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A Maximal Covering Model for Helicopter Emergency Medical Systems

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Abstract We propose a maximal covering location problem for helicopter emergency medical service system. In the system, the transportation time of each patient to a hospital depends on both locations of helipads for helicopters to land and take off and hospitals for deploying helicopters. The aim of this paper is to provide an optimal location strategy of both the helipads and the hospitals so as to maximize the number of covered demand which can be transported to a hospital within a preset time. The problem can be formulated as an integer linear program. We present numerical examples using an idealized city model and examine optimal solutions obtained.

Keywords Facility Location Theory; Maximal Covering Location Problem; Emergency Medical System; Helicopter

1 Introduction

In some major cities in Japan, emergency medical services are provided by helicopters with physicians, not only by a fleet of ambulances. These are dispatched from a hospital or an emergency department to a limited number of high-emergency calls requiring the presence of a specially-trained physician, which include car accidents involving major injuries, cardiac arrest cases, and so on. The helicopters must be located at appropriate points in order to provide adequate coverage. Thus we have to consider that the helicopters need landing locations without obstacles around them since they cannot always land at the accident points. The patients are transported by an ambulance to a place where a helicopter can land and then be transported by a helicopter to a hospital or an emergency department. In designing effective helicopter emergency medical service system, it is vital to design optimal locations where helicopters can land and take off (helipads) as well as hospitals for deploying them.

To the best of our knowledge, most available studies on such problems apply to ambulances only and there is no prior research that address the helicopter cases. These have been studied in the development of ambulance location models and algorithms. One of the earliest models are the maximal covering location problem [2] in which the aim is to maximize the number of covered demands with a given number p of ambulances. In

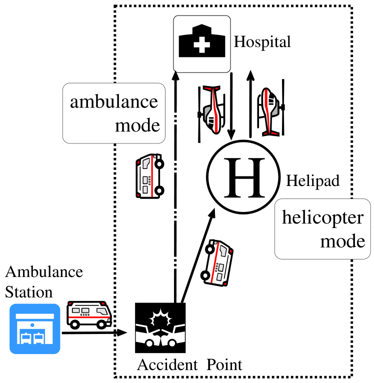


Figure 1: A helicopter emergency medical service system

this model, demands are considered to be covered if it can be reached by at least one ambulance within a given time T. Many static models and probabilistic models have been proposed as extensions of the model [1, 3, 4, 5].

Most of such models consider only transportation by ambulances. In helicopter emergency medical service systems, ambulance and helicopter transportation routes and helipad and hospital locations are important factors to ensure effective system operation. When patients are transported from helipads to hospitals both patients and helicopters need to arrive at a helipad. Both their locations affect the service level provided. Therefore when designing the systems, it is very important to consider both locations simultaneously.

Our aim is to formulate and solve a location problem arising in the system. The objective is to maximize the total covered demands which can be transported from their accident points to a hospital within a preset time, when the numbers of helipads and hospitals for deploying helicopters are given. We will introduce and formulate our model in the next section, followed by computational results on idealized city models, and by summary and future works.

2 Model description and formulation

In this section, we introduce the helicopter emergency medical service system assumed in this paper. We focus on how many people can arrive at a hospital within a given time T in the system. It is assumed that demands are discretely distributed over the target area. Let us consider the situation in which each patient has two modes to be transported to a hospital (Figure 1): by a low speed ambulance (ambulance mode) and by a high speed helicopter (helicopter mode). We assume the following:

- The transportation from each demand to a helipad is carried out by an ambulance.
- The travel distance between any two points is measured by Euclidean distance.
- The helicopter's speed v is much faster than the ambulance's speed w.

In explaining the travel time for these two modes, we introduce the Cartesian coordinates system to denote the demand location by (x, y). The helipad and hospital locations are denoted by (h_x, h_y) and (e_x, e_y). Then, in ambulance mode, the travel time from an accident point to the hospital is given as follows:

Travel Time by Ambulance Mode

$$t_{\mathrm{a}} = \frac{\sqrt{(x-e_x)^2+(y-e_y)^2}}{w}$$

In the case of helicopter mode, the travel time of each transportation period is given as:

- Travel time by an ambulance from an accident point to a helipad: t_1

$$t_1 = \frac{\sqrt{(h_x-x)^2+(h_y-y)^2}}{w}$$

- Travel time by a helicopter between a hospital to a helipad: t_2

$$t_2 = \frac{\sqrt{(e_x-h_x)^2+(e_y-h_y)^2}}{v}$$

Since both the ambulance and the helicopter have to arrive at the helipad, the total travel time in helicopter mode from each accident point to the hospital is given as follows:

Travel Time by Helicopter Mode

$$t_{\mathrm{h}} = \max\{t_1,\ t_2\} + t_2.$$

As indicated by this equation, the definition of the travel time from an accident point to a hospital is more complex than that of the travel time for ambulance mode only. Thus this travel time is affected by both the helipad and hospital locations.

We develop a maximal covering model of the helipad and hospital location problem and provide an integer programming formulation. The problem is how to locate p helipads and q hospitals so as to maximize the number of demands which can be transported to one of the hospitals within a given time T either by helicopter mode or by ambulance mode ($t_{\mathrm{h}} \le T \vee t_{\mathrm{a}} \le T$). To formulate the problem, the following notations are introduced:

Parameters

D : set of demand points
H : set of candidate locations for helipads
S : set of candidate locations for hospitals
p : number of helipads to be located
q : number of hospitals to be located
w_i : demand at node i
a_{ijk} : 1 if the transportation time by helicopter mode from demand i to hospital k via helipad j is within T hours, otherwise 0
b_{ik} : 1 if the transportation time by ambulance mode from demand i to hospital k is within T hours, otherwise 0

The following are decision variables:

Decision Variables

x_j : 0-1 variable that takes 1 if the helipad is located at node j, otherwise 0
y_k : 0-1 variable that takes 1 if the hospital is located at node k, otherwise 0
z_{jk} : 0-1 variable that takes 1 if the helipad is located at node j and the hospital is located at node k, otherwise 0
u_i : 0-1 variable that takes 1 if demand point i is transported by helicopter mode, otherwise 0
v_i : 0-1 variable that takes 1 if demand point i is transported by ambulance mode only, otherwise 0

The problem is formulated as follows:

Maximal Covering Model

$$\text{max.} \quad \sum_{i \in D} w_i(u_i + v_i), \tag{1}$$

$$\text{s. t.} \quad z_{jk} \le x_j, \qquad j \in H\ k \in S, \tag{2}$$

$$z_{jk} \le y_k, \qquad j \in H\ k \in S, \tag{3}$$

$$u_i \le \sum_{j \in H} \sum_{k \in S} a_{ijk} z_{jk}, \qquad i \in D, \tag{4}$$

$$v_i \le \sum_{k \in S} b_{ik} y_k, \qquad i \in D, \tag{5}$$

$$u_i + v_i \le 1, \qquad i \in D, \tag{6}$$

$$\sum_{j \in H} x_j = p, \tag{7}$$

$$\sum_{k \in S} y_k = q, \tag{8}$$

$$x_j \in \{0,1\}, \qquad j \in H, \tag{9}$$

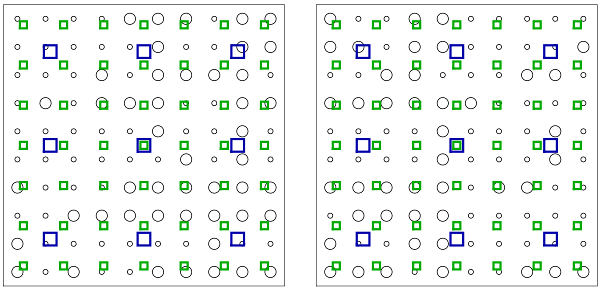
$$y_k \in \{0,1\}, \qquad k \in S, \tag{10}$$

$$z_{jk} \in \{0,1\}, \qquad j \in H,\ k \in S, \tag{11}$$

$$u_i \in \{0,1\}, \qquad i \in D, \tag{12}$$

$$v_i \in \{0,1\}, \qquad i \in D. \tag{13}$$

The objective (1) maximizes the sum of covered demands. Constraints (2) and (3) are combined to mean that a pair of helipad j and hospital k cannot be used unless the pair is located. Constraints (4) stipulate that demand i cannot be covered by a helicopter unless at least one pair, $(j,\ k)$, is located within the time T. Constraints (5) state that demand i cannot be covered by an ambulance unless at least one hospital is located within the time T. Constraints (6) represent that each demand is covered by either a helicopter or an ambulance. Constraints (7) and (8) mean that the numbers of helipads and hospitals are p and q, respectively. Constraints (9), (10), (11), (12), and (13) are standard binary constraints.



(a) Pattern 1 (b) Pattern 2

Figure 2: Two demand distribution patterns in a square target area

3 Numerical Examples

This section presents some optimal solutions of our proposed model by using an idealized square city model and provides some insights obtained in this analysis. The integer program (1)–(13) was run on CPLEX 12.1 under an Intel Core2 Duo 2.5 GHz and 2 GB RAM environment. As shown in Figure 2 we consider a square target area with 200 km side lengths. There are 49 candidate helipad locations (small squares in Figure 2) and nine candidate hospital locations (large squares in Figure 2). In the area, accident points are distributed regularly and uniformly. We use two patterns of demands distributions which are represented by proportional circles: larger demands are dominant in the right-hand side of the area in Pattern 1, and those are dominant in the left-hand side in Pattern 2. We show six examples of various values of p and q. We use $T = 0.5$ hour in each example.

Table 1 shows the percentages of the covered demands. Figures 3 and 4 show the optimal solutions in Pattern 1 and Pattern 2, respectively. Colored squares denote the selected locations for each optimal solution. Demand points shown as black colored circle and as gray colored circle represent helicopter mode users and ambulance mode users, respectively. To cover the demands effectively, hospitals and helipads are located in the east in Pattern 1 (Figure 3) and in the west in Pattern 2 (Figure 4) in response to the biased demand distributions.

4 Summary and Future Works

We have formulated and solved a maximal covering location problem arising in the management of the helicopter emergency medical service systems. The aim is to provide an optimal location of helipads and hospitals for deploying helicopters, both of which are important in determining the systems efficiency. The problem was formulated as an 0-1 integer programming problem and analyzed some optimal solutions obtained by CPLEX.

Future research on this topic may focus on several areas. Our model focused only on the transportation time from demand points to a hospital. However, other constraints and restrictions may be required depending on each environment. For instance, the travel time of helicopters from a hospital to a helipad is also important to provide first aid by physicians. Another attractive topics are to devise sophisticated solution algorithms for

Table 1: Computational Results

p	q	Pattern 1	Pattern 2
6	1	20.14 %	21.32 %
6	2	29.86 %	33.09 %
6	3	37.50 %	39.71 %
12	2	40.28 %	41.18 %
12	3	48.26 %	50.74 %
12	4	54.86 %	57.35 %
18	3	56.25 %	58.09 %
18	4	65.28 %	65.81 %
18	5	71.18 %	72.43 %

large instances and to apply our model and the algorithms to real-world scenarios using actual data.

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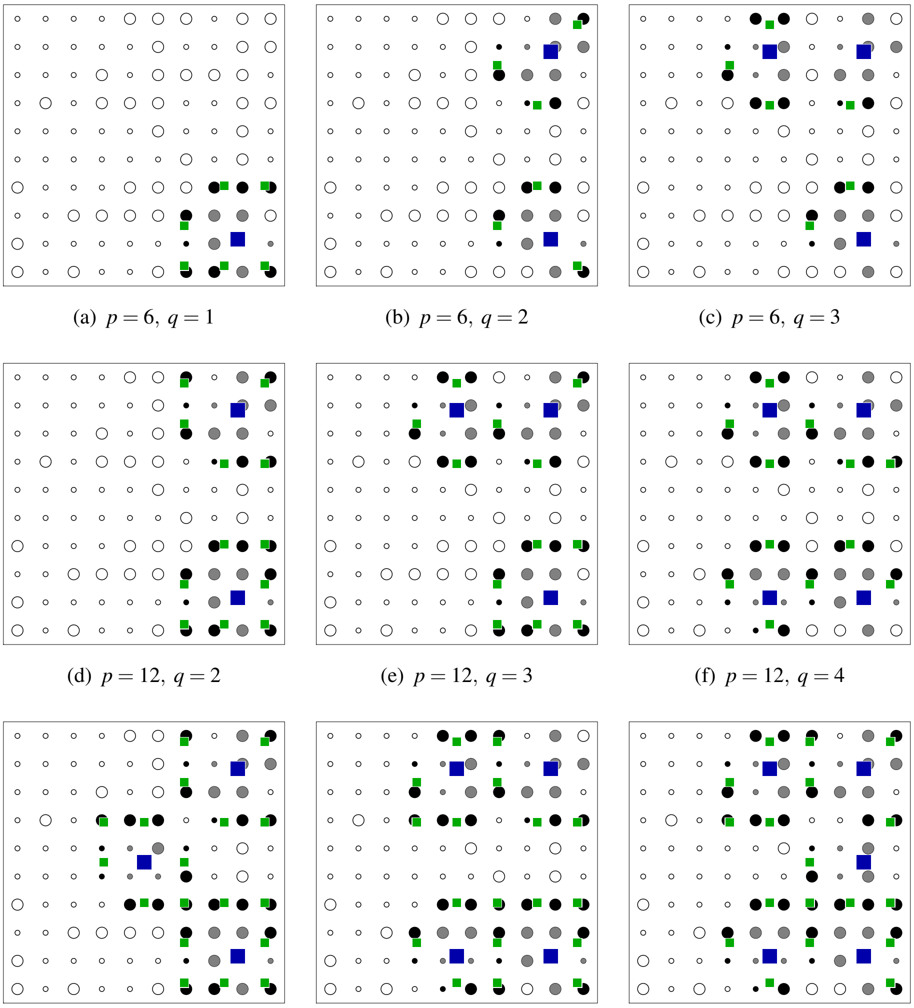
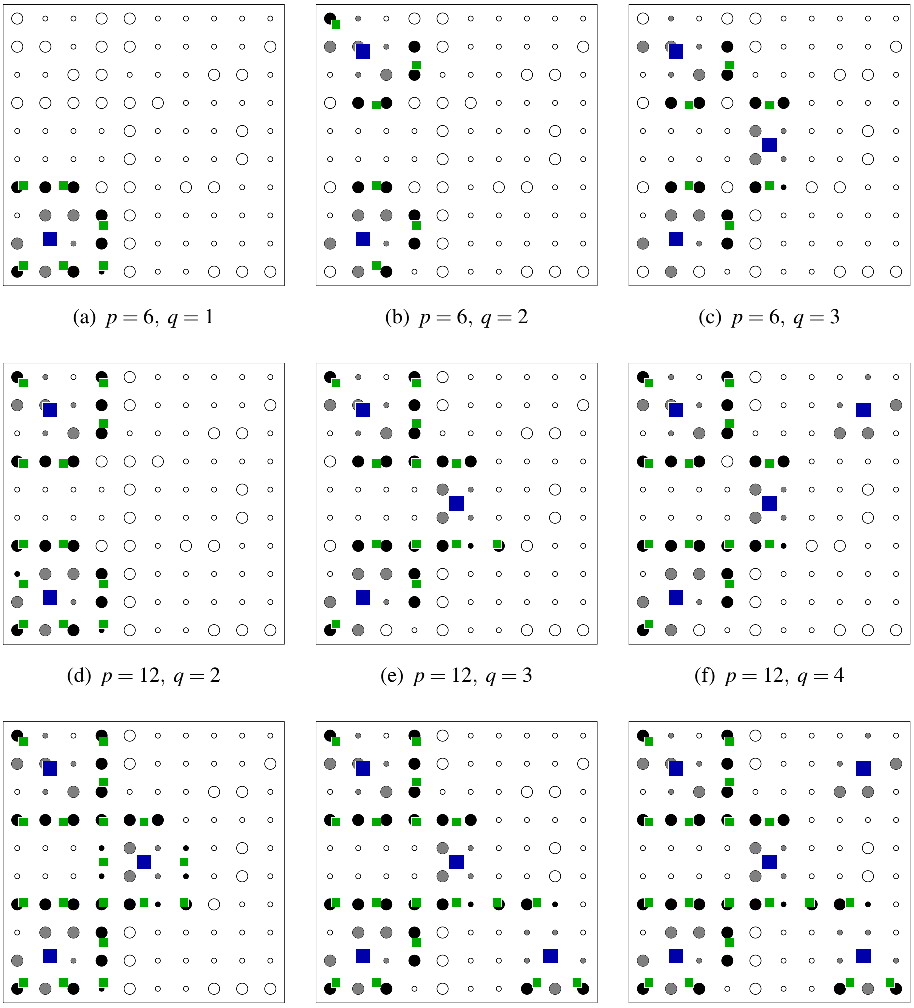


(a) $p = 6,\ q = 1$ (b) $p = 6,\ q = 2$ (c) $p = 6,\ q = 3$

(d) $p = 12,\ q = 2$ (e) $p = 12,\ q = 3$ (f) $p = 12,\ q = 4$

(g) $p = 18,\ q = 3$ (h) $p = 18,\ q = 4$ (i) $p = 18,\ q = 5$

Figure 3: Optimal solutions for Pattern 1



(a) $p = 6,\ q = 1$ (b) $p = 6,\ q = 2$ (c) $p = 6,\ q = 3$

(d) $p = 12,\ q = 2$ (e) $p = 12,\ q = 3$ (f) $p = 12,\ q = 4$

(g) $p = 18,\ q = 3$ (h) $p = 18,\ q = 4$ (i) $p = 18,\ q = 5$

Figure 4: Optimal solutions for Pattern 2