The Tenth International Symposium on Operations Research and Its Applications (ISORA 2011)
Dunhuang, China, August 28–31, 2011

Sensitivity Analysis for Wealth Process

Laouar Amel [1] Boukhetala Kamel [2]

1 ENSSMAL, Algiers, Alegria
2 USTHB, Algiers, Alegria
amel.laouar@gmail.com kboukhetala@usthb.dz

Abstract We consider an insurance business with alpha-stable claims process, we assume that claims are large, i.e. that the distribution of their sizes is heavy tailed and investment portfolio consisting of a non risky and risky asset. The models are described and then the sensitivity of the models with respect to changes in parameters is studied by simulation.

Keywords α-stable process; interest rate models; stochastic control

1 Introduction

Brownian motion has been studied for a long time and its usefulness in stochastic modeling is well accepted. However, Gaussian process and variables do not allow for large fluctuations and may sometimes be inadequate for modeling high variability, especially in finance and non life insurance. Both stable random variable's and processes arise naturally as alternative modeling tools. For this we propose to study the behavior of the wealth for an insurance company, with a claims process described by a stochastic differential equation governed by α -stable motion, and the company is also allowed to invest in stock market. A part of the capital is invested in non risky asset, and the rest in a risky asset, modeled by diffusion process; with stochastic drift and we propose for this latter the Vasicek and CIR model's, we make a comparison of investment process for these models. Finally we compare by simulation with Maple the behavior of the risk and wealth processes with respect to change in parameters of α -stable claim process.

2 Main results

The risk process is given by the following equation: $X(t) = u + ct - S(t)$ where u>0 is the initial capital, c>0 is the constant premium rate over time and $S(t)$ is the claim process, supposed α -stable and described by the following differential equation:

$$\frac{dS(t)}{S(t)} = \mu\, dt + \gamma L_{\alpha}(t), \quad S(0) = s$$

The insurance portfolio is composed of one riskless asset B_t and risky asset $Z_{t,}$:

$$\frac{dB(t)}{B(t)} = \tau\, dt, \quad B(0) = 1 \qquad \text{and} \qquad \frac{dZ_t}{Z(t)} = (r_t - q)dt + \sigma dw_t^1, \quad Z(0) = z$$

r_t is the stochastic drift. We propose the Vasicek and Cox-Ingersoll-Ross models ;

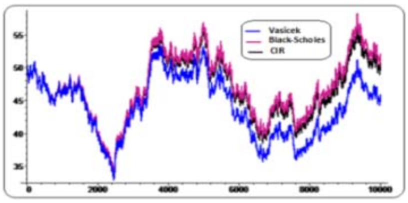

Fig. 1: Investment process for different drift coefficient

If at time t the insurer has wealth R_t and invests an amount A_t of money in the stock and the remaining reserve $(R_t - A_t)$ in the bond, the process of wealth is giving by:

$$\begin{cases} dR_t = [c - \mu S_t + \tau (R_t - A_t) + (r_t - q) Z_t]dt + \sigma_1 A_t Z_t dw_t^1 - \gamma S_t L_\alpha(t) \\ R_0 = u, \quad S_0 = s, \quad Z_0 = z \quad , r_0 = r \end{cases}$$

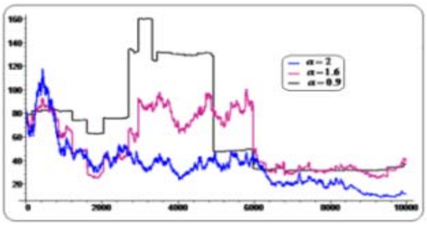

Fig.2: Risk process

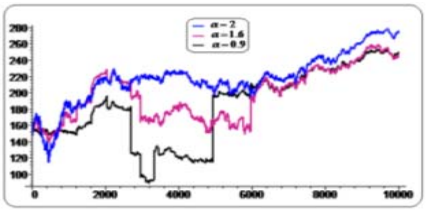

Fig.3: Reserve process

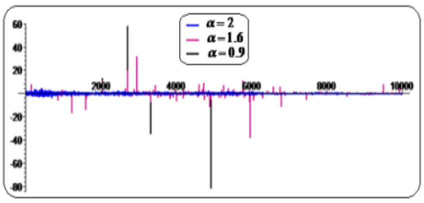

Fig.4: Increments of the risk process

In this part we analyze the effect of the index parameter:
we present in figure 2 some simulations of trajectories of claim and reserve processes for a few values of the parameter α , We see clearly that trajectories have almost the same behaviour, however when a parameter α taking smaller values, the “jumps” of trajectories become bigger, Where we can explain that by realization of rare event. We can better visualize this difference in the increment representation of claim process in figure **4.**

α -Stables processes present a big jumps in their trajectories when alpha is small (close to 0), and the jumps become more and more smaller when α near to 2, as we can see in Fig.7, for α=2 the increments of risk process are between -20 and 20, for α=1.6, the increments are between -30 and 30, finally for α=0.9, the increments are between -80 and 60. We recall that for $\alpha=2$, the standard alpha stable motion is the Brownian motion, hence the risk process become the classical risk process (Cramer-Lundberg model), we observe clearly the difference between these two models, the stable process produce more fluctuations.

3 Conclusion

The choice of the stable processes is of for the fact that they consider the extreme events, thus there are more adapted for the modeling wealth of insurance. All simulations are going to allow us to make a comparative study of the probability of ruin of the various models according to various parameters, and we envisage an analytical research to resolve this problem, for determine the optimal strategy of investment, and minimize ruin probability.

References

[1] Brigo, D. and Mercurio, F. *Interest Rate Models - Theory and Practice*. ISBN 978-3-540-22149-4 2nd ed.Springer-Verlag Berlin Heidelberg, 2007.

[2] Hipp, C. and Plum, M. *Optimal investment for insurers*. Insurance, Mathematics and Economics 27, 2000.

[3] Janiki, A. and Weron, A. *Simulation and chaotic behaviour of α-stable process*. Marcel Dekker, INC, New York .1994.