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The Team Orienteering Problem with Capacity Constraint and Time Window

Zhenping Li[1,*] Xianman Hu[2]

[1]School of Information, Beijing Wuzi University, Beijing 101149,China
[2]Department of Postgraduate, Beijing Wuzi University, Beijing 101149,China

Abstract The Team Orienteering Problem (TOP) is defined as to find a set of vehicle walks such that the total collected reward received from visiting a subset of customers is maximized and the length of each vehicle walk is restricted by a pre-specified limit. The team orienteering problem with capacity constraint and time window(TOPCT) is an extension of the TOP where each customer has a demand and a time window, each customer must be visited in the given time window and the capacity of each vehicle is limited. In this paper, we first formulate the TOPCT problem into an integer linear programming based on network flow theory and solve it to obtain the exact optimal solution. Then we give computational results which demonstrate the efficient of the model and algorithm.

Key words: Team Orienteering Problem with Capacity Constraint and Time Window; Integer Linear Programming; Computational Result.

1 Introduction

The orienteering problem (OP) is a well established problem in combinatorial optimization which is introduced in [4]. In the orienteering problem, we are given an edge-weighted graph $G(V,E)$, where $V = \{v_1, v_2, \cdots, v_n\}$, each node $v_i \in V$ has an associated non-negative score w_i, each edge $e_{ij} \in E$ has a weight d_{ij} representing the length or time taken to travel from node v_i to node v_j. Given a starting node $s \in V$ and an ending node $t \in V$, a length or time limit D_{lim}. The goal is to find a walk beginning at s and ending at t of total length at most D_{lim} that maximizes the total score of distinct vertices visited by the walk.

In the past decades, researchers have proposed a large number of heuristics algorithms for the OP. Tsiligrides [11] proposed a stochastic algorithm for the OP. Wang et al [7] applied artificial neural networks to the OP and obtained high-quality results. Chao et al [1] applied deterministic annealing to the OP and also obtained high-quality results. Gendreau et al. [10] applied tabu search to the OP and obtained near-optimal solutions to instances with up to 300 nodes.

The Team Orienteering Problem (TOP) is a generalization of the OP in which a set of vehicle walks are constructed such that the total collected reward received from visiting

*corresponding author: lizhenping66@163.com

a subset of customers is maximized and the length of each vehicle walk is restricted by a pre-specified limit.

The team orienteering problem with capacity constraint and time windows(TOPCT) is an extension of the TOP where each customer has a demand and a time window, each customer must be visited in the given time window and the capacity of each vehicle is limited.

The TOPCT arises in many applications. Consider, for instance, there are m transport vehicles to deliver goods for customers at geographically distributed locations. The capacity of each vehicle is limited. The demand of each customer is given. If the goods are delivered within a specified time window, the vehicle can get some rewards from the customer, otherwise, it can get no reward. As there is often a limitation on capacity of each vehicle, and each customer has different demand and service time window. Thus, it may not be possible to include all customers requiring service in the service schedules for a given day. Instead, a subset of the customers will be selected. The problem is which customers to choose for inclusion in each of the service vehicle schedules such that the total rewards obtained can be maximal.

Although there are many algorithms proposed for the OP or TOP, they can not be used for the TOPCT since their constraints are different. In this paper, the TOPCT is formulated into an integer linear programming model based on the network flow theory in section 2, then the optimal solution is obtained by solving the integer linear programming. Section 3 gives the computational results. Our model and algorithm are the first approaches for the TOPCT, and high-qualified solutions can be obtained by these model and algorithm on small size of example. Finally, the conclusions are given in section 4.

2 The Team Orienteering Problem with Capacity Constraint and Time Window

The Team Orienteering Problem with Capacity Constraint and Time Window (TOPC-T) can be formulated as follows: Given an undirected weighted graph $G = (V,E)$ with node set $V = \{v_1, v_2, \cdots, v_n\}$, edge set E. Each node v_i has a demand q_i, a time window $[e_i, l_i]$, a serving time τ_i and a weight $w_i \geq 0$. Where τ_i can be interpreted as the time needed to serving for node v_i and w_i can be interpreted as reward or benefit obtained by serving for node v_i. The starting node is v_1 and the ending node is v_n. Each edge e_{ij} has weight t_{ij} which can be interpreted as time for a vehicle to travel through edge e_{ij}. The TOPCT is to find m walks $P_1, P_2, \cdots, P_m$ in graph G which satisfies the following conditions. (1) P_i $(i = 1,2,\cdots,m)$ is starting from node v_1 and ending at node v_n; (2) the total demands of each walk P_i $(i = 1,2,\cdots,m)$ is no more than the given capacity C_i $(i = 1,2,\cdots,m)$;(3) the total distance of each walk P_i $(i = 1,2,\cdots,m)$ is no more than the given limit L_i $(i = 1,2,\cdots,m)$;(4) for each node in a walk, the time arriving at it satisfies its time window constraints; (5)the total reward or benefit of nodes in all walks P_i $(i = 1,2,\cdots,m)$ are maximal.

Using the network flow theory, suppose there are m flows of size $n-1$ entering into the network from source node v_1. Then each flow flows through one edge and enters into another node. Whenever a flow flows into a node, a unit of flow is absorbed by the node, the other units will flow out through another edge. Finally, when a flow enters into node v_n, all the units will be absorbed which means that node v_n is the ending node where all the flows entering into it will be absorbed. The nodes and edges along each flow consists of a walk from v_1 to v_n. The TOPCT can be formulated into an integer linear programming model.

Let $K = \{1, 2, \cdots, m\}$ be the set of vehicles; $D = \{v_1, v_2, \cdots, v_n\}$ be the set of nodes, where v_1 is the starting node and v_n is the ending node; t_{ij} is the time passing through edge e_{ij}; τ_i is the serving time of node v_i where $\tau_1 = \tau_n = 0$; C_k is the capacity of vehicle k; q_i is the demand of node v_i; w_i is the reward obtained from serving for node v_i; $[e_i, l_i]$ is the time window of node v_i; L_k is the time limit for vehicle k to arrive at node v_n.

Set x_{ik} be a binary variable, $x_{ik} = 1$ indicates that vehicle k serving for node v_i, while $x_{ij} = 0$ otherwise. y_{ijk} be a binary variable, $y_{ijk} = 1$ indicates that after vehicle k serve for node v_i then it directly go to serve for node v_j; $y_{ijk} = 0$ otherwise. r_{ik} be the time of vehicle k arriving at node v_i. If vehicle k do not serve for node v_i, then $r_{ik} = 0$. M is a very large positive number.

The TOPCT can be formulated into the following integer linear programming model.

$$\max \sum_{i=2}^{n-1} \sum_{k=1}^{m} w_i x_{ik} \quad (1.1)$$

$$s.t. \begin{cases} \sum_{j=2}^{n} y_{1jk} = 1, k = 1, 2, \cdots, m & (1.2) \\ \sum_{i=1}^{n-1} y_{ink} = 1, k = 1, 2, \cdots, m & (1.3) \\ \sum_{i=1}^{n-1} y_{ijk} = \sum_{l=2}^{n} y_{jlk} = x_{jk}, j = 2, 3, \cdots n-1, k = 1, 2, \cdots, m & (1.4) \\ \sum_{k=1}^{m} x_{ik} \le 1, i = 2, 3, \cdots, n-1 & (1.5) \\ \sum_{i=2}^{n-1} q_i x_{ik} \le C_k, k = 1, 2, \cdots, m & (1.6) \\ \sum_{i=1}^{n-1} \sum_{j=2}^{n} y_{ijk}(t_{ij} + \tau_i) \le L_k, k = 1, 2, \cdots, m & (1.7) \\ r_{1k} = 0, k = 1, 2, \cdots, m & (1.8) \\ r_{ik} + \tau_i + t_{ij} - r_{jk} \le M(1 - y_{ijk}), i, j = 1, 2, \cdots, n; k = 1, 2, \cdots, m & (1.9) \\ e_j x_{jk} \le r_{jk} \le l_j x_{jk}, j = 2, \cdots, n-1; k = 1, 2, \cdots, m & (1.10) \\ r_{nk} \le L_k, k = 1, 2, \cdots, m & (1.11) \\ x_{ik} \in \{0, 1\}, y_{ijk} \in \{0, 1\}, i, j = 1, 2, \cdots, n; k = 1, 2, \cdots, m & (1.12) \\ r_{jk} \ge 0, j = 1, 2, \cdots, n; k = 1, 2, \cdots, m & (1.13) \end{cases}$$

The objective function (1.1) maximizes the total reward in all walks.

Constraints (1.2) and (1.3) means that every walk must start from node v_1 and end at node v_n. Constraint (1.4) means that for every vehicle k, if it enters into node v_j, then it must leave node v_j; Constraint (1.5) means that for every node v_i, there is at most one vehicle provide serve for it; Constraint (1.6) is the capacity constraint of vehicle. Constraint (1.7) means that for vehicle k, the total time used for passing through its walk is no more than L_k; Constraint (1.8) means that for every vehicle, the start leaving time is 0; Constraint (1.9) means that the arriving time relationship for two sequence visiting node in a vehicle's walk; Constraint (1.10) means that for every node v_j, if vehicle k passing through it, then the arriving time of vehicle k must satisfy its time window constraint; Constraint (1.11) means that for vehicle k, the arriving time at node v_n should not exceed L_k; Constraint (1.12) indicates that x_{ij},y_{ijk} are the binary variables; Constraint (1.13)indicates that r_{jk} are nonnegative continuous variables.

Table 1: The demand , serve time, time window and the reward of 14 customers

Customer	Demand	Server time	The earlist serve time	The latest serve time	Benefit
1	0.12	0.3	1	5	80
2	0.32	0.5	1	3.5	250
3	0.15	0.5	2	6	100
4	0.2	0.5	1.5	5	180
5	0.06	0.4	6	8.5	50
6	0.3	0.5	0.5	3	200
7	0.25	0.5	5	8	200
8	0.35	0.5	4	7	400
9	0.1	0.3	1	5	80
10	0.08	0.2	5	7	60
11	0.1	0.2	0	5	70
12	0.12	0.3	0.5	4	80
13	0.2	0.4	3	6	200
14	0.1	0.4	6	8.5	100

3 Computational Results

For the small size of the TOPCT problem (for example the number of network nodes is no more than 30), we can directly solve the integer linear programming model to obtain global optimal solutions.

We use an example of 2 transportation vehicles to deliver goods from one distribution center to 14 customers. The sites of the distribution center and 14 customers are illustrated in figure 1. The capacities of two vehicles are 1 and 1.5 tons respectively. For each customer, the demand , the serve time, the time window, the reward are listed in table 1. The time distance between any pair of distribution center and customers are listed in table 2.

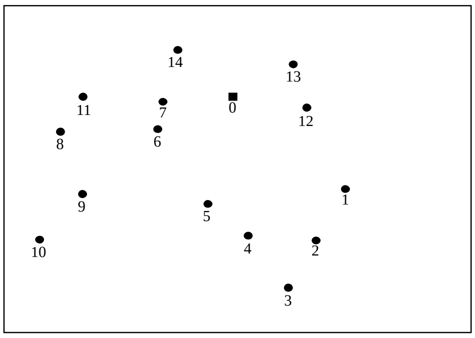

Figure 1: The site of distribution center and customers, where node 0 is the distribution center and nodes 1-14 are customers.

Table 2: The time distance between distribution center and 14 customers

Node	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	1.04	1.24	1.5	1.1	0.84	0.56	0.48	1.18	1.26	1.66	1	0.52	0.48	0.52
1	1.04	0	0.42	0.84	0.74	0.9	1.32	1.38	1.92	1.7	2	1.91	0.69	1.05	1.58
2	1.24	0.42	0	0.42	0.46	0.78	1.36	1.46	1.88	1.56	1.78	1.96	1.15	1.4	1.79
3	1.5	0.84	0.42	0	0.44	0.82	1.48	1.62	1.9	1.48	1.62	2.06	1.43	1.77	2.03
4	1.1	0.74	0.46	0.44	0	0.38	1.04	1.18	1.48	1.1	1.32	1.08	1.52	1.43	0.73
5	0.84	0.9	0.78	0.82	0.38	0	0.66	0.84	1.12	0.8	1.1	1.2	1.01	1.24	1.24
6	0.56	1.32	1.36	1.48	1.04	0.66	0	0.2	0.64	0.7	1.12	0.57	1.03	1.05	0.65
7	0.48	1.38	1.46	1.62	1.18	0.84	0.2	0	0.7	0.88	1.28	0.54	0.98	0.93	0.43
8	1.18	1.92	1.88	1.9	1.48	1.12	0.64	0.7	0	0.52	0.82	0.33	1.68	1.66	1.03
9	1.26	1.7	1.56	1.48	1.1	0.8	0.7	0.88	0.52	0	0.4	0.79	1.66	1.75	1.31
10	1.66	2	1.78	1.62	1.32	1.1	1.12	1.28	0.82	0.4	0	1.17	2.08	2.19	1.76
11	1	1.91	1.96	2.06	1.08	1.2	0.57	0.54	0.33	0.79	1.17	0	1.52	1.43	0.73
12	0.52	0.69	1.15	1.43	1.52	1.01	1.03	0.98	1.68	1.66	2.08	1.52	0	0.36	1
13	0.48	1.05	1.4	1.77	1.43	1.24	1.05	0.93	1.66	1.75	2.19	1.43	0.36	0	0.79
14	0.52	1.58	1.79	2.03	0.73	1.24	0.65	0.43	1.03	1.31	1.76	0.73	1	0.79	0

In this section, we select the distribution center (node 0) as both the starting and ending nodes of two vehicles' walks. Solving the integer linear programming model by Lingo software, we can obtain the optimal walks of both vehicles, which are $0-12-6-11-9-13-14-0$ for vehicle 1 and $0-1-2-4-3-10-8-7-0$ for vehicle 2. The maximum total reward is 2000.

4 Conclusion

In this paper, we proposed the team orienteering problem with capacity constraint and time window (TOPCT), and formulated it into an integer linear programming model. By solving the integer linear programming, we can obtain the global optimal solution of the TOPCT. We applied our model and algorithm to an example and obtain the optimal result.

Although we have used our model and algorithm on an example and obtained the optimal solution. We have not test the algorithms on large size of examples with more than 30 nodes. In the future, we will explore some more algorithms for the large size of TOPCT examples.

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