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A New Approach for Scheduling Problem in Multi-Hop Sensor Networks

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Abstract In this paper, we consider a multi-hop wireless sensor network, where the network topology is a tree, time division multiple access (TDMA) is employed as medium access control, and all data generated at sensor nodes are delivered to a sink node located on the root of the tree through the network. It is reported that if a transmission schedule that avoids interference between sensor nodes completely can be computed, TDMA is preferable to others in performance. However, solving the scheduling problem for TDMA is difficult, especially, in large-scale multi-hop sensor networks. In this paper, based on graph theory, we propose new formulation for the TDMA scheduling problem.

Keywords Multi-Hop Sensor Network, State Transition, Shortest Path

1 Introduction

A wireless sensor network (WSN) consists of wireless sensor nodes, which contain devices for sensing, wireless communication, information processing and electric source. Sensor nodes are spatially distributed and cooperatively monitor physical or environmental conditions, such as temperature, sound, vibration and so on. WSNs are paid much attention due to their rich applications such as environmental, military and civilian area. Depending on applications, a wide variety of network sizes can be used. In applications such as environmental monitoring, for example, hundreds or thousands of sensor nodes are deployed in a large monitoring field. When sensor nodes are distributed in a field, a network is first organized in ad-hoc manner. After that, sensor nodes can communicate with each other.

Medium access control (MAC), a way of multiple nodes communicating, is a crucial factor. Kiri et al. [3] study multi-hop communication between sensor nodes in large-scale sensor networks. They investigate the characteristics of sensor networks by comparing two types of MAC. According to the result of their study, TDMA, a type of MAC, is the most preferable if positional informations of all nodes are available and if a transmission schedule that avoids interference between nodes completely can be obtained.

In general, the scheduling problem is formulated as a combinatorial optimization problem, where each combination corresponds to a schedule [4]. However, it is known that such a problem is NP-hard. Therefore, a technique to find near-optimal solutions in the scheduling problem within a practical CPU time for large-scale sensor networks where TDMA is employed should be developed. Furuta et al. [1] formulate the scheduling problem for TDMA as an integer programming problem.

In this paper, we formulate the scheduling problem as the shortest path problem on directed graph on which we can consider available transmission schedule. We consider a multi-hop sensor network, where the network topology is a tree, TDMA is employed as MAC. All data monitored at sensor nodes are gathered to a sink node through the network. Based on this framework, we construct a graph. A vertex of the graph represents a set of of sensor nodes which have already transmitted the data. Consider two vertices. If the set of sensor nodes represented by one of the vertices is a subset of the set of sensor nodes by the other vertex, we connect the two vertices by an edge. The edge of the graph is equivalent to a set of sensor nodes, which is difference set of two sets which are represented by starting vertex and the end vertex, respectively. The elements of the set represented by the edge are the sensor nodes which newly transmit the data. The initial vertex of the graph represent the an empty set of sensor nodes which means none of the sensor nodes has transmitted the data. The terminal vertex represent the set of all sensor nodes which means all the sensor nodes have transmitted the data. A path from the initial vertex to the terminal vertex is equivalent to a transmission schedule. The shortest path gives us the optimum transmission schedule.

The rest of this paper is organized as follows. We introduce the model of the multi-hop sensor network in section 2. In Section 3, we present the graph which is equivalent to transmission schedule. The necessary conditions to ease calculation are provided in section 4, and we show their validation. Finally, conclusion is drawn in section 5.

2 Model Description

In this section, we introduce the model of the multi-hop sensor network, a centralized multi-hop sensor network, which is considered in this paper. The centralized network means that the base station (BS) organizes a network and controls all sensors' activities. Given the tree topology organized by the BS, the BS informs each sensor node of its parent node (the sensor node which it should send a packet to) and child nodes (the sensor nodes which it should receive packets from).

The aim of the sensor network under consideration is to periodically collect all data generated by sensor nodes at a special node called a sink node. All sensor nodes monitor event and generate data periodically. A sensor node having child nodes must wait sending a packet until it receives data from all its child nodes. After receiving data from all its child nodes, the sensor node sends the packet to its parent node. This process is repeated, and all data generated at sensor nodes are finally collected at the sink node, which is located on the root of the tree. We call the elapsed time to collect all data generated by sensor nodes at the sink node as collecting time. Achieving a short collecting time is crucial, especially, for real-time applications.

As for medium access control, we assume that time division multiple access (TDMA) is employed. TDMA allows several nodes to share the same frequency channel by allocating their transmissions to divided time slot. We suppose that one time slot is needed for the packet transmission. Since sensor nodes share a single channel, a suitable transmission schedule is required to avoid interference between sensor nodes. Once interference occurs, packets are not correctly sent and sensor nodes must retransmit the packets. This causes delay and reduces throughput of network. We assume that transmissions always succeed as long as no interference occurs.

We here specify the condition that an interference occurs between sensor nodes in our model. The tree network organized by the BS can be considered as a subgraph of the unit-graph [2]. We assume that all sensor nodes have the same and fixed transmission radius r. Then a unit-graph is generated by drawing arcs between nodes if their distance is less than or equal to r. Two nodes are called one hop apart with each other if there is an arc between them in the unit-graph. If two nodes are not one hop apart but they have a common node which is one hop apart from the two nodes, they are called two hop apart with each other. Then, the condition that an interference occurs between sensor nodes is specified as follows: Suppose that node i transmits a packet to node j in a time slot. If and only if there exists a node which is one hop apart from node j and transmits a packet other than node i in the time slot, an interference against the transmission from node i to node j occurs.

3 State Transition Graph

In the following, we introduce a graph for describing the sensor network scheduling problem. First, we introduce two kinds of sensor node set that are used as vertices and edges of the graph afterward. One of the sets is a state of network, which reflects states of all the sensors in the network. The other is a transition between two network states. After we explain the relation between those two sets and transmitting schedule, we introduce the graph where vertices are network state and edges are state transitions. Then, we show that a transmitting schedule corresponds to the path between particular vertices in such the graph.

Before we start describing, let us denote the definition of notations of the sensor network:

- N: the set of sensor nodes. $|N| = n$.
- T: the set of time slots ($T = 1, \cdots, n$).
- r: the transmission radius.
- $a(i)$: parent node of sensor $i \in N$. The node which sensor i transmits.
- B_i: the set of sensor nodes which transmit to sensor i. The set of child sensor nodes of i.
- C_i: the set of sensor nodes where distance from sensor i is less than r.

Let us consider a state of network, which describes states of all sensor nodes. At the beginning of schedule, no sensor has transmitted data to its parent node. Each sensor transmits data during a certain time slot, and all sensor nodes finally complete transmission by the collecting time. At the end of each time slot, sensor nodes can be classified into two disjoint sets, i.e., the sensors that have already completed transmission and the others that have not yet. Thus, we can represent an arbitrary state of network as the set of transmitted sensor nodes. Let set Y denote a subset of sensor nodes N. Assume that sensor node $i \in Y$ means that node i has already transmitted. Now, we regard Y as a state of the sensor network. For example, the state associated with empty set, that is $Y = \emptyset$, illustrates

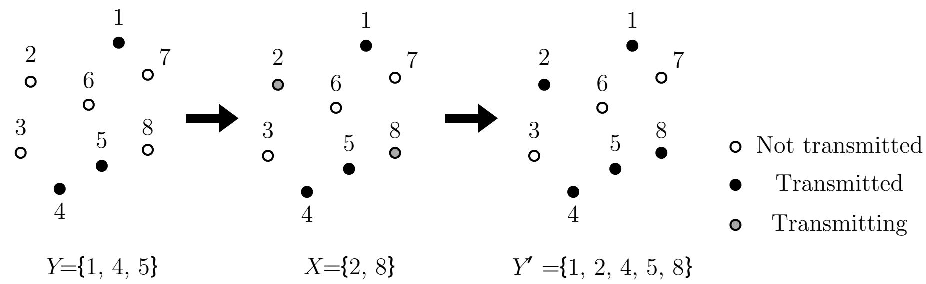

Figure 1: Transition X from state Y to state Y'

no sensor node has transmitted, which corresponds to the initial state. The state associated with the set of all nodes, that is $Y = N$, illustrates every sensor node has completed transmission, which corresponds to the final state.

The network state repeats a change as a transmission schedule advances. We call this change of network state a transition. When some sensor nodes transmit during a time slot, the network state changes according to those sensor nodes. That is, the transmitting sensors are added to network state, which consists of transmitted sensors, after their transmission completed. Therefore, we define the transition as a set of sensor nodes that transmit during a certain time slot simultaneously. Consider the transition from state Y to state Y', and suppose that set X denotes that transition. Then, we can describe X as the complement of Y in Y':

$$X = Y' \setminus Y. \tag{1}$$

Each element in set X represents transmitting node. Node $i \in X$ means that sensor node i transmits while the state changes from Y to Y'. Due to an interference and/or structure of network, a transition is possible at not all time or network state. It depends on a network state and structure for transitions to be possible or not. The conditions if a transition X is possible or not are as follows:

$$\bigcup_{i \in X} B_i \subseteq Y, \tag{2}$$

$$\bigcup_{i \in X} \left(C_{a(i)} \setminus \{i\}\right) \cap X = \emptyset. \tag{3}$$

In condition (2), all the set of sensor nodes that transmit to sensor i is included in the set Y, the set of transmitted sensor nodes. Therefore, it means that sensor nodes should wait for transmission completion of its child nodes. Condition (3) expresses avoiding interference. Interference occurs when a sensor node within distance r from sensor node i transmits simultaneously as i. $C_{a(i)}$ is a set of sensor nodes that exist within distance r from sensor node i, and it is not allowed to transmit at one time with set X. We mention that the definition X also represents that every node transmits only once.

Relation between network state and state transition is illustrated in figure 1. In that figure, we do not consider the interference between sensor nodes for the purpose of simpfying. There, state Y includes three sensor nodes, 1, 4 and 5, where they have already

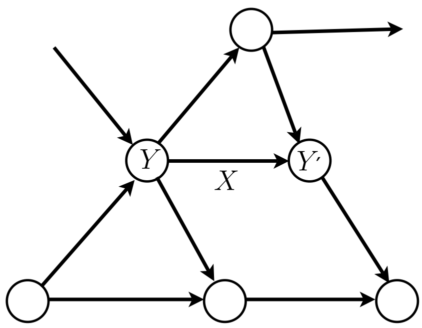

Figure 2: State transition graph

transmitted. After transition X, which means the two sensor nodes 2 and 8 transmit, network state becomes Y'.

By using the state transitions, we can describe a transmission schedule. It is represented as the continual state transitions where each transition corresponds to the transmitting sensors and position of transition denotes time slot. The schedule is also expressed by the sequence of network states from the initial state to the final state. In that way, each state represents transmitted sensors by that time slot, and transmitting sensors at a certain time slot are represented as the difference of two states lying next to each other.

To depict the scheduling problem, we introduce a graph $G(V,E)$. At such the graph, we assume that the set of vertices V is the set of all possible network states, that is the power set of N. Note that from the definition of network state, there are 2^n possible states when the number of sensor nodes is n. However, there are a number of network states that are never realized from a structure of network. Then, we establish a directed edge between vertices when the transition of states corresponding to the vertices is possible within one time slot. Figure 2 illustrates a part of state transition graph. In the figure, two vertices Y and Y' represent network states, and the edge X from Y to Y' represent the transition between them.

Let Y_I be the initial state that all the nodes have not transmitted, and Y_F be the final state that they have already done, i.e., $Y_I = \emptyset$ and $Y_F = N$. Consider a path S from Y_I to Y_F. S is expressed by the permutation of vertices that correspond to network states, that is $S = (Y_0, Y_1, \cdots, Y_t, \cdots, Y_\tau)$ where $Y_0 = Y_I$ and $Y_\tau = Y_F$. Regarding Y_t as the network state after transmission at time slot t, the path S means the process of network state changes, and we can suppose that the path S is equivalent to one transmitting schedule. In such the path S, let set X_t be the complement of Y_{t-1} in Y_t:

$$X_t = Y_t \setminus Y_{t-1}, \tag{4}$$

then X_t represents the set of sensor nodes whose transmission is scheduled at time slot t. So, transmitting schedule $(X_1, \cdots, X_\tau)$ is lead from the path S, and its collecting time is equal to τ corresponding to the number of edges included in S. Therefore, the problem which obtains the optimal schedule is equivalent to finding the shortest path S^* between Y_I and Y_F in the graph $G(V,E)$.

4 Maximal Sensor Nodes Set

In this section we focus on features of the shortest path which represents optimal schedule proposed in section 3. Although the shortest path problem yields the shortest time schedule, its calculation is still difficult. The reason of difficulty is a size of the graph which consists of states of sensor network and their transitions in section 3. To solve the problem is to find the shortest path between the initial state Y_I and the final state Y_F. As is well known, complexity of the algorithm for the shortest path problem, such as the Dijkstra method, depends on the size of the graph. Therefore, to reduce the number of edges representing the transitions which should be considered is important to avoid the difficulty of the problem. In the following, we analyze the structure of the shortest path of the graph, and derive the conditions about states and/or transitions which are necessary in searching the optimal schedule.

To deal redundant states, we consider a transition from a certain state. Suppose that we obtain the temporary shortest path through the transition from the state. When there exists other path, of which length is less than or equal to it, from the state without including that transition, we recognize that its path is no longer the shortest between the state and the final state. In our searching for the shortest path, if we know existence of the other path in advance, it has no necessity to be examined that the state which is the end of the transition and following states through it. Inversely, we can find the shortest path by examining the states which are the ends of the transitions satisfying the conditions that there exists no other shorter path without including its transition.

Now, we introduce maximal sensor node set (MSNS) that represents the transitions satisfying above conditions. MSNSs are sensor node sets that no other sensor node is simultaneously transmittable, and they are achieved corresponding to states. Note that transition is defined as a set of sensor nodes and it means sensor nodes in it transmit at one time. MSNSs include as many sensor nodes as possible. This does not mean that MSNS is the transition which has the biggest number of simultaneously transmittable sensor nodes. MSNS is the set to which new element can't add without eliminating any element belonging to it.

We explain that it is unnecessary to consider the transitions except MSNS as follow manner: We consider two transitions from a state and paths through them. As described above, transitions represent sets of sensor nodes transmitting simultaneously, and we suppose that one is a subset of the other. Then, we show that there is a path through the

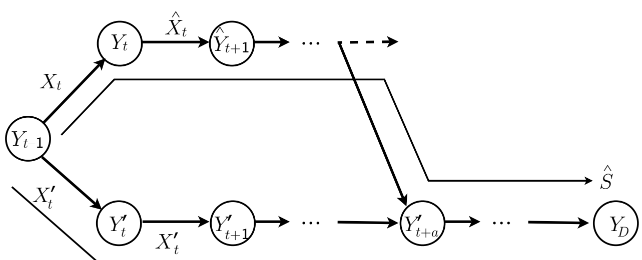

Figure 3: Shortest path searching

superset, of which length is smaller than or equal to the length of the temporary shortest path through the subset. Since there is no superset of MSNS from its definition, we can obtain the shortest path by examining transitions represented as MSNSs.

Let set X_t be a set of simultaneously transmittable sensor nodes at time slot t. State Y_{t-1} changes state Y_t through transition X_t:

$$Y_{t-1} \cup X_t = Y_t. \tag{5}$$

And, let X'_t be an arbitrary subset of X_t:

$$X'_t \subset X_t \tag{6}$$

In the following, we discuss two paths from state Y_{t-1} and their lengths. One is the path S' that is the temporary shortest path through transition X'_t. It is expressed as the permutation of network state: $(Y_{t-1}, Y'_t, Y'_{t+1}, \cdots, Y_F)$. The other one is the path $\hat{S}$ through transition X_t. It is also expressed as $(Y_{t-1}, Y_t, \hat{Y}_{t+1}, \cdots, Y_F)$. In figure 3, path S' is illustrated as a sequent of states at lower part of figure, and path $\hat{S}$ is similarly at upper part. We show that there always exists a particular path $\hat{S}$ through transition X_t satisfying conditions (2) and (3), where the length of it is equal to the length of the temporary shortest path S' through transition X'_t.

Assume that $\hat{X}_{t+k}$ is a complement of transmitted X_t in X'_{t+k}, and $\hat{Y}_{t+k-1}$ is a union of Y'_{t+k-1} and transmitted X_t.

$$\hat{X}_{t+k} = X'_{t+k} \setminus X_t \subseteq X'_{t+k}, \tag{7}$$

$$\hat{Y}_{t+k-1} = Y'_{t+k-1} \cup X_t \supseteq Y'_{t+k-1}. \tag{8}$$

In path $S' = (Y_{t-1}, Y'_t, Y'_{t+1}, \cdots, Y_F)$, each X'_{t+k}satisfies follow conditions:

$$\bigcup_{i \in X'_{t+k}} B_i \subseteq Y'_{t+k-1}, \tag{9}$$

$$\bigcup_{i \in X'_{t+k}} \left(C_{a(i)} \setminus \{i\}\right) \cap X'_{t+k} = \emptyset. \tag{10}$$

All the sensor nodes in B_i, which is a child of sensor node $i \in \hat{X}_{t+k}$, has already transmitted by time slot $t+k-1$. From equation (7), (8) and (9), $\hat{X}_{t+k}$ satisfies

$$\bigcup_{i \in \hat{X}_{t+k}} B_i \subseteq \bigcup_{i \in X'_{t+k}} B_i \subseteq Y'_{t+k-1} \subseteq \hat{Y}_{t+k-1}. \tag{11}$$

Also because of the definition of $\hat{X}_{t+k}$, sensor nodes which belong to X'_{t+k} can transmit simultaneously. Then, from equation (7) and (10), $\hat{X}_{t+k}$ satisfies

$$\bigcup_{i \in \hat{X}_{t+k}} \left(C_{a(i)} \setminus \{i\}\right) \cap \hat{X}_{t+k} = \emptyset. \tag{12}$$

The length of path $\hat{S}$ is as same as the length of path S'. In the path $\hat{S}$, equation (11) shows that sensor nodes transmit when its all child nodes have transmitted, and equation

(12) shows that sensor nodes which exist within distance r from a receiver don't transmit at one time. Since above conditions are satisfied by arbitrary set X_t, the shortest path can be found in the transitions that have no superset. Note that there remains the possibility that the path of which length is equal to the length of the shortest path obtained in above way. From definition of MSNS, it does not have any superset. Therefore, we can find one of the shortest paths by examining MSNSs, which include as many number of sensor nodes as possible.

5 Conclusion and Remarks

In this study, we proposed a new approach to solve a scheduling problem arisen in wireless networks. In our approach, we introduced a graph where vertices correspond to states of sensor network and edges correspond to transitions from one state to another. We showed that finding optimal schedule is equivalent to finding a shortest path on the graph. In addition, we further discussed how to reduce the number of states that should be examined, which lead to significant reduction of the size of the graph. For future research, we further improve our approach to solve large problems. To propose a good heuristic approach is also an important work.

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