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Analysis of Toll Setting for Japanese Highway Networks Based on Social Optimization

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Abstract In recent years in Japan, there have been many debates of setting highway toll charges as a part of policy discussions. However, although these debates should be analyzed objectively based on quantitative analysis, it cannot be denied that the psychological impact of the proposals is being given more priority. Considering that highways are a type of public infrastructure, it is appropriate that tolls be set so that both highways and normal roads are effectively utilized. In this research, we propose a simple toll setting model for highways to minimize the overall travel time in cities. In addition, the proposed model is applied to Japanese highway network, and the optimum toll setting for the highway network is discussed from the perspective of social optimization.

Keywords highway toll, social optimization, travel time, Japanese highway network

1 Introduction

In recent years there have been many lively debates regarding setting highway toll charges as a part of wide-ranging policy discussions. From March 28th 2009, an upper limit on the toll charges for ETC vehicles has been set at 1000 yen, and the debate by the present Government to basically make highways free is still fresh in our memory.

However, although these debates should be analyzed objectively from the perspective of the economic ripple effect, environmental measures, etc., it cannot be denied that the psychological impact of the proposals is being given more priority. Therefore, the uniform 1,000 yen toll caused greater than expected traffic congestion, and the inconsistency of making the highways toll-free with other policy measures to reduce CO2 emissions has also been pointed out.

Against this background of increasing focus on setting tolls for highways, in many areas an imbalance between roads with a toll and normal roads has been observed-although the highways are empty, there is congestion on the normal roads parallel to it. Considering that highways are a type of public infrastructure, it is appropriate that tolls be set so that roads with a toll as well as normal roads are effectively used.

In this regard of view, in this paper, we propose a toll setting model for roads with a toll from the perspective of social optimization. In particular, we focus on the optimization of the overall travel time in the network as an objective to be achieved. In addition, the proposed model is applied to the nationwide highway network, and the optimum toll setting for the highway network is discussed from the perspective of social optimization.

2 Formulation

In this section, we formulate a toll setting model based on social optimization.

2.1 Toll setting and traveling speed

First, we consider the relationship between toll setting for a road with a toll and the traveling speed. let us consider that two certain points are connected by the different types of roads: a road with a toll and a normal road. It is also considered that there is a close relationship between the toll setting of the toll road and the number of vehicles using it. In other words, if the toll is low it is likely that many vehicles will use that road, and conversely, if the toll is high it is likely that only a small number of vehicles will use it. Generally, the traveling speed on the road with a toll is faster than on the normal road, thus in order to minimize the overall traveling time of all users, the very common-sense conclusion that the tolls should be set low is obtained.

However, it is necessary to pay attention to the relationship between the number of vehicles using the road and the traveling speed. If we blindly increase the number of vehicles using the road, the traveling time on the road with a toll will reduce, and as a result there is a drawback that the traveling time of the users will increase. In other words, in order to minimize the traveling time of users, it is necessary to set the tolls taking into consideration the trade-off between the number of vehicles using the road with a toll and the traveling time.

2.2 Setting tolls to minimize the overall traveling time

Keeping this in mind, a toll setting model is constructed that minimizes the overall traveling time. Here it is assumed that two certain points are connected by two types of roads: a road with a toll and a normal road and as an urban model it is assumed that the two points separated by l are connected by the two types of roads (a road with a toll and a normal road). It is also assumed that the number of vehicles traveling between the two points (the demand) is Q vehicles. Also, the number of vehicles using the road with a toll is expressed as $q_H\,(\leq Q)$, and the time required when using the road with a toll is given by the function

$$t^H(q_H) = \frac{l}{v_H}\left\{1+\alpha\left(\frac{q_H}{C}\right)^{\beta}\right\} \tag{1}$$

Equation (1) is normally referred to as the BPR function [2], v^H is the free traveling speed on the road with a toll, C is the traffic capacity, and α, β are positive parameters. On the other hand, the time required when using the normal road is assumed to not depend on the number of cars using the road, and is obtained from the traveling speed on a normal road v^L, and given by $t^L = l/v_L\,(=\text{constant})$.

The toll per unit distance on the road with a toll is given by P_H yen, and we want to determine the relationship between q_H and P_H. Based on the principle of equilibrium of users [1], there is no advantage for either the road with a toll nor the normal road, so the following equation is established (when $0 < q_H < Q$):

$$t^H (q_H) + \frac{l \cdot P_H}{w} = t^L. \tag{2}$$

Here, w is a time evaluation parameter, defined as the monetary value [yen] per unit time. Therefore, solving (2) for P_H we obtain

$$P_H (q_H) = w \left[\frac{1}{v_L} - \frac{1}{v_H} \left\{ 1 + \alpha \left(\frac{q_H}{C} \right)^{\beta} \right\} \right]. \tag{3}$$

Equation (3) can be interpreted as the equation for calculating the toll setting P_H to achieve the number of vehicles q_H using the road with a toll.

Now, the total time T required for Q vehicles to travel between the two points is

$$T = t^H (q_H) \cdot q_H + t^L \cdot (Q - q_H). \tag{4}$$

From the perspective of social optimization, it is desirable that the toll of the road with a toll be set to minimize T. Therefore focusing on the fact that T is a function of q_H, to obtain qH that minimizes T, we should specifically solve

$$T' (q_H^*) = t^{H'} (q_H^*) \cdot q_H^* + t^H (q_H^*) - t^L = 0 \tag{5}$$

If specifically solved

$$q_H^* = C \left\{ \frac{v_H}{\alpha (1 + \beta)} \left(\frac{1}{v_L} - \frac{1}{v_H} \right) \right\}^{\frac{1}{\beta}} \tag{6}$$

(for $q_H^* \leq Q$). Therefore using Equation (3), the optimal toll P_H^* that should be set to minimize T can be calculated from

$$P_H^* = P_H (q_H^*) > 0. \tag{7}$$

In other words, when there is a demand greater than q_H^*, we can understand that the toll on the road with a toll should be set to P_H^* yen per unit distance (it should not be free).

On the other hand, when $q_H^* > Q$, the toll P_H^* to minimize T may be freely set within the range that satisfies

$$t^H (Q) + \frac{l \cdot P_H^*}{w} < t^L. \tag{8}$$

$P_H^* = 0$ clearly satisfies Equation (8), so when the demand is less than q_H^*, it can be seen that the total traveling time is minimized even when the toll is free.

3 Extension to a Network Model

From the discussion in the previous section, it has been shown that there is a threshold q_H^* for setting the toll to minimize the overall traveling time, and the decision that the toll should be free depends on whether the demand between the two points exceeds q_H^* or not. In this section, the model is extended to a network, and a numerical example for an imaginary network is given.

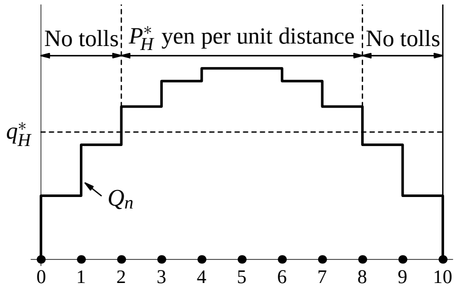

Figure 1: Toll setting in the assumed network ($N = 10$)

3.1 Formulation

Consider a network $G(V,E)$ consisting of several cities (represented by the group V) which are connected by road links (represented by the group E). Between a pair of cities $V \times V$ there are several toll road links, and there are also normal road links between all city pairs. Under these circumstances we want to set the toll for each toll road link to minimize the overall traveling time required.

In this case, for each city pair O→D, there are several toll road links that can be used, and therefore, the route traveled is given by O→ $v_1 \rightarrow \cdots \rightarrow v_\Lambda$ →D. Presently, assuming that the traveling route itself does not change depending on the density of traffic on each link, in each city $v_i \rightarrow v_{i+1}$, travelers compare toll roads with normal roads, and accordingly select their route. Therefore, in order to minimize the total traveling time required on the whole network, it is clear that the argument in Section 2 should be applied to each pair of cities (v_i, v_j) which have a toll road link.

3.2 Numerical example

Consider a chain network of N+1 nodes as a example network, and assume that there are toll road links and normal road links between the nodes. Assuming there is a demand of Q vehicles for all node pairs, the total amount of traffic on link n between nodes $(n-1,n)$ is as follows:

$$Q_n = Q \cdot n \cdot (N - n + 1). \tag{9}$$

Furthermore, assuming the link capacity C_n of each toll road link is a constant, the threshold q_H^* is determined to be uniform on the network. Therefore, in the central part where Q_n exceeds q_H^*, the toll is set at P_H^* yen per unit distance, and for peripheral parts where Q_n is less than q_H^*, it is possible to set the tolls to be free (Figure 1).

4 Application of the model to the Highway Network

In this section, the proposed model was applied to the Japanese highway network, and the optimum toll setting at each point was calculated.

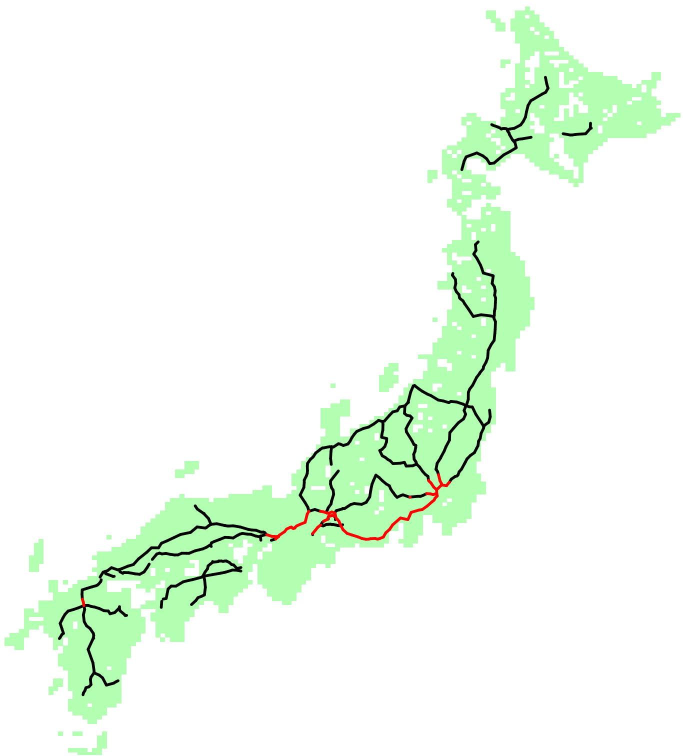

Figure 2: Example of application of the model to the nationwide highway network

4.1 Parameter setting

Before the calculation, it was necessary to determine the capacity and demand on each link. For this research, first, for all links, the number of lanes, the average speed, and the amount of traffic in 24 hours was extracted from the Highway Traffic Census for 2005 [4]. In addition, to reproduce the relationship between the average speed and the amount of traffic, the parameters of the BPR function were set to $C=$16,250 [vehicles/lane-day], $\alpha = 0.54$ and $\beta = 3.3$. The actual measured amount of traffic in 24 hours was also taken to be the amount of demand. In reality the latent demand should be greater than the measured amount of traffic, so this assumption sets the amount of demand a little low (in other words, making it easier for the highways to be toll-free).

4.2 Estimation

The results of the application to the nationwide highway network are shown in Figure 2. These results suggest that apart from the three major urban areas and the Tomei and Meihan Highways connecting these areas, most of the areas could be made toll-free. Especially in the case of the rural highways, the measured amount of traffic is frequently less than 50% of the threshold value, so in order to promote the use of the highway, it has been shown that a further reduction in the tolls would be effective.

5 Conclusion

In this paper, we proposed a toll setting model for roads with a toll from the perspective of social optimization. In particular, focus on the optimization of the overall travel time in the network, we clarified a coherent rule to set the optimal toll for highway networks. Furthermore, the proposed model was applied to the nationwide highway network, and the optimum toll setting for Japanese highway network was discussed from the perspective of social optimization.

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