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Two-stage Model of Vehicle Routing Problem with Fuzzy Demands and its Ant Colony System Algorithm

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Abstract In this paper, we deal with the vehicle routing problem where vehicles have finite capacities and demands of customers are uncertain. We represent the uncertain demands by triangular fuzzy numbers and interpret them as possibility distributions. According to the same consideration as the stochastic programming with recourse, we treat the influence of the fuzziness of customers' demands as recourse cost and formulate the problem as a two-stage possibilistic programming model. Defining the Fuzzy Mean of a fuzzy number as its the generalized mean value, the proposed model is equivalent to an ordinary programming problem and then a solution method based on Ant Colony System (ACS) can be proposed to give the best solution of the problem. Finally, some examples are given to illustrate the two-stage model and the solution algorithm.

Keywords Fuzzy Vehicle Routing, Possibilistic Programming, Recourse Cost, Ant Colony System

1 Introduction

Efficient fleet and vehicle planning, scheduling and dispatching are essential for transportation service providers that want to improve service and increase reactivity, particularly when cost is a primary factor. In the field of operations research, the problems of fleet and vehicle planning, scheduling and dispatching are recognized as Vehicle Routing Problems (VRP). A typical vehicle routing problem requires one to design least cost routes from one depot to a set of geographically scattered points (cities, stores, warehouses, schools, customers etc.) [1].

Although a great number of models and solution methods for solving vehicle routing problems, almost of them arise from deterministic mathematical models and all the factors involved in the models must be known exactly. Unfortunately, real world situations are often not so deterministic. There are cases that the imprecision/ uncertainty concerning demand, location, distance, timing, travel time, etc. must be taken into account. Fuzzy set theory has provided efficient and meaningful concepts and methodologies to formulate and solve mathematical programming and decisionmaking problems of real world. Fuzzy approaches have been applied to solve a few kinds of fuzzy vehicle routing problems. Dubois and Prade [2] first introduced the fuzzy shortest-path problem in 1980. This problem has further been investigated by other researchers and generalized to a variety of situations [3]. Cheng and Gen [4] proposed a genetic algorithm to solve the fuzzy vehicle routing problem that fuzzy due-time is given as a triangular fuzzy number and the objectives are to minimize the fleet size of vehicles, maximize the average grade of satisfaction over customers, and minimize total travel distance and total waiting time for vehicles. Dusan and Goran [5] incorporated the rules of fuzzy arithmetic and fuzzy logic into the heuristic sweeping algorithm, and proposed two approximate reasoning algorithms to solve the vehicle routing problem with fuzzy demands.

In this paper, we deal with the vehicle routing problem that vehicles have finite capacities and demands of customers are uncertain. We represent the uncertain demands by triangular fuzzy numbers and interpret them as possibility distributions. We propose a two-stage model with recourse cost for the problem, and then show that this model is equivalent to an ordinary 0-1 integer programming problem. To obtain the best solution of the problem, we further propose a solution method based on Ant Colony System (ACS). Finally, some examples are given to illustrate the two-stage model and the solution algorithm.

2 Problem Description

The fuzzy vehicle routing problem considered in this paper is specified as follows:

(1) There is one central depot. m vehicles in the fleet start from the central depot, traverse n (n >1) customers to pick up passengers or products, and return to the central depot. Customers are indexed from 1 to n and the index 0 stands for the central depot.

(2) The cost (distance or time, etc.) of a vehicle travelling from customer i (or depot 0) directly to customer j (or depot 0) is c_{ij}, which is an exact number and also $c_{ii}=0$ (i, j =0, 1, 2, …, n).

(3) The capacity of vehicle k is Q_k (k =1, 2, …, m), which is an exact number.

(4) The demands of customers cannot be known exactly. The demand of customer i is described by a normalized triangular fuzzy number $\boldsymbol{D}_i$, its membership function is $\mu_{D_i}(x)$ (i =1, 2, …, n).

(5) A picking up must be made to each customer exactly once and vehicles are always available.

(6) The objective is to find the optimal routing plan for vehicles to minimize the total cost, which is the weighted sum of the total travel cost of vehicles and penalties for less utilization or lack of capacity of vehicles.

3 Two-Stage Model

3.1 Notation and the Mode of fuzzy number

In this paper, index i and j are used to denote a customer (i,j =1, 2, …, n) and

i,j= 0 stand for depot, and further index k is used to denote a vehicles (k =1, 2, …, m). A 0-1 variable x_{kij} is introduced that x_{kij}=1 if vehicle k travels directly from customer i to customer j and otherwise x_{kij}=0. Because the demand of customer i is a fuzzy number $\boldsymbol{D}_i$, total requirement of picking up for vehicle k is also a fuzzy number and denoted as $\boldsymbol{R}_k$ (k =1, 2, …, m).

Generally, the mode is the most frequently occurring value in a set of discrete data. The mode of a fuzzy number is the value or the median of values corresponding to the membership function value 1. Let M($\boldsymbol{t}$) be the operator to calculate the mode of fuzzy number $\boldsymbol{t}$ and t_a be its mode, i.e., t_a=M($\boldsymbol{t}$), $\boldsymbol{t}$ can then be rewritten as:

$$\boldsymbol{t}= t_a +\boldsymbol{t_f} \tag{1}$$

where t_a is a crisp value and $\boldsymbol{t_f}$ is a fuzzy number with mode 0 (M($\boldsymbol{t_f}$)=0).

According to equation (1), the fuzzy demand $\boldsymbol{D}_i$ can be described as:

$$\boldsymbol{D}_i = D_{ai}+ \boldsymbol{D}_{fi} \tag{2}$$

where $\boldsymbol{D}_{fi}$ satisfies M($\boldsymbol{D}_{fi}$)=0, and $\boldsymbol{D}_{fi}$ represents a measure of the degree of fuzziness of demand $\boldsymbol{D}_i$. $\boldsymbol{R}_k$, the total requirement of picking up for vehicle k, can also be described as:

$$\boldsymbol{R}_k= R_{ak}+ \boldsymbol{R}_{fk} \tag{3}$$

where $\boldsymbol{R}_{fk}$ satisfies M($\boldsymbol{R}_{fk}$)=0.

3.2 First Stage Problem (FSP)

If we take only D_{ai} into account, the less utilization of capacity of vehicle k, denoted by S_{ak} and the lack of capacity of vehicle k, denoted by L_{ak} can be defined as

$$S_{ak} =\mathrm{Max}\,(0, Q_k - R_{ak}),\ L_{ak} =\mathrm{Max}(0, R_{ak} - Q_k) \tag{4}$$

Considering only S_{ak} and L_{ak}, we can formulate a crisp vehicle routing problem as the following equations (5)-(11).

(First Stage Problem FSP)

$$\text{Minimize}\quad C = \sum_{k=1}^{m}\sum_{i=0}^{n}\sum_{j=0}^{n} x_{kij}c_{ij} + \sum_{k=1}^{m}(d_k S_{ak} + e_k L_{ak}) \tag{5}$$

Subject to

$$R_{ak} = \sum_{i=0}^{n}\sum_{j=1}^{n} x_{kij} D_{aj} \quad ; \quad k=1, 2, \dots, m \tag{6}$$

$$S_{ak} =\mathrm{Max}\,(0, Q_k - R_{ak}),\ L_{ak} =\mathrm{Max}(0, R_{ak} - Q_k)\ ;\ k=1, 2, \dots, m \tag{7}$$

$$\sum_{k=1}^{m}\sum_{j=0}^{n} x_{kij} = 1 \quad ; \quad i=1, 2, \dots, n \tag{8}$$

$$\sum_{j=1}^{n} x_{k0j} = 1,\quad \sum_{i=1}^{n} x_{ki0} = 1 \quad ; \quad k=1, 2, \dots, m \tag{9}$$

$$\sum_{j=0}^{n} x_{kij} - \sum_{j=0}^{n} x_{kji} = 0 \quad ; \quad i=1, 2, \dots, n \tag{10}$$

$$x_{kij} \in \{1, 0\}\ ;\ i,j=0, 1, 2, \dots, n;\ k=1, 2, \dots, m \tag{11}$$

3.3 Recourse Cost and Two-Stage Model

Based on the same consideration as the stochastic programming with recourse [6], we can first solve the first stage problem (FSP) described above and obtain a delivery plan. Then, at the second stage, once the delivery plan obtained at the first stage is carried out, we need make corrections or recourse actions on the delivery plan, because the realization of the fuzzy demand differs from its mode or most possible values. These corrections or recourse actions should be made at minimal costs.

(1) Recourse cost

For the fuzzy vehicle routing problem considered here, The recourse cost is the penalty due to less utilization of vehicles' capacity and the additional cost to cover the failures that vehicles are not able to serve some customers on the planned route due to insufficient capacity. Due to the spread $\boldsymbol{D}_{fi}$ of the fuzzy demand $\boldsymbol{D}_i$, the total requirement for vehicle k, $\boldsymbol{R}_k$ differs from its mode R_{ak} and this difference, denoted as $\boldsymbol{R}_{fk}$ in equation(3), can be calculated as:

$$\boldsymbol{R}_{fk} = \sum_{i=0}^{n} \sum_{j=1}^{n} x_{kij} \boldsymbol{D}_{fj} \quad ; \quad k=1, 2, \ldots, m \tag{12}$$

The difference $\boldsymbol{R}_{fk}$ in the requirement for vehicle k leads to the difference in the less utilization of capacity or the lack of capacity of vehicle k. Denoting the difference in the less utilization of capacity and the lack of capacity of vehicle k by $\boldsymbol{S}_{fk}$ and $\boldsymbol{L}_{fk}$ respectively, we define $\boldsymbol{S}_{fk}$ and $\boldsymbol{L}_{fk}$ as:

$$\boldsymbol{S}_{fk} = \mathrm{Max}\,(0, -\boldsymbol{R}_{fk}) \, ; \quad \boldsymbol{L}_{fk} = \mathrm{Max}(0, \boldsymbol{R}_{fk}) \tag{13}$$

Let u_k (u_k>0) be the penalty for a unit less utilization of capacity of vehicle k, and v_k (v_k>0) be the additional cost for a unit lack of capacity of vehicle k, the recourse cost can then be calculated as:

$$\sum_{k=1}^{m} (u_k \boldsymbol{S}_{fk} + v_k \boldsymbol{L}_{fk}) \tag{14}$$

(2) Two-Stage Model

For each possible realization of fuzzy demands, we can decide a correction or a recourse action of the minimum recourse cost; weighted with their respective possibilities, a mean cost can be computed. This mean cost is indeed the mean recourse cost corresponding to all of routing plan obtained at the first stage. The criterion to choose a routing plan thus becomes the minimal total cost, consisting of the direct cost in the first stage and the mean recourse cost at the second stage. We propose Two-Stage Model (TSM) for the fuzzy vehicle routing problem as follows.

(Two-Stage Model , TSM)

$$\text{Minimize} \quad C = \sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=0}^{n} x_{kij} c_{ij} + \sum_{k=1}^{m} (d_k S_{ak} + e_k L_{ak})$$

$$+ \text{Fuzzy mean}\Big[\sum_{k=1}^{m} (u_k \boldsymbol{S}_{fk} + v_k \boldsymbol{L}_{fk}) \Big] \tag{15}$$

Subject to

$$R_{ak} = \sum_{i=0}^{n} \sum_{j=1}^{n} x_{kij} D_{aj} \quad ; \quad k=1, 2, \ldots, m \tag{16}$$

S_{ak} =Max (0, Q_k - R_{ak}), L_{ak} =Max(0, R_{ak} - Q_k) ; k=1, 2, …, m (17)

$$\boldsymbol{R}_{fk} = \sum_{i=0}^{n}\sum_{j=1}^{n} x_{kij}\boldsymbol{D}_{fj} \quad ; \quad k=1, 2, \ldots, m \tag{18}$$

$\boldsymbol{S}_{fk}$ =Max (0, -$\boldsymbol{R}_{fk}$) ; $\boldsymbol{L}_{fk}$ =Max(0, $\boldsymbol{R}_{fk}$) (19)

$$\sum_{k=1}^{m}\sum_{j=0}^{n} x_{kij} = 1 \quad ; \quad i=1, 2, \ldots, n \tag{20}$$

$$\sum_{j=1}^{n} x_{k0j} = 1, \quad \sum_{i=1}^{n} x_{ki0} = 1 \quad ; \quad k=1, 2, \ldots, m \tag{21}$$

$$\sum_{j=0}^{n} x_{kij} - \sum_{j=0}^{n} x_{kji} = 0 \quad ; \quad i=1, 2, \ldots, n \tag{22}$$

$$x_{kij} \in \{1, 0\} \; ; \; i,j=0, 1, 2, \ldots, n; \; k=1, 2, \ldots, m \tag{23}$$

3.4 Equivalent Problem of Two-Stage Model

As the two-stage model of equations (15)-(23) includes the "Fuzzy mean", we have to decide how to define it before discussing the solution procedure of the problem. Here, we define the "Fuzzy mean" of a fuzzy number as its the generalized mean value (GMV) proposed by Lee and Li [7]. If $\boldsymbol{t}$ is a normalized triangular fuzzy number, $\boldsymbol{t}=\langle\langle a,b,c \rangle\rangle$, its GMV can be obtained as:

GMV($\boldsymbol{t}$)=(a + b + c) / 3 (24)

As $\boldsymbol{D}_{fi}$ and $\boldsymbol{R}_{fk}$ are normalized triangular fuzzy numbers with mode 0, we have

$$\boldsymbol{D}_{fi} = \langle\langle D_i^L, 0, D_i^R \rangle\rangle, \; \boldsymbol{R}_{fk} = \langle\langle R_k^L, 0, R_k^R \rangle\rangle \tag{25}$$

According to equations (18), $\boldsymbol{R}_{fk}$ can be rewritten as:

$$\boldsymbol{R}_{fk} = \langle\langle R_k^L, 0, R_k^R \rangle\rangle = \langle\langle \sum_{i=0}^{n}\sum_{j=1}^{n} x_{kij} D_j^L, 0, \sum_{i=0}^{n}\sum_{j=1}^{n} x_{kij} D_j^R \rangle\rangle \tag{26}$$

Notice that $\boldsymbol{R}_{fk}$ is a fuzzy number with mode 0 and satisfies $R_k^L \le 0$, $R_k^R \ge 0$, we can rewrite $\boldsymbol{S}_{fk}$ and $\boldsymbol{L}_{fk}$ as:

$$\boldsymbol{S}_{fk} = \mathrm{Max}\,(0, -\boldsymbol{R}_{fk}) = \langle\langle 0, 0, -R_k^L \rangle\rangle \tag{27}$$

$$\boldsymbol{L}_{fk} = \mathrm{Max}\,(0, \boldsymbol{R}_{fk}) = \langle\langle 0, 0, R_k^R \rangle\rangle \tag{28}$$

According to equation (24), the GMV of $\boldsymbol{S}_{fk}$ and $\boldsymbol{L}_{fk}$ can be obtained as:

$$\mathrm{GMV}\,(\boldsymbol{S}_{fk}) = -R_k^L/3, \; \mathrm{GMV}\,(\boldsymbol{L}_{fk}) = R_k^R/3 \tag{29}$$

Then, we can obtain the equivalent problem of TSM as:

(Equivalent Problem of TSM)

$$\text{Minimize} \quad C = \sum_{k=1}^{m}\sum_{i=0}^{n}\sum_{j=0}^{n} x_{kij} c_{ij} + \sum_{k=1}^{m} (d_k S_{ak} + e_k L_{ak}) + \sum_{k=1}^{m} (-u_k R_k^L + v_k R_k^R)/3 \tag{30}$$

Subject to

$$R_{ak} = \sum_{i=0}^{n}\sum_{j=1}^{n} x_{kij} D_{aj} \quad ; \quad k=1, 2, \ldots, m \tag{31}$$

$$S_{ak} = \mathrm{Max}\,(0, Q_k - R_{ak}),\ L_{ak} = \mathrm{Max}(0, R_{ak} - Q_k)\ ;\ k=1, 2, \ldots, m \tag{32}$$

$$R_k^L = \sum_{i=0}^{n}\sum_{j=1}^{n} x_{kij} D_j^L,\ R_k^R = \sum_{i=0}^{n}\sum_{j=1}^{n} x_{kij} D_j^R \quad ; \quad k=1, 2, \ldots, m \tag{33}$$

$$\sum_{k=1}^{m}\sum_{j=0}^{n} x_{kij} = 1 \quad ; \quad i=1, 2, \ldots, n \tag{34}$$

$$\sum_{j=1}^{n} x_{k0j} = 1,\quad \sum_{i=1}^{n} x_{ki0} = 1 \quad ; \quad k=1, 2, \ldots, m \tag{35}$$

$$\sum_{j=0}^{n} x_{kij} - \sum_{j=0}^{n} x_{kji} = 0 \quad ; \quad i=1, 2, \ldots, n \tag{36}$$

$$x_{kij} \in \{1, 0\}\ ;\ i,j=0, 1, 2, \ldots, n;\ k=1, 2, \ldots, m \tag{37}$$

It is obvious that the two-stage model (15)-(23) of the fuzzy vehicle routing problem reduces to an ordinary 0-1 integer programming problem. .

4 Ant Colony System method

We propose an algorithm to obtain solution of the equivalent problem (30)-(37), it is based on the Ant Colony System (ACS) algorithm proposed by Dorigo and Gambardella [8]. The procedures of our algorithm are described as follows:

[Step1] Set parameters: q_0 ($0 \le q_0 \le 1$), α and β ($0 \le \alpha, \beta \le 1$), ψ and ρ ($0 < \psi, \rho <$ 1).

[Step2] Initialize solution.

[Step2.1] Let $\tau_0 = f_0$ (f_0 is a given value) and set the initial pheromone level $\tau(i, j) = \tau_0$ (i,j=0, 1, 2, …, n).

[Step2.2] Place m ants to the central depot 0, and define the set of customers that remain to be visited by ant k (k=1, 2, …, m) as $N_k=\{i, i=1, 2, \ldots, n\}$.

[Step3] Construct solution.

For each ant k, set i= 0 (denoting the customer where ant k is currently in), repeat the following Step3.1 to Step3.3 until ant k has completed its solution ($N_k = \Phi$).

[Step3.1] Generate a random number q($q \in [0,1]$), choose the customer j to be visited after customer i by applying the pseudo-random-proportional rule given by following equation (38):

$$j = \begin{cases} \arg\max\limits_{h \in N_k} \{ [\tau_{ih}]^{\alpha} \cdot [\eta_{ih}]^{\beta} \}, & \text{if } q \le q_0 \\ r & \text{, otherwise} \end{cases} \tag{38}$$

where η_{ih} is the visibility of customer h from customer i given by in following equation (39):

$$\eta_{ih} = [c_{ih} + d_k S_{ak} + e_k L_{ak} + (-u_k R_k^L + v_k R_k^R)/3]^{-1} \tag{39}$$

r ($r \in N_k$) is a customer selected according to the probability p_{ir} given in following equation (40):

$$p_{ir} = \begin{cases} \dfrac{[\tau_{ir}]^{\alpha} \cdot [\eta_{ir}]^{\beta}}{\sum_{u \in N_k} [\tau_{iu}]^{\alpha} \cdot [\eta_{iu}]^{\beta}}, & \text{if } r \in N_k \\ 0, & \text{otherwise} \end{cases} \tag{40}$$

[Step3.2] Change the pheromone level by applying the local updating rule of equation (41):

$$\tau_{ij} \leftarrow (1-\rho) \cdot \tau_{ij} + \rho \cdot \tau_0 \tag{41}$$

[Step3.3] Remove customer j from N_k and set i=j as the current customer. Then return to Step3.1.

[Step4] Denote the global best solution as X_{gb} and the objective value of equation (30) corresponds to X_{gb} is f_{gb}. Update the pheromone level by applying the global updating rule of equation (42):

$$\tau_{ij} \leftarrow (1-\psi) \cdot \tau_{ij} + \psi \cdot \Delta\tau_{ij} \tag{42}$$

where $\Delta\tau_{ij} = \begin{cases} \dfrac{1}{f_{gb}} & \text{if link}(i,j) \in X_{gb} \\ 0 & \text{otherwise} \end{cases}$

[Step5] If the stopping criterion (maximum number of iterations G, in this paper) is met, then stop and output the best solution. Otherwise, go to Step 2.

5 Computational Experiment

To illustrate the proposed model and algorithm, we generated computational examples that the direct cost between every two customers or depot and the fuzzy demands of customers are given randomly from some uniform discrete distributions.

(1) Compared to the chance-constrained model [9], it was demonstrated that the two-stage model has no infeasible solution because non-satisfaction of the capacity constraints is taken into the objective value as recourse costs. On the other hand, chance-constrained models provide very tight constraint to capacity of vehicles and therefore it has very few feasible solutions.

(2) Applying the proposed ACS algorithm, the optimal solutions of 19 out of 20 3-customer and 2-vehicle fuzzy vehicle routing problems were obtained and so the proposed algorithm is effective to small problems. Although iteration number to repeat step2 to step4 was set to 500, the proposed algorithm gave the optimal or best solutions at only the first iteration. It is very efficient to give problems' solution and on the other hand, it is necessary to improve the algorithm to escape from the local optimal solution.

6 Concluding Remarks

For those situations when the manager cannot exactly specify customers' demand as either deterministic numbers or probabilistic random variables, it is natural and realistic to express the demand as fuzzy numbers. This study dealt with fuzzy vehicle routing problem and formulated a two-stage possibilistic programming model where the influence of the fuzziness of demands was treated as recourse cost. Defining fuzzy mean as the generalized mean value, the two-stage model is equivalent to an ordinary crisp vehicle routing problem with a linear objective function. Thus, its solution can be obtained easily by applying the proposed Ant Colony System algorithm.

The proposed two-stage approach can not only take the fuzziness of customers' demand into account, but also can avoid the computational complexity involved with calculating the summation of non-normalized fuzzy variables and comparing the non-normalized fuzzy objective values. The proposed Ant Colony System algorithm is effective to small problems and competitive with respect to computational effort.

Like other Ant Colony System algorithms, the proposed Ant Colony System algorithm must be improved to be able to escape more effectively from local optimal solutions and solution quality should be confirmed by comparing the proposed algorithm with some optimization methods or meta-heuristic like Genetic Algorithms (GA) .

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