. The second is to evaluate the intrinsic value of each selected equity. In this paper, the emphasis is laid on the first phase, and we focus on the overall performance evaluation of equities based on five criteria suggested by Buffet.

PROMETHEE is one of the most recent MCDM methods that was proposed by Brans et al.[1] and has successively been applied in many fields[2], especially in the investment analysis and performance evaluation. Mareschal and Brans[3], Vranegl et al.[4], Babic and Plazibat[5], Bouri et al.[6] and Albadvi et al.[7] all applied PROMETHEE as a decision making tool to solve the different problems in the field of finance. In this paper, we apply PROMETHEE II to select firms with outstanding performance in the decision process of value investing.

In the above mentioned application in finance, the performances parameters are all represented by single value numbers. However, finance investment decision making is a complex process due to the uncertain nature of financial markets, or because the markets are not well understood which is called non-random uncertainty. it is hard to convey the profitability information just by a single-valued number or a simple average of the past. So in this paper, we introduce interval number to model the uncertainty in value investing decision making problem based on the PROMETHEE method.

The rest of this paper is organized as follows: in section 2 the original PROMETHEE II method is recalled and then it is generalized to interval-PROMETHEE in section 3; in section 4, the multi-criteria ranking problem of ranking outstanding firms are presented and the application of interval-PROMETHEE method is performed for stocks based on 5 criteria in Shanghai Stock Exchange; finally the conclusions are drawn in section 5.

2 PROMETHEE II

$A = \{a_1, a_2, \ldots, a_n\}$ is a set of alternatives to rank, $F = \{f_1, f_2, \ldots, f_m\}$ is a set of criteria, which have to be optimized according to their potential contributions to the final results. If the higher of the performance evaluation for a criterion, the higher of the ranking, this criterion will be maximized; otherwise this criterion should be minimized. A pair-wise comparison between any two alternatives a_i and a_j is implemented and the intensity of the preference of an alternative a_i over another alternative a_j denoted by $P_k(d_k)$, and here

$$d_k = f_k(a_i) - f_k(a_j) \tag{1}$$

is determined firstly, where $f_k(a_i)$ is the evaluation of alternative a_i corresponding to the criterion f_k. Six different types of the preference function for the k-th criterion P_k are recommended by Brans et al. The decision makers can also define their own preference function. A linear preference function [1] is selected in this paper:

$$P_k(d_k) = \begin{cases} 0, & if\ d_k < 0 \\ \dfrac{d_k}{q_k}, & if\ 0 \le d_k \le q_k \\ 1, & if\ d_k > q_k \end{cases} \tag{2}$$

This shows that the intensity of the decision maker's preference between the alternatives a_i and a_j which increases linearly with the growth of d_k up to q_k. After the threshold q_k, the preference will be equal to 1. For ranking purposes, q_k can be set according the real situation. The value of the preference scales varies from 0 (no preference) to 1 (strong preference).

The preference of alternative a_i and a_j is evaluated for each criterion and the preference index is determined by

$$\pi(a_i, a_j) = \sum_{k=1}^{m} w_k P_k(d_k) = \sum_{k=1}^{m} w_k P_k(f_k(a_i) - f_k(a_j)),\ \forall a_i, a_j \in A \tag{3}$$

where $w_k \in W = \{w_1, w_2, \ldots, w_m\}$ is a weight for k-th criterion which is a measure for the relative importance of each criterion. The leaving flow and the entering flow of a_i are respectively given by

$$\phi^+(a_i) = \frac{1}{n-1}\sum_{\substack{j=1 \\ j\ne i}}^{n} \pi(a_i, a_j), \qquad \phi^-(a_i) = \frac{1}{n-1}\sum_{\substack{j=1 \\ j\ne i}}^{n} \pi(a_j, a_i) \tag{4}$$

The basic premise is that the higher the leaving flow and the lower the entering flow, the better the alternative. PROMETHEE II method is a total ranking method based on the evaluation of the net flow

$$\phi(a_i) = \phi^+(a_i) - \phi^-(a_i), \forall a_i \in A \tag{5}$$

The higher the net flow the better the alternative.

3 Interval-PROMETHEE

As we can see above, the original PROMETHEE method is designed for a single-valued number. When some uncertainties inherent are represented as interval numbers, the interval-PROMETHEE method is then required. This comes from the fact that in most cases the input data cannot be defined within a reasonable degree of accuracy. This imprecision is sometimes treated as an interval number. So the regular PROMETHEE algorithm will be generalized to the interval-PROMETHEE.

3.1 Interval number

An interval number x has such a form: x = [a, b], a < b , where a and b are all real number. The interval number set are recorded as I(R). Obviously for x = [a, b] ∈ I(R), if a = b , then x = a = b is an ordinary real, so R⊂ I(R). The basic operations with interval numbers are summarized in Table1.

Table 1: the basic operations with interval numbers

Addition	[a, b]+[c, d] = [a+c, b+d]
Subtraction	[a, b]- [c, d] = [a- d, b-c]
Multiplication	[a, b] {c, d] = [min{ac, bd , ad, bc}, max{ ac, bd , ad, bc }]
Division	[a, b]/ [c, d] = [min{a/c, b/d , a/d, b/c}, max{ a/c, b/d , a/d, b/c }] , 0∉[c, d]

3.2 Interval-PROMETHEE

When the performance of a_i and a_j corresponding to the criterion f_k are represented by interval numbers, the $d_k = f_k(a_i) - f_k(a_j)$ is the interval number (u , v), and $P_k(d_k)$ in (2) between a_i and a_j based on f_k is expressed as:

$$P_k(d_k) = \begin{cases} 0, \ if \ u \le 0 \\ \dfrac{(u,v)}{q_k}, if \ 0 \le u, v \le q_k \\ 1, \ if \ v > q_k \end{cases} \tag{6}$$

where q_k could be expressed as interval, but for simplicity we take single value numbers. Then the procedure of the PROMETHEE method described in (3)-(5) are followed step by step by the interval number calculations. Similarly, other parameters, are all considered as regular data with precise numerical values.

Finally, when interval data are included, the net flows are all interval numbers, so the final ranking problem is boiled down to the ranking of interval numbers. Thus the comparison of two intervals plays a crucial role. An enhanced ranking approach for interval numbers presented by Li (2004)[8] was used in this paper which is based on the possibility degree which represents the degree of one interval is greater than another interval. This methodology is briefly described below.

Let $I_a = [\underline{a}, \overline{a}]$ and $I_b = [\underline{b}, \overline{b}]$ are two intervals, if $\underline{a} = \underline{b}$ and $\overline{a} = \overline{b}$, then $I_a = I_b$; if the possibility degree of $I_a > I_b$ denoted as $P_{I_a > I_b}$ is greater than 0, then $I_a > I_b$; if $P_{I_a > I_b} < 0$, then $I_a < I_b$. The possibility degree of $I_a > I_b$ is defined as

$$P_{I_a > I_b} = \begin{cases} 1, & if \quad \underline{a} \geq \overline{b} \\ \dfrac{(\overline{a} - \overline{b}) - (\underline{b} - \underline{a})}{\overline{a} - \underline{a}}, & if \quad \underline{a} \leq \underline{b} < \overline{b} \leq \overline{a} \\ \dfrac{\overline{a} - \overline{b}}{\overline{a} - \underline{a}} + \left(\dfrac{\overline{b} - \underline{a}}{\overline{a} - \underline{a}} \right) \cdot \left(\dfrac{\underline{a} - \underline{b}}{\overline{b} - \underline{b}} \right), & if \quad \underline{b} \leq \underline{a} < \overline{b} \leq \overline{a} \end{cases} \tag{7}$$

If $I_b = [\underline{b}, \overline{b}]$ degrades to a single value, i.e., $I_b = [\underline{b}, \overline{b}] = b$, the possibility degree of $I_a > I_b$ is given by

$$P_{I_a > I_b} = \begin{cases} 1, & if\ b < \underline{a} \\ \dfrac{(\overline{a} - b) - (b - \underline{a})}{\overline{a} - \underline{a}}, & if\ \underline{a} < b \leq \overline{a} \\ -1, & if\ b > \overline{a} \end{cases} \tag{8}$$

If $I_a = [\underline{a}, \overline{a}] = a$ and $I_b = [\underline{b}, \overline{b}] = b$, the comparison of two intervals become ranking two real numerical values. In this scenario, the possibility degree is then defined as

$$P_{I_a > I_b} = \begin{cases} 1, & if \quad a > b \\ 0, & if \quad a = b \\ -1, & if \quad a < b \end{cases} \tag{9}$$

The comparison matrix of possibility degree for m interval numbers is determined by

$$P = \begin{pmatrix} 0 & P_{12} & \cdots & P_{1m} \\ P_{21} & 0 & \cdots & P_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ P_{m1} & P_{m2} & \cdots & 0 \end{pmatrix} \tag{10}$$

where P_{ij} is the possibility degree of $I_{a_i} > I_{a_j}$ and meets $-1 \leq P_{ij} \leq 1$ and $P_{ij} + P_{ji} = 0$. Let

$$r_i = \sum_{k=1}^{m} P_{ik}, i = 1, 2, \cdots, m \tag{11}$$

One gets $R = (r_1, r_2, \cdots, r_m)^T$, the comparison of interval numbers then becomes ranking r_i, i.e., the higher the r_i, the higher the interval number.

4 Application to Value investing

The philosophy behind value investing is that the intrinsic value determines the stock price of a firm and the stock price fluctuates around the value; the outstanding performance suggests that the firm has great potential and ability to grow and profit more, which lead to a higher intrinsic value, and thus the current below-intrinsic-value price is expected to rise, therefore if an investor buys and holds it now and he or she will surely make money in a long time horizon.

The five criteria for measuring the growth potential of a firm suggested by Warren Buffet are return on assets(denoted as f_1), increasing ratio of sales(f_2), increasing ratio of equity(f_3), increasing ratio of earnings per share(f_4), increasing ratio of free cash flow(f_5). These five criteria provide a relatively overall evaluation of the ability of a firm to grow and profit. The five criteria are listed in Table 2.

Table 2: Five criteria for selecting stocks in value investing

	Criterion	Definition	Min /Max	Unit
$f1$	Return on assets	$\frac{\text{Earnings before interest and taxes}}{\text{total assets}}$	Max	Percentage
$f2$	Increasing ratio of sales	$\frac{\text{Sales}_t - \text{Sales}_{t-1}}{\text{Sales}_{t-1}}$	Max	Percentage
$f3$	Increasing ratio of equity	$\frac{\text{Equity}_t - \text{Equity}_{t-1}}{\text{Equity}_{t-1}}$	Max	Percentage
$f4$	Increasing ratio of earnings per share	$\frac{[\text{earningspershare}]_t - [\text{earningspershare}]_{t-1}}{[\text{earningspershare}]_{t-1}}$	Max	Percentage
$f5$	Increasing ratio of free cash flow	$\frac{[\text{freecashflow}]_t - [\text{freecashflow}]_{t-1}}{[\text{freecashflow}]_{t-1}}$	Max	Percentage

The performance can be calculated based on the firms' financial statements. In this paper, the finance data are obtained through CSMAR4.0. We randomly select 20 firms from the Shanghai Stock Exchange. We compute the five criteria annual performances for each firm for the financial years 2007, 2008, 2009. And we use the past data to develop an interval number representation for each criterion and each firm. The interval representations are listed in Table 3.

Table 3 Five criteria performance for 20 randomly selected firms (percentage)

600375	(4.67,15.06)	(-11.35,41.23)	(-42.68,265.32)	(0.81,13.19)	(-251.72,419.13)
600252	(7.77,13.17)	(-11.02,119.81)	(29.41,122.69)	(8.96,27.65)	(-111.95,240.16)
600031	(16.02,27.58)	(20.01,99.91)	(-48.74,60.74)	(22.54,83.33)	(-360.05,178.91)
600712	(15.44,34.78)	(10.56,118.26)	(15.45,79.25)	(12.74,21.34)	(7.33,12.27)
600600	(9.50,13.23)	(12.50,17.40)	(22.70,71.44)	(3.91,33.43)	(-3.59,125.90)
600195	(14.52,18.41)	(6.01,15.95)	(6.88,43.76)	(2.03,15.93)	(-128.68,290.35)

600233	(9.72,19.24)	(-2.66,26.23)	(-44.26,126.46)	(10.63,17.15)	(55.79,469.65)
600750	(11.46,17.27)	(-1.06,29.66)	(-37.33,53.86)	(10.68,15.69)	(14.55,42.40)
600528	(7.94,12.94)	(29.87,73.11)	(-14.97,133.41)	(10.54,94.02)	(-507.79,39.58)
600859	(8.84,17.39)	(9.09,39.93)	(12.80,34.72)	(-31.90,74.79)	(-46.60,91.76)
600101	(6.73,10.62)	(-18.57,35.52)	(-136.11,89.41)	(6.67,28.88)	(-52.27,29.26)
600697	(9.96,14.11)	(14.42,60.82)	(3.48,47.02)	(6.89,12.47)	(-190.01,352.76)
600708	(12.53,14.78)	(-1.22,20.71)	(-22.20,9.84)	(-12.29,26.32)	(-67.62,204.84)
600835	(11.64,15.75)	(5.16,26.08)	(-16.58,34.90)	(9.20,9.47)	(-51.52,59.45)
600664	(11.31,15.29)	(-9.86,16.95)	(2.97,45.82)	(1.81,14.54)	(7.89,29.40)
600829	(16.83,18.93)	(1.69,14.38)	(3.25,26.24)	(0.47,29.76)	(-21.89,45.08)
600269	(11.98,13.79)	(5.39,35.27)	(-43.94,35.77)	(14.05,21.40)	(-126.30,55.72)
600611	(10.40,12.23)	(-29.69,19.83)	(-42.51,58.64)	(-4.97,51.03)	(-265.10,147.93)
600519	(31.05,5.44)	(13.88,47.82)	(13.8,88.4)	(27.37,39.60)	(3.12,90.24)
600066	(14.58,30.72)	(5.35,59.06)	(-7.5,93.75)	(-39.86,107.63)	(-113.08,407.29)

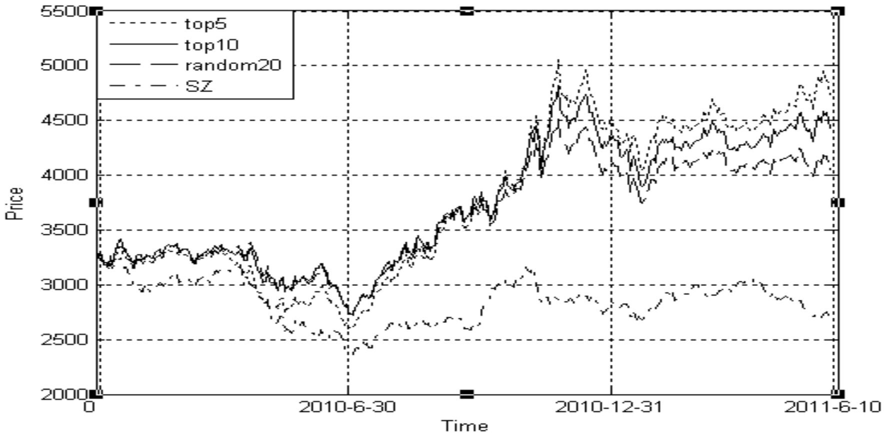
With the interval-PROMETHEE methods presented above, we rank the above stocks and get the order listed in Table 4. Here we list only the top10

Table 4 The ranking result of the top10 firms

Code	Interval of net flow		Value of r_i	Rank
600712	-0.27885	0.514948	30.52631	1
600252	-0.30656	0.555963	30.39054	2
600375	-0.38101	0.520593	16.65066	3
600031	-0.32511	0.433371	15.11768	4
600519	-0.29406	0.348474	9.074004	5
600600	-0.33066	0.332213	1.346427	6
600750	-0.32896	0.316332	-0.63525	7
600195	-0.3301	0.315249	-0.96069	8
600528	-0.33835	0.321953	-1.23726	9
600101	-0.38084	0.360621	-2.02337	10

The higher the value of r_i , the higher the rank of the firm and the higher the potential of the stock price to grow. The price with rank 1 is supposed to have higher potential to grow than that with rank 2. For the complex nature of finance market, it is not realistic to expect stock with rank n to grow faster than that with that with rank n+1. But it is natural to expect that the portfolio value with top m stocks grows faster than that with top n stocks, where m is less than n. Thus, the effectiveness of this interval ranking method can be verified by the investment effect of portfolios with top m stocks and top n stocks.

For verifying the feasibility of the interval-PROMETHEE method, we construct portfolios with top 5, top 10 and 20 stocks randomly selected, and each stock with 1000 shares. Suppose we invested these three portfolios right after the end of 2009, and on the data Jan 04, 2010. We kept them until June 10, 2011 without any changing of the portfolios. For comparing the investment effect, the portfolios are adjusted to be equal at the right beginning and adjusted with the same ratio in the following. For example, suppose we take the Shanghai Index value 3000, on Jan 04 2010 as the base point, the portfolio value was 30000, then we divide the portfolio value by 10(30000/3000=10), and thus the adjusted portfolio value is 3000 equal to the Shanghai index at the very beginning, then for each day after, the adjusted portfolio is set to the real value divided by 10, so that they are comparable. We adjusted the three portfolios as above mentioned and the evolution of the portfolios are plotted in Graph 1.



Graph 1 The portfolio values

As we can see from the Graph1, the portfolio value with the top 5 stocks stays above that with the top 10 stocks in the long run. Similarly, the top10 stocks stays above the 20 randomly selected stocks throughout the investment period. This fact suggests that the ranking methods we used is effective. By the way, the first two portfolios perform better than the Shanghai index which represents the average performance of the Chinese stock market.

Table 5 The distributions of return rates

	2010-6-30		2010-12-31		2011-6-10	
	mean	std	mean	std	mean	std
top5	-0.428817	0.223669	0.362508076	0.273947	0.3625081	0.261254
top10	-0.293117	0.215117	0.319870755	0.248416	0.3198708	0.238435
random20	-0.30083	0.221058	0.263547856	0.221058	0.2635479	0.070005
SZ	-0.639363	0.229973	-0.07600934	0.219195	-0.076009	0.122188

Furthermore, we compute the distribution of the annual rate of the portfolios from the beginning up to 2010-06-30, 2010-12-31 and 2011-06-10 respectively. The corresponding investment periods are 6 months, 12 months and 17 months respectively. The mean value and the standard deviation for the annual return distribution are listed in Table 5. Up to 2010-06-30, the average value of the return of the top 5 portfolio is -42.88%, the top 10 portfolio has the average return ratio -29.31%, although they are both loss and the top 5 loss is higher than the top 10 loss, but this states clearly that the market is instability. Up to 2010-12-31, the top-5 portfolio has the average return rate 36.25%, which is higher than that of the top-10 portfolio which is 31.98%. Similarly we can see the better performance of the top-5 portfolio up to 2011-06-10 than the top 10 , random20 portfolio and the Shanghai index. On the other hand, the risk of a portfolio is measure by the standard deviation (std) of the return rate. The smaller the std, the less risky the portfolio. For the above mentioned 3 investment periods, the stds of the top-5 portfolio are almost the same with the top10. All in all, the top 5portfolio brings higher return in the same risk with the top 10 portfolio in the long run. This empirical study shows the effectiveness of the application of interval-PROMETHEE method to value investing.

5 Conclusions

This paper applied the interval-PROMETHEE method to the investment decision-making process of value investing. Five criteria were used to evaluate the growth-potential performance suggested by Buffet and were represented by interval numbers based on the real financial data gathered from the data base CSMAR4.0. The interval-PROMETHEE method was applied to rank 20 randomly selected stocks in Shanghai Stock Exchange. The portfolio with the top 5 stocks was proven to have higher return than the top 10 stocks within the whole 17-month period. The empirical study showed the effectiveness of the interval-PROMETHEE method in the decision making process of value investing.

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