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An Emergency Evacuation Planning Model using the Universally Quickest Flow

Atsushi Takizawa[1] Masaki Inoue[1] Naoki Katoh[1]

[1]Department of Architecture and Architectural Engineering, Kyoto University, Japan

Abstract In recent years, catastrophic disasters by massive earthquakes are increasing in the world, and disaster management is required more than ever. In the case of disasters such as tsunami from which the slight delay of evacuation deprives of life. In this article, we formalize the emergency evacuation planning model for the evacuation from tsunami etc. based on the idea of the universally quickest flow. We will show that there does not always exist a universally quickest flow when the capacity constraint of refuges is taken into account. Therefore, we will propose an alternative criterion that approximates a universally quickest flow, and presents an algorithm for finding an optimal flow for this criterion. Numerical experiments of the evacuation aimed at a local city in Japan are carried out where tsunami damages are assumed to occur when a big earthquake occurs in the ocean nearby.

Keywords Emergency Evacuation Planning Model; Universally Quickest Flow; Tsunami

1 Introduction

In recent years, catastrophic disasters by massive earthquakes are increasing in the world, and disaster management is required more than ever. For example, Tohoku-Pacific Ocean Earthquake happened in Japan on March 11, 2011, and serious damage was caused by tsunami. Although disaster prevention in civil and architectural engineering fields of Japan had been considered mainly from physical aspects previously, it becomes clear that it is difficult to prevent large tsunami physically. Therefore, the disaster prevention from non-physical aspect, such as city planning and evacuation planning, is considered to become more important from now on.

The mathematical approach seems to be useful in order to ensure the evidence of the planning. The authors have formalized the evacuation planning problem by using the idea of network flow, and have proposed the evacuation model to find the fastest completion time of evacuees [6]. In our previous research, minimizing the evacuation time by which all evacuees completed the evacuation is the criterion for the evacuation planning. Let us denote this time by Θ^*. This problem can be reduced to the quickest flow problem of a given dynamic network. However, in the case of disasters such as tsunami in which the slight delay of evacuation deprives of life, we need to minimize not only Θ^* the evacuation completion time but also to take into account the other criterion that maximizes the number of evacuees who have already evacuated at an arbitrary time before Θ^*. The evacuation planning problem with this criterion can be formalized by so-called the universally quickest flow [7].

In contrast with the quickest flow, the universally quickest flow simultaneously maximizes the cumulative number of evacuees at an arbitrary time until Θ^*. Although Minieka [7] firstly proved that the universally quickest flow always exists, the running time of the algorithm for solving it remained to be pseudo-polynomial. Recently, a polynomial time algorithm for solving it was proposed by Baumnn and Skutella [1]. When we apply this model to the emergency evacuation problem, we have to consider capacity constraints of refuges. However, we will show an counterexample in which there does not exist a universally quickest flow in the presence of such constraints. We then propose an alternative criterion which in some sense approximates a universally quickest flow and will present an algorithm for finding an optimal flow with this criterion. Finally, numerical experiments of the evacuation planning are carried out aimed at a local city in Japan where tsunami damages are predicted when Nankai earthquake occurs.

2 Preliminaries

We denote by $\mathbb{R}_+$ and $\mathbb{Z}_+$ the sets of nonnegative reals and nonnegative integers, respectively. Before explaining our algorithm, we explain the basis of related network flow models.

2.1 Dynamic networks

Let $D = (V,A)$ be a directed graph which may have parallel arcs. A node v is said to be reachable from a node u when there is a directed path from u to v. We denote by $e = uv$ an arc e whose tail and head are u and v, respectively. If $e = uv$ has no parallel arc, we may simply write uv. Furthermore, for each $e \in A$ we denote by $t(e)$ and $h(e)$ the tail and the head of e, respectively. For each $X \subseteq V$, let $\delta_D(X)$ (resp. $\rho_D(X)$) be the set of arcs $e \in A$ with $t(e) \in X$ and $h(e) \notin X$ (resp. $t(e) \notin X$ and $h(e) \in X$).

Let $\mathcal{N} = (D = (V,A), c, \tau, b, S)$ be a *dynamic network* which consists of a directed graph D, a capacity function $c\colon A \to \mathbb{R}_+$, a transit time function $\tau\colon A \to \mathbb{Z}_+$, a supply function $b\colon V \to \mathbb{R}_+$ and a set of sinks $S \subseteq V$. Since we consider the evacuation to $s \in S$, we assume without loss of generality that for any $s \in S$ $\delta_D(s) = \varnothing$ and $b(s) = 0$, and s is reachable from every node.

We define a *dynamic flow* $f\colon A \times \mathbb{Z}_+ \to \mathbb{R}_+$ as follows. For each $e \in A$ and $\theta \in \mathbb{Z}_+$, we denote by $f(e,\theta)$ the flow rate entering e at time step θ which arrives at $h(t)$ at the time step $\theta + \tau(e)$. We call f *feasible* if it satisfies the *capacity constraint*

$$0 \le f(e,\theta) \le c(e) \quad (\forall e \in A \text{ and } \theta \in \mathbb{Z}_+),$$

the *flow conservation*

$$\sum_{e \in \delta_D(v)} \sum_{\theta=0}^{\Theta} f(e,\theta) - \sum_{e \in \rho_D(v)} \sum_{\theta=0}^{\Theta-\tau(e)} f(e,\theta) \le b(v) \quad (\forall v \in V \text{ and } \Theta \in \mathbb{Z}_+),$$

and the *demand constraint*

$$\sum_{s \in S} \sum_{e \in \rho_D(s)} \sum_{\theta=0}^{\Theta-\tau(e)} f(e,\theta) = b(V) \quad (\exists \Theta \in \mathbb{Z}_+). \tag{1}$$

For a feasible dynamic flow f, we define the *evacuation time* of f by the minimum time step Θ^* satisfying (1) and call it the *quickest flow*.

2.2 Time-expanded networks

Let $\mathcal{N}_s = (D_s = (V_s, A_s), c_s, b_s, S_s)$ be a *static network* which consists of a directed graph $D_s = (V_s, A_s)$, a capacity function $c_s \colon A_s \to \mathbb{R}_+$, a supply function $b_s \colon V_s \to \mathbb{R}_+$ and a set of sinks $S_s \subseteq V_s$. We call $f_s \colon A_s \to \mathbb{R}_+$ a *feasible flow* if it satisfies the *capacity constraint*

$$0 \leq f_s(e) \leq c_s(e) \quad (\forall e \in A_s)$$

and the *flow conservation*

$$\sum_{e \in \delta_{D_s}(v)} f_s(e) - \sum_{e \in \rho_{D_s}(v)} f_s(e) = b_s(v) \quad (\forall v \in V_s \setminus S_s)$$

In order to solve the decision version of the evacuation problem with time horizon Θ, Ford and Fulkerson [2, 3] introduced the *time-expanded network* $\mathcal{N}(\Theta)$ which is a static network in which for each $v \in V$ and $i \in \{0, \ldots, \Theta\}$ there exists a node $v(i)$, and for each $e = uv \in A$ and $i \in \{0, \ldots, \Theta - \tau(e)\}$ there exists an arc $e(i) = u(i)v(i + \tau(e))$ whose capacity is $c(e)$, and for each $v \in V$ and $i \in \{0, \ldots, \Theta - 1\}$ there exists a holdover arc $v(i)v(i+1)$ with infinite capacity. For each $v \in V$, the supply of $v(0)$ is set to $b(v)$ and the supplies of all the other nodes $v(i) (i \in \{1, \ldots, \Theta\})$ are set to zero. $\{s(i) \mid s \in S\}$ is aggregated into a super sink node $s^*(i)$ as well as an arc $(s(i), s^*(i))$ with infinite capacity (see Figure 1).

It is known [3] that there exists a feasible dynamic flow f whose evacuation completion time is at most Θ if and only if there exists a feasible flow in $\mathcal{N}(\Theta)$. Thus, the quickest flow can be found by testing the feasibility of $\mathcal{N}(\Theta)$ for different Θ's $\log \Theta^*$. Although we can decide if there exists a feasible flow in $\mathcal{N}(\Theta)$ by solving the maximum flow problem, the running time to test the feasibility of $\mathcal{N}(\Theta)$ is pseudo-polynomial because the size of the time-expanded network is proportional to Θ. In our experiments, we used the time-expanded network to find an optimal dynamic flow.

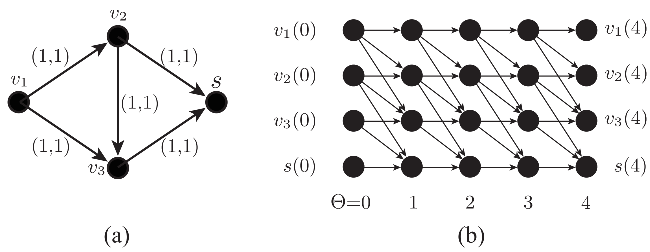

Figure 1: Illustration of a dynamic network (a) and its time expanded-network (b). All arcs are directed from left to right. (1,1) on each arc represents the capacity and transit time of the arc, respectively.

2.3 The lexicographic maximum flow

In this subsection, we will define a lexicographic maximum flow. Suppose we are given a static network $N_s = (D_s = V_s, A_s), c_s, b_s, S, T)$, where c_s is a capacity function from A to $\mathbb{R}^+$, b_s is a supply function from S to $\mathbb{R}^+$, a source node set $S \subseteq V$, and a sink

Algorithm 1

Let F be a zero flow and let $i = 0$.
For each $i = 0, 1, \dots, k$ **do**
 Find a maximum flow f_i from a super source node to x_i under the constraint that flow F is fixed.
 Let $F = F + f_i$.
End for
Output F.

Figure 2: An algorithm to find a lexicographic maximum flow.

set $T \subseteq V$ with $S \cap T = \emptyset$. When there are two or more sources, they can be aggregated into a single source by newly creating a super source node and adding arcs whose capacity is infinite from sources to the super source. Let $T = \{x_1, x_2, \dots, x_k\}$ denote the ordered set of sinks and $M(T')(T' \subset T)$ denote the maximum flow value of a feasible flow f which enters T'. Let $T(i) = \{x_1, x_2, \dots, x_i\}(i = 1, 2, \dots, k)$. Now, it is known [7] that there exists the maximum flow whose flow value is $M(T(i))$ for each $i = 1, \dots, n$, and it is called the *lexicographic maximum flow*. The lexicographic maximum flow can be obtained by Algorithm 1 described in Figure 2.

2.4 The universally quickest flow

For the set F of the feasible dynamic flows f, the flow $f^* \in F$ which satisfies

$$\sum_{s\in S}\sum_{e\in\delta^-(s)}\sum_{\theta=0}^{\Theta-\tau(e)} f^*(e,\theta) \geq \sum_{s\in S}\sum_{e\in\delta^-(s)}\sum_{\theta=0}^{\Theta-\tau(e)} f(e,\theta) \quad (\forall\Theta \text{ and } f \in F). \tag{2}$$

is called the *universally quickest flow*. An example of the universally quickest flow on the dynamic network in Figure 1 (a) is shown in Figure 3.

The time expanded network $\mathcal{N}(\Theta)$ for the quickest flow is extended in order to be adopted to find the universally quickest flow. As shown in Figure 4, super sink $s^*(i)$ is added at each $i \in \{0, \dots, \Theta\}$ and connected to each sink $s(i)(s \in S)$. Here, the lexicographic maximum flow considering the ordered set of sinks $\{s^*(0), \dots, s^*(i), \dots, s^*(\Theta)\}$ is the universally quickest flow when the evacuation completion time is Θ. Meanwhile, in this example, multiple source nodes are aggregated to a single one v^*, and arcs are drawn from it to each source node in order to change the original problem to a single source problem.

3 The emergency evacuation planning model

Based on the above preparation, we formulate the emergency evacuation model. Here we propose two models without/with the capacity constraint of sinks (refuges).

3.1 The model without the capacity constraint of refuges

The emergency evacuation planning problem without the capacity constraint of refuges is modeled as a dynamic network problem and is solved by (1) first finding a minimum

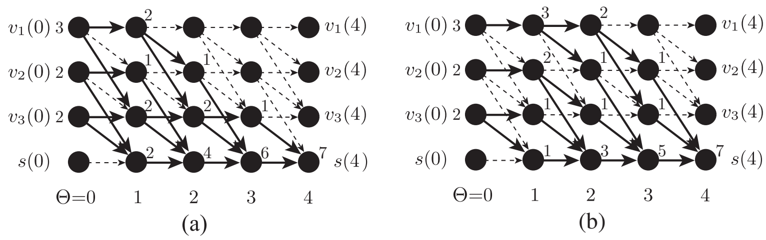

Figure 3: The universally quickest flow (a) and one of the quickest flows (b) on the dynamic network in Figure 1 (a) in which $b(v_1) = 3$, $b(v_2) = 2$ and $b(v_3) = 2$. Solid lines represent a flow, and a number attached to each node represents the amount of supply that passes that node (for a sink node it represents the amount of supply that reaches it).

evacuation completion time Θ^* and by finding a lexicographic maximum flow for a time-expanded network $N(\Theta^*)$.

3.2 The model with the capacity constraint of a refuge

The previous definition of the universally quickest flow does not consider the capacity of a refuge. However, in the actual situation of evacuation, there is an upper limit of refuge capacity, and if a refuge has no remaining capacity for evacuees that arrive later, they have to go to another refuge. This makes the evacuation time longer.

Therefore we discuss the existence of the universally quickest flow under the constraint that evacuees does not exceed the capacity of each refuge. Let $r\colon V \to \mathbb{Z}_+$ denote the capacity function of a refuge. This constraint is formulated as

$$\sum_{\theta=0}^{\Theta-\tau(e)} \sum_{e\in\delta^-(s)} f(e,\theta) \le r(s) \quad (\forall s \in S). \tag{3}$$

Figure 4 shows a counterexample for which there is no universally quickest flow where it takes two time units until completing the evacuation if two evacuees located at left and middle nodes go to the left sink and the right evacuee goes to the upper sink, respectively. However, if the evacuee located at the middle node goes to the top node, the evacuation completion time takes three time units.

Since there does not always exist a universally quickest flow, we will compute instead a lexicographically quickest flow which can be obtained greedily by computing the maximum flow with a single super sink $s^*(\theta)$ at every time step θ in the order of $\theta = 1,2,\ldots,\Theta^*$. Let $r_{remaining}(s)$ be the remaining capacity of a refuge s, and connect $s(\theta)$ to $s^*(\theta)$ by an arc with capacity $r_{remaining}(s)$. It is easy to see that at time θ this algorithm computes the flow which maximizes the cumulative number of evacuees at each time. Since this flow differs from the universally quickest flow, we call it the *lexicographically quickest flow* (lexico-quickest flow).

Since the completion time of a lexicographically quickest flow may become longer than that of a quickest flow, it is natural to ask whether there exists a lexicographically quickest flow with the constraint that its completion time is equal to that of a quickest

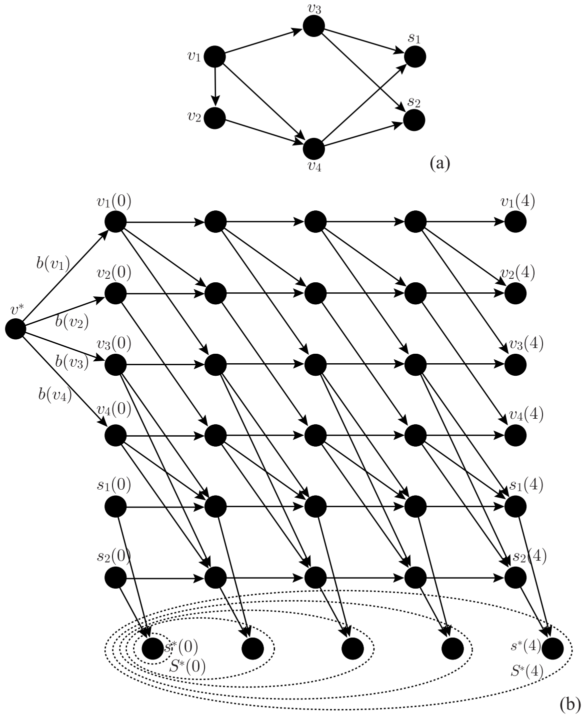

Figure 4: Illustration of a dynamic flow (a) and its time-expanded network (b) used to find a universally quickest flow where the transit time of all arcs is 1. $S^*(i)(i = 0,1,\ldots,4)$ represents the set of super sinks at time i.

flow. Such flow can be computed as follows. Let Θ^* denote the evacuation completion time of a quickest flow. The computation of a lexicographically quickest flow is done by sequentially computing a maximum flow (denoted by $f_{\max}(\theta)$) for $\theta = 0,1,\ldots$ in this order from a super source node to a super sink node $s^*(\theta)$ by taking refugee capacity constraints into account. Assuming that at time step $\theta - 1$, it is guaranteed that all evacuees who has not yet finished evacuation can complete the evacuation by the time Θ^*, we will explain how to compute a flow which arrives at supersink $s^*(\theta)$.

Notice that the maximum flow value obtained for time θ (denoted by $v_{\max}(\theta)$) stands for the maximum possible number of evacuees that complete the evacuation at time step θ. However, this may cause the delay of the evacuation completion time for the remaining evacuees who has not yet completed the evacuation at time θ. Thus, we need to test whether those evacuees can complete evacuation by the time Θ^*. This is done by again computing the maximum flow on time-expanded network $\mathcal{N}(\Theta)$ under the constraint that flows $\hat{f}(0),\hat{f}(1),\ldots,\hat{f}(\theta-1),f_{\max}(\theta)$ computed so far are fixed and the capacities of all arcs directed towards all super sinks $s^*(t)$ for $0 \le t \le \theta$ are assumed to be zero (thus a flow goes into super sinks $s^*(\theta+1), s^*(\theta+2)\ldots,s^*(\Theta^*)$. If the obtained maximum flow value is less than the number of remaining evacuees, we can conclude that the evacuation

Problem	The emergency evacuation planning problem without the capacity constraint of refuges
Input	A dynamic network $\mathcal{N}$
Output	A feasible flow f^* on $\mathcal{N}$ which satisfies (2)

Figure 5: The emergency evacuation planning problem without the capacity constraint of refuges.

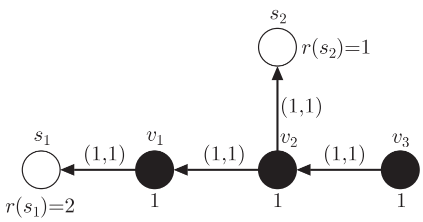

Figure 6: A counterexample of asking for the universally quickest flow on the dynamic network with the capacity constraint of a refuge by Algorithm 2. A black node represents an evacuee, and white node represents a refuge. The transit time of all arcs is 1.

completion time becomes longer than Θ^*. In this case, we need to find a maximum possible flow value that guarantees that the evacuation completion time is equal to Θ^*. This is done by a binary search over $[0, v_{\max}(\theta)]$. In each step of the binary search, we test whether the flow value $v(\theta) \in [0, v_{\max}(\theta)]$ guarantees that the evacuation completion time is equal to Θ^* or not. This is done by computing a maximum flow on time-expanded network $\mathcal{N}(\Theta)$ under the constraint that (i) flows $\hat{f}(0), \hat{f}(1), \ldots, \hat{f}(\theta-1)$ computed so far are fixed, (ii) the capacities of all arcs directed towrds all super sinks $s^*(t)$ for $0 \leq t \leq \theta - 1$ are assumed to be zero, and the flow value that goes into $s^*(\theta)$ is fixed to $v(\theta)$.

In the computational experiments, we have not implemented this algorithm.

4 Case study

4.1 Experimental settings

The proposed model was applied to Okisu area of Tokushima City in Japan where tsunami of two meters high is predicted to come when Nankai Earthquake occurs. Some strong and tall buildings have been designated for the evacuation building for tsunami. The details of those buildings are published through the website of Tokushima City Government [8]. Especially, in case of Tokushima City, since there exists information of the capacity of evacuation buildings, we adopted this area for applying the proposed model.

GIS databases used for the experiment are as follows: fundamental map information (1/2500, by Geospatial Information Authority of Japan), population census (2005, by Ministry of Internal Affairs and Communications of Japan), housing map (ZmapTown II, 2005, by Zenrin Corporation), and Japan digital road map (by Japan Digital Road Map Association). The target area is shown in Figure 7. The road network has 860 nodes and 1,106 arcs, and the population is 9,810 in this area. The population in a small area for the census unit is proportioned in the total floor space of each residential building located there and distributed to them. We suppose that dwellers in residential buildings whose number of stories is at most three need to evacuate. The number of evacuees becomes 7,445 under this assumption. Each population in a building from which residents have to leave for evacuation in case of tsunami is summed up to the nearest node as the supply of the dynamic network. The velocity of an evacuee is set to 1 meter per second and the interval of the time step is set to 5 seconds. The capacity of an arc is determined to be one of 2, 4 and 6 meters according to the width of the road where 2 meters width is used for the approach from a public road to an evacuation building. There are 11 refuges which are numbered from 1 to 11 in figure 7. They are mid-rise apartments and have been designated as evacuation buildings from tsunami. Other refuges numbered from 12 to 14 represents not buildings but gateways to the outside of Okisu area and their capacity is set to be infinite.

Four experiments listed in the Table 1 are defined by combining the flow model and existence or nonexistence of the capacity constraint of a refuge. The experimental program is written in C++ and the max-flow function of LEDA [9] is used in order to compute the maximum flow of the time expanded network.

Table 1: Combination of the flow model and the capacity constraint of a refuge.

Flow model	Capacity of a refuge	
	Considered	Not considered
The quickest flow	QC	QN
The universally quickest flow	LC	UN

4.2 Result

In the computational experiments, let us explain four cases used in the computational experiments. In the case where capacity constraint is not considered (denoted by capital letter "N"), we tested two flow models; the quickest flow model (denoted by capital letter "Q") and the universally quickest flow model (denoted by capital letter "U") while in the case where capacity constraint is considered, we tested two flow models; the quickest flow (denoted by capital letter "Q") and the lexicographic quickest flow (denoted by capital letter "C") .

Figures 8 and 9 show the cumulative number of evacuees in each case. We can see that the cumulative number of evacuees increases more rapidly at an early stage in case of UN and LC compared with QN and QC. Especially, since as will be seen from Figures 9 and 10 the difference becomes remarkable when the capacity constraint of refuges is considered, this implies that the capacity of refuges much influences on the evacuation time when the refuge capacity constraints are considered. Meanwhile the evacuation completion time of LC is longer than that of QC by about 150 seconds. However, almost all evacuees have already completed their evacuation before the final evacuee completes the evacuation in case of LC, and thus it can be said that a merit of the rapid increase of the cumulative number of evacuees in the early stage exceeds the delay of the evacuation completion time in this example. Therefore the solution by using the lexicographically

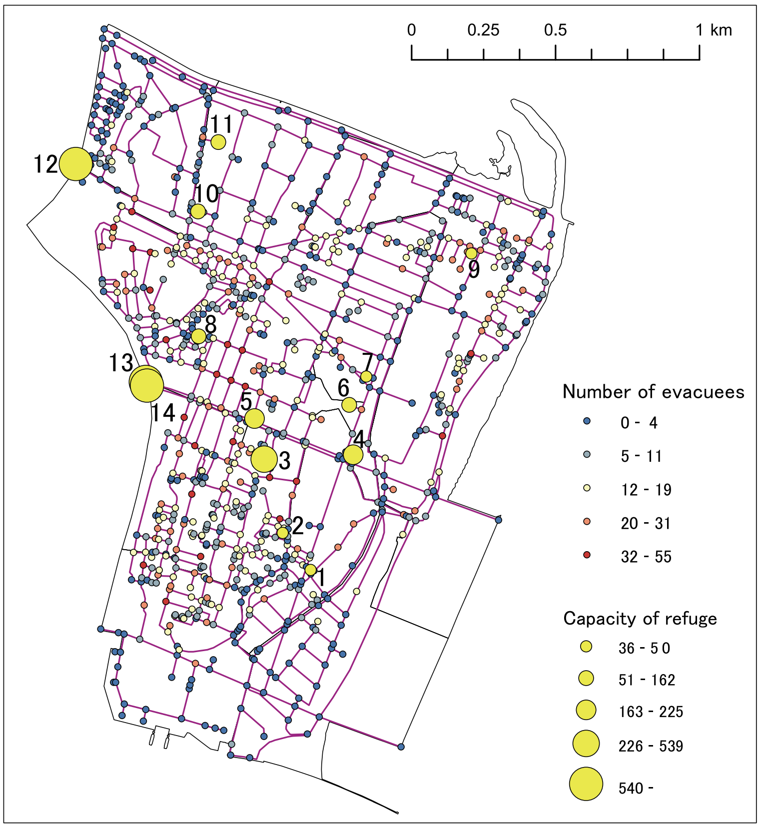

Figure 7: Okisu area of Tokushima City.

quickest flow has rationality.

Table 2 lists the number of evacuees in each refuge in case of QN and UC. If the capacity constraint is considered, all refuges with finite capacity become full except refuge 11. The reason that Refuge 11 did not become full is that it is located near the node 12 having the infinite capacity of evacuees, and population density around it is not so high.

The computation time of LC took about four minutes (Intel Core2 Duo E8400, 4GB memory, WindowsXP Professional SP3, Visual C++ 2008). Since the computation time was unsure before the numerical experiment, we limited the area for computation. However, it became clear that the computation time was not so long. Therefore, the numerical experiment considering wider area is possible and it will enable us to evaluate the location, number and capacity of refuges based on the results.

5 Conclusion

We advanced the previous evacuation planning model [6] by the quickest flow and proposed the emergency evacuation planning model by the universally quickest flow and

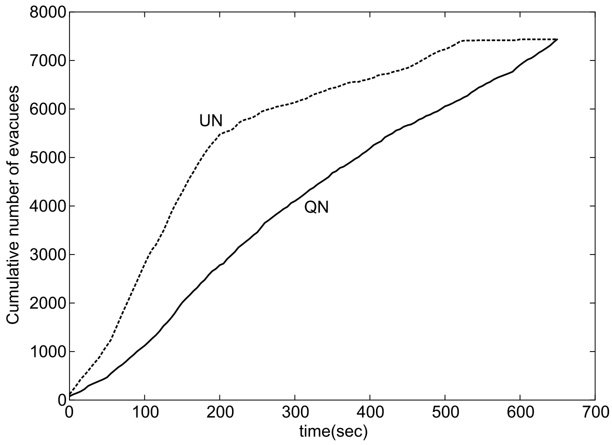

Figure 8: Comparison of cumulative numbers of evacuees without the capacity constraint of a refuge for quickest and universally quickest flow models.

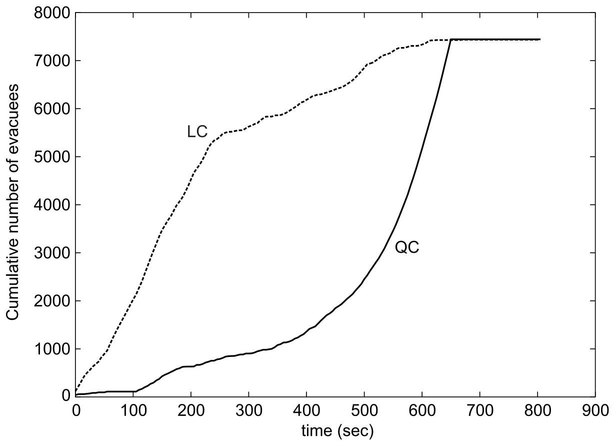

Figure 9: Comparison of cumulative numbers of evacuees with the capacity constraint of a refuge for quickest and lexicographically quickest flow models.

Table 2: Capacity of refuges and the number of evacuated people by UN and LC.

No.	Capacity	UN	LC	No.	Capacity	UN	LC
1	36	894	36	7	49	232	49
2	50	1	50	8	138	344	138
3	539	433	539	9	42	181	42
4	225	274	225	10	104	194	104
5	202	368	202	11	162	1	50
6	127	220	127	12-14	∞	3010	5883

the lexicographically quickest flow. From the result of numerical experiments, we can conclude that the proposed model can maximize the cumulative number of evacuees at the early stage of the evacuation, even if considering the capacity constraint of a refuge. Moreover, this model also can suggest the validity of existing evacuation buildings from tsunami.

Acknowledgements

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