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Locating Flow Capturing Facilities on A Railway Network with Two Levels of Coverage

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Abstract This paper presents a flow capturing location problem which focuses on railway passenger flows. The flow capturing location problem (FCLP), originally proposed by Hodgson (1990), aims to locate a given number of facilities on a network to maximize the total flow that can obtain service at facilities along their pre-planned routes. For railway users, facilities located at the origin station and the destination station are more easily accessible than facilities located midway along the path. Therefore, we differentiate these two cases by introducing two levels of coverage; a given flow is fully captured when captured at the origin station or the destination station, or partially captured when captured at a midway station along the path. We provide an integer programming formulation of the proposed problem. We apply the model to analyze the optimal locations of flow capturing facilities on Keio Line using commuter traffic flow data, which is extracted from transportation census data of Tokyo Metropolitan area.

Keywords Facility Location Problem; Flow Capturing Location Problem; Railway Network

1 Introduction

In traditional network facility location models, demands for services occur at nodes of the transportation network. Typical examples assuming point-based demands are classical location models such as the p-median model (Hakimi 1964) and the maximal covering location model (Church and ReVelle 1974). There is, however, another type of demand which is more appropriately represented as flows traveling along various paths of the network. Typical examples include stopping at an automatic teller machine or a convenience store on a pre-planned tour, such as the daily commute to work. Focusing on this aspect, Hodgson (1990) and Berman, Larson, and Fouska (1992) proposed the flow capturing location problem (FCLP). The aim of the FCLP is to locate a given number of facilities so as to maximize the number of flow-based customers who encounter at least one facility on their pre-planned trips. There are various applications for which the flow capturing approach is appropriate, such as advertising billboards (Averbakh and Berman 1996; Hodgson and Berman 1997), vehicle inspection stations (Hodgson, Rosing, and Zhang 1996) and alternative-fuel stations (Kuby and Lim 2007). Because of this wide applicability, various types of extended models of FCLP have been proposed. Extensions of the basic FCLP include a model in which deviation from pre-planned trips is allowed (Berman, Bertsimas and Larson 1995), a model considering the time spent in the facility including possible waiting time (Berman 1995), and the multi-counting model in which consumers can be captured multiple times when there are several facilities on their paths (Avebakh and Berman 1996). More details can be found in the review article by (Berman, Hodgson, and Krass 1995).

We present a new FCLP focusing on train passengers over the railway network. In the original FCLP, a given flow is either fully captured or not captured at all. In some applications, however, there can be an intermediate coverage level between these two extreams. For railway users, facilities located at the origin station and the destination station are more easily accessible than facilities located midway along the path. Therefore, we differenciate these two cases by introducing two levels of coverage; a given flow is fully captured when captured at the origin station or the destination station, or partially captured when captured at a midway station along the path. We call the proposed problem FCLP with two levels of coverage.

This paper is organized as follows. First, we describe the original FCLP and its formulation. We then propose FCLP with two levels of coverage and provide an integer programming formulation. We apply the model to analyze the optimal location of flow capturing facilities on Keio Line, one of the major commuting railway lines in Tokyo Metropolitan area, using real passenger flow data. In the final section, some conclusions are given and directions for future work are discussed.

2 Formulation

In this section, the original flow capturing location problem (FCLP) is introduced. Then, we propose the FCLP with two levels of coverage and formulate the problem as a 0-1 integer programming problem.

The original flow capturing problem

Hodgson (1990) and Berman, Larson, and Fouska (1992) independently developed the original flow capturing problem. The aim is to maximize the number of consumers who encounter at least one facility along their pre-planned travel paths. Let us consider a network with node set K. We denote by Q the set of positive flow paths between origin-destination pairs of the network with elements q. The volume of flow q is given by f_q. The aim is to locate p facilities among the nodes in the network so as to maximize the total flow captured. To formulate the FCLP as an integer programming problem, two sets of decision variables are introduced:

$$x_k = \begin{cases} 1 & \text{if a facility is located at node } k \\ 0 & \text{otherwise} \end{cases}$$

$$y_q = \begin{cases} 1 & \text{if flow } q \text{ is captured} \\ 0 & \text{otherwise} \end{cases}$$

The problem is formulated as follows:

$$\max \quad \sum_{q\in Q} f_q y_q \tag{1}$$

$$\text{s.t.} \quad \sum_{k\in K} x_k = p \tag{2}$$

$$y_q \le \sum_{k\in K} \alpha_{qk} x_k \quad \forall q \in Q \tag{3}$$

$$x_k \in \{0,1\} \quad \forall k \in K \tag{4}$$

$$y_q \in \{0,1\} \quad \forall q \in Q \tag{5}$$

In this formulation, the parameters α_{qk} are

$$\alpha_{qk} = \begin{cases} 1 & \text{if node } k \text{ is included in the travel path of flow } q \\ 0 & \text{otherwise} \end{cases}$$

The objective function (1) seeks to maximize the number of customers traveling on the network who have at least one facility on their paths. Constraint (2) stipulates that the number of facilities to be located is p. Constraints (3) state that flow q cannot be captured unless at least one facility is located in the nodes that can capture q. It should be noted that flow q is not counted as captured more than once; capturing the same flow multiple times does not make any contribution to the objective value. Constraints (4) and (5) are standard binary constraints. The flow coverage variable y_q can be relaxed to $0 \le y_q \le 1$ instead of $y_q \in \{0,1\}$ because of the form of constraints (3) and the objective function.

2.1 A flow capturing location problem with two levels of coverage

In the original FCLP, a given flow is either captured or not captured at all. In some cases, however, it is more appropriate to consider several different coverage levels depending on the way flows are captured. The present paper considers one such situation in which railway passenger flows are captured. For railway users, facilities located at the origin station and the destination station are more easily accessible than facilities located midway along the path. Therefore, it is important to differenciate these two cases. We define that a given flow is fully captured (or covered) when a facility is located at the origin station or the destination station or is partilly captured when at least one facility is located at a midway station along the path. To describe this structure, we introduce the following coefficients which represent the benefit of coverage:

a_{f} : coefficient for fully-captured flow

a_{p} : coefficient for partially-captured flow

We introduce coverage indexes corresponding to two types of coverage:

$$\alpha_{qk} = \begin{cases} 1 & \text{if node } k \text{ is either the origin station or the destination station of flow } q \\ 0 & \text{otherwise} \end{cases}$$

$$\beta_{qk} = \begin{cases} 1 & \text{if node } k \text{ is included in the travel path of flow } q \\ 0 & \text{otherwise} \end{cases}$$

To differenciate the two types of coverage, fully captured or partially captured, we introduce two types of coverage variables.

$$y_q = \begin{cases} 1 & \text{if flow } q \text{ is fully captured} \\ 0 & \text{otherwise} \end{cases}$$

$$z_q = \begin{cases} 1 & \text{if flow } q \text{ is partially captured} \\ 0 & \text{otherwise} \end{cases}$$

The proposed problem can be formulated as follows.

$$\max \quad \sum_{q \in Q} \left(a_{\mathrm{f}} f_q y_q + a_{\mathrm{p}} f_q z_q \right) \tag{6}$$

$$\text{s. t.} \quad \sum_{k \in V} x_k = p \tag{7}$$

$$y_q + z_q \leq 1 \quad \forall q \in Q \tag{8}$$

$$y_q \leq \sum_{k \in V} \alpha_{qk} x_k \quad \forall q \in Q \tag{9}$$

$$z_q \leq \sum_{k \in V} \beta_{qk} x_k \quad \forall q \in Q \tag{10}$$

$$x_k \in \{0,1\} \quad \forall k \in K \tag{11}$$

$$y_q, z_q \in \{0,1\} \quad \forall q \in Q \tag{12}$$

The objective function (6) is the total number of potential customers weighted by the full coverage coefficient and the partial coverage coefficient. Constraint (7) stipulates that the number of facilities to be located is p. Constraints (8) require that flow q be either fully captured or partially captured or not captured at all. Constraints (9) state that flow q cannot be fully captured unless at least one facility is located at the origin station or the destination station. Constraints (10) state that flow q cannot be partially captured unless at least one facility is located at the travel path of flow q. Constraints (11) and (12) are standard binary constraints. The flow coverage variables y_q and z_q can be relaxed to $0 \leq y_q \leq 1$ and $0 \leq z_q \leq 1$ instead of $y_q \in \{0,1\}$ and $z_q \in \{0,1\}$ because of the form of constraints (9) and (10), and the objective function.

3 Application

In this section, we apply the model to the analysis of optimal locations of flow capturing facilities on Keio Line using railway passenger flow data. Keio Line is one of the major commuting railway lines in Tokyo Metropolitan area, which has 34 stations and is connected to Shinjuku station as shown in Fig. 1. In this analysis, census data conducted in 2005 for commuter traffic using public railway lines in Tokyo Metropolitan area is used. We focus on flows which use Keio Line as part of the trip. As illustrated in Fig. 2, there are four types of travel patterns using Keio Line dependeing on whether the origin station or the destination station is on the Keio Line or not. We identify the four types of travel patterns as the same OD pattern on Keio Line when the endpoint stations on Keio Line are the same. The OD table for Keio Line which identify flows between each pair of

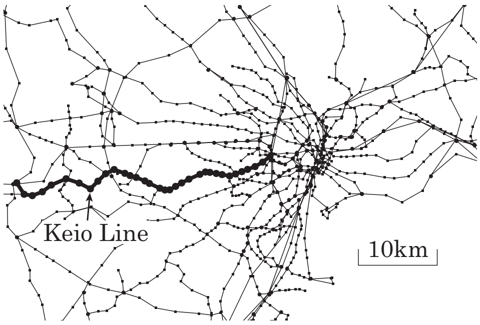

Figure 1: Railway network of Tokyo Metropolitan area and Keio Line

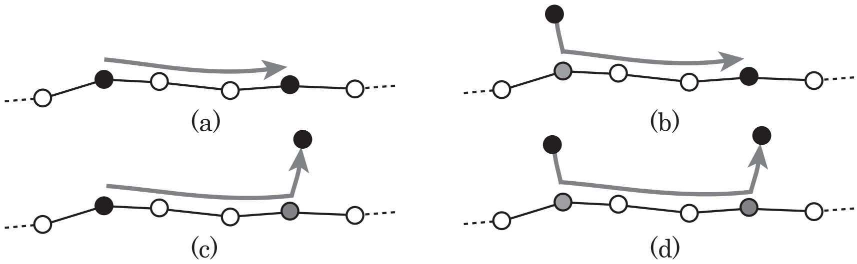

Figure 2: Four travel patterns using Keio Line

stations is constructed as indicated above. The total flow using Keio Line as part of the trips is 482,820. Using the OD data, we analyze charecterictics of flow for each station. Fig. 3 shows the number of flows departing from and arriving at each station. This is equal to the number of fully captured flows by a single facility at each station. Fig. 4 shows the number of flows passing through each station. This is equal to the number of partially captured flows by a single facility at each station. In these figures, the number of flows are indicated by the area of the circle centered at each station. As can be seen from Fig. 3, the number of flows departing from and arriving at is the largest at Shinjuku station (266,438) followed by Chofu station (107,794) and Meidaimae station (85,976). We can also see from Fig. 4, that the number of flows passing through each station becomes larger near Shinjuku station. The largest value is observed at Hatsudai station (248,011) which is next to Shinjuku station.

We next analyze how much flow can be captured by a single facility located at each station. Fig. 4 shows the number of captured flow for each station when the full coverage coefficient and the partial coverage coefficient are set equal to $a_{\mathrm{f}} = 1.0$ and $a_{\mathrm{p}} = 0.2$. The value can be obtained by summing the volume of flow at each station in Fig. 3 multiplied by 1.0 and that of Fig. 4 multiplied by 0.2. This case supposes that capturing flow at the origin or destination staion is five times as desirable as capturing at a midway station along the path. Shinjuku station observes the largest value of the objective value followed by Chofu station and Meidaimae station.

Next, we compare some optimal solutions for the Hodgson's original FCLP and the proposed FCLP. Optimal solutions are obtained using a mathematical programming software: NUOPT Ver. 12. Fig. 6 shows the optimal solutions for $p = 1$, $p = 2$, $p = 3$ and $p = 4$ for the Hodgson's original FCLP. Fig. 7 shows the optimal solutions for $p = 1$, $p = 2$, $p = 3$ and $p = 4$ for the FCLP with two levels of coverage in the case of $a_{\mathrm{f}} = 1.0$ and $a_{\mathrm{p}} = 0.2$. Facilities in Fig. 6 are more dispersed than those of Fig. 7. We can also see that stations having large number of arrival and departure flows are more likely to be selected in the proposed model than in the original FCLP. Shinjuku station is selected in all optimal solutions of FCLP with two levels of coverage. Meidaimae station and Chofu station are also large stations having large number of departing and arriving flows, both of which are selected in many of the optimal solutions.

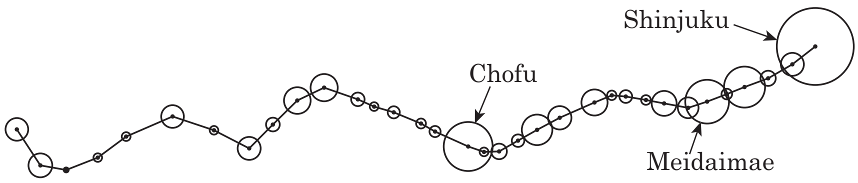

Figure 3: Flows arriving at and departing from each station

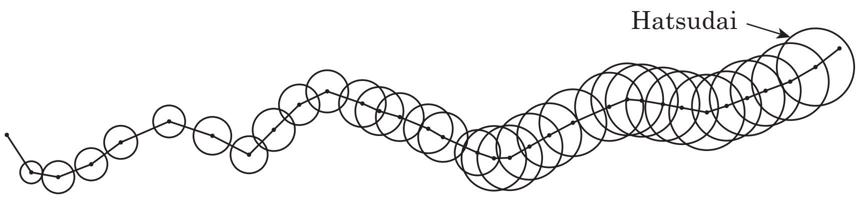

Figure 4: Flows passing through each station

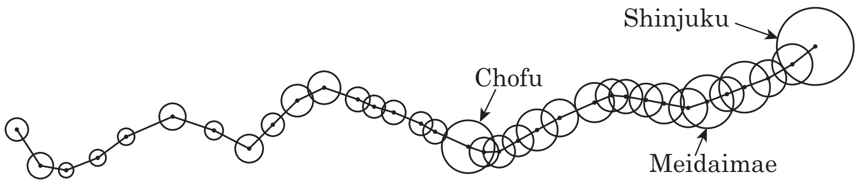

Figure 5: Objective value obtained by a single facility at each station

4 Conclusion and future work

This paper presented an extended model of the flow capturing location problem which focused on railway passenger flows. For railway users, facilities located at the origin station and the destination station are more easily accessible than facilities located midway along the path. Therefore, we constructed the model differenciating these two cases and

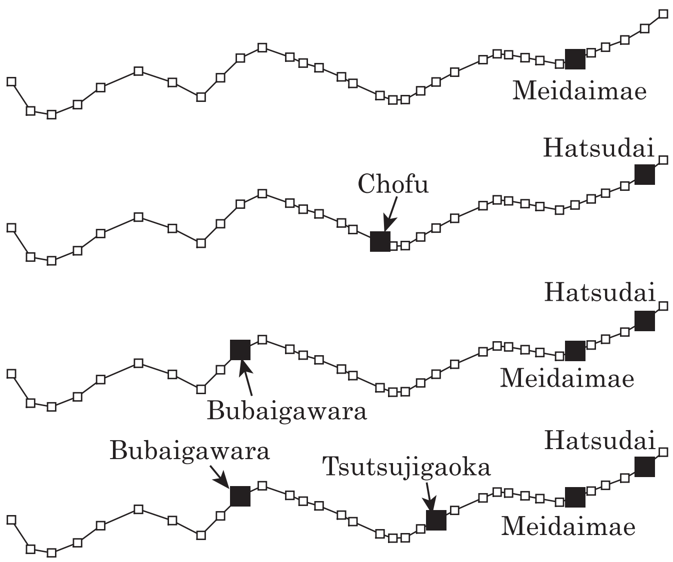

Figure 6: Optimal locations of flow capturing facilities for the original FCLP

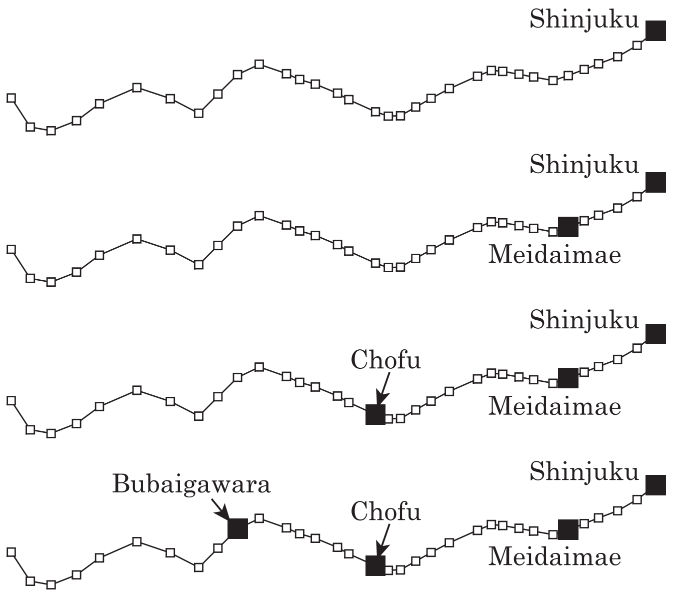

Figure 7: Optimal locations of flow capturing facilities for the FCLP with two levels of coverage in the case of $a_{\mathrm{f}} = 1.0$ and $a_{\mathrm{p}} = 0.2$

assumed that a given flow is more preferable to be captured at the origin station or the destination station than to be captured at a midway station along the path. We provided the integer programming formulation of the proposed problem and applied the model to analyze the optimal locations of flow capturing facilities on Keio Line using real passenger flow data.

There are various possible directions for future research. It is important to extend the number of coverage levels from two to multiple levels and find various applications. Another interesting topic is to incorporate the effect of multi-counting which assumes that the same flow can be captured multiple times when there are several facilities along the path.

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