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New Method for Basic Road Network Programming

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Abstract In this paper, a new road network programming model and strategy are proposed. First, the definition of basic road network is introduced. Then a new mathematical model and method on solving the basic road network programming are proposed. Last, a typical example with relevant evaluation are carried out, which testified the feasibility and efficiency of the proposed model and method.

Keywords Road Network Programming; Operational Research; Basic Road Network

1 Introduction

With the development of the national economy and society, the road transportation is playing a more and more significant role in the integrated transport system. While the road transportation is an absolutely dominated mode of transport in developed countries, the proportion of road transport volume in the whole integrated transport system is growing year by year. However, the "bottleneck" of the road transport has not been well solved, which can be largely contributed to the present road network layout. Since the basic road network didn't function very well on the premise that the trunk lines of road network has been fixed, a great amount of transport demand can not achieve the standard of "accessibility"[1,2]. Therefore, it is necessary to explore more advanced methods and strategies to settle the newly occurred problems.
In this paper, the road network is classified into 2 categories, one is the trunk road network and the other is the basic one. The classification stands on the research organization. For example, if the research is conducted by a provincial organization, then the trunk road network contains the state highway, the basic road network contains the provincial highway, prefectural road, country road, village road and linking road. If the research organization is a country-level one, then the basic road network merely consists of country road, village road and linking road. The rest are all belongs to the trunk road network. The existing theories and methods[3–6] such as OD traffic flow method, important degree-traffic location method are mainly aimed at solving trunk road network programming. So, based on our research, a new method based on "Four Factors" is proposed. In this paper, the four factors are defined as the road network construction investment, land expropriation cost, transportation cost and network transportation efficiency since they are the fundamental elements must be considered in all road network programming and construction.

The four factors are utilized as the main components in mathematical modeling and result evaluations.

2 Basic Road Programming Based on Four Factors Method

Suppose there are n points in the basic road network. For any given 2 given demand points v_i and v_j, they have two relationships, one is they are connected by road, the other is isolated from each other. Denote them as follows:

$$a_{ij} = \begin{cases} 1 & road \quad existing \\ 0 & else \end{cases}$$

Then an adjacency matrix A_G can be utilized to describe the entire connective relationships of the road network.
Before further discussion, we outline some hypothesis. Suppose there are p ranks of road, the unit construction cost of each rank is z_i , designed width of each rank is k_i, designed traffic volume is y_i, unit transport cost is $h_i,(i = 1,...,p)$. Besides, suppose there are r ranks of the land, unit land expropriation cost is $t_i,(i = 1,...,r)$, l_{ij}represents the length of the road between any given demand points $v_i$and $v_j,(i \neq j)$, corresponding road transportation quantity is$\overline{y}_{ij} = F(a_{ij})$. Q is the total cost of the road network, f_{ij} describe the road in function between any given two demand points $v_i$and $v_j(i \neq j)$.

2.1 Basic Road Network Construction

First, let's suppose all the roads in the plan are straight lines, that is, all f_{ij} are linear functions. Thus, the basic road network programming problem can form an constrained optimization model as follows[7]:

$$min \quad Q = \alpha_1 Q_1 + \alpha_2 Q_2 + \alpha_3 Q_3 + \beta \alpha_4 Q_4$$

$$s.t. \begin{cases} \overline{y}_{ij} > \gamma \widetilde{y}_{ij} & (i, j = 1,...,s, i \neq j) \\ a_{ij} \in \{0,1\}, d_{i'} > 0, i' = 1,...,p \end{cases}$$

where: $\alpha_i \in [0,1], i = 1,...,4, \sum_{i=1}^{4} \alpha_i = 1$, β is conversion factor, converting the efficiency into currency value,$\gamma > 1$is a given constant, $\widetilde{y}_{ij}$is the certain designed traffic volume when road f_{ij} is determined, that is, it take in certain y_i value, c_{ij} represents the total cost of road f_{ij}.
Q_1represents the total road network construction cost:

$$Q_1 = \sum_{j=1,j\neq i}^{s} \sum_{i=1}^{s} \sum_{i'=1}^{p} a_{ij} l_{ij} z_{i'}$$

Q_2represents the total road network land expropriation cost:

$$Q_2 = \sum_{j=1,j\neq i}^{s} \sum_{i=1}^{s} a_{ij} l_{ij} (\sum_{j'=1}^{r} \sum_{i'=1}^{p} \delta_{j'} k_{i'} t_{j'})$$

where $\delta_{j'} \in [0,1]$, $\sum_{j'=1}^{r} \delta_{j'} = 1$ are undetermined coefficients.
Q_3represents the total road network transport cost:

$$Q_3 = \sum_{j=1,j\neq i}^{s} \sum_{i=1}^{s} a_{ij} l_{ij} \bar{y}_{ij} (\sum_{i'=1}^{p} \tau_{i'} h_{i'})$$

where $\tau_{i'} \in [0,1]$, $\sum_{i'=1}^{p} \tau_{i'} = 1$ are undetermined coefficients.
Q_4represents the total road network transportation efficiency:

$$Q_4 = D(\varepsilon_{ij})^{\zeta} |\bar{\varepsilon} - \eta|^{1-\zeta}$$

where $D(\varepsilon_{ij})$is the variance of $\varepsilon_{ij}, i, j = 1, ..., s$,$\varepsilon_{ij}, (i, j = 1, ..., s)$ is the specific road transportation efficiency index. Usually, it falls in $[0.6, 0.8]$; $\bar{\varepsilon}$ is the mean of $\varepsilon_{ij}, (i, j = 1, ..., s)$,$\eta, \zeta \in [0,1]$are given coefficients.

2.2 Road Shape Optimization

In 2.1, all roads are supposed straight lines, that is, all$f_{ij}$are linear functions. Nevertheless, this is not true in reality. Due to the restrictions of the natural ground features, such as landform, mountains, rivers, the road shapes are varied. Thus, f_{ij} can take in other types of functions, such as exponential function, trigonometric function. Noting that many functions can be expressed into polynomial function by Taylor Series Expansion, then f_{ij} can be described as follows without losing generosity:

$$f_{ij} = g_m x^m + g_{m-1} x^{m-1} + \cdots + g_1 x + g_0$$

where m is the frequency of the polynomial,$g_i, i = 0, ..., m$ are coefficients. Suppose f_{ijk} is a curve function,$x \in [x_0, x_q]$, $(x_0, f_{ijk}(x_0))$ and $(x_q, f_{ijk}(x_q))$ are the coordinates of points v_i, v_j, $\int_{x_0}^{x_q} f_{ijk}(x)dx$ is the curve length.
If the total cost of the curve road is lower than corresponding straight line obtained in part 2.1, then f_{ij} is replaced byf_{ijk},that is, a curved road is preferred. Otherwise, the straight line is retained. The detailed method is outlined as Algorithm 1:
Step 1: Let $c_{ij0} = c_{ij}, k = 1$
Step 2: Compute the total cost of curve f_{ijk}:

$$c_{ijk} = l_{ijk} (\sum_{i'=1}^{p} z_{i'} + \sum_{j'=1}^{r} \sum_{i'=1}^{p} \eta_{j'} k_{i'} t_{j'} + \sum_{i'=1}^{p} \theta_{i'} \bar{y}_{ij} h_{i'})$$

where is $\eta_{j'} \in [0,1]$, $\sum_{j'=1}^{r} \eta_{j'} = 1$ and $\theta_{i'} \in [0,1]$, $\sum_{j'=1}^{r} \theta_{i'} = 1$ undetermined coefficient.
Other hypothesizes are reserved.
Step 3: If $c_{ijk} < c_{ij0}$ and $|c_{ijk} - c_{ij0}| < \lambda$, then go to step 4;
If $c_{ijk} < c_{ij0}$ but $|c_{ijk} - c_{ij0}| > \lambda$, then let $c_{ij0} = c_{ijk}, k = k + 1$, go to step 2;

If $c_{ijk} \geq c_{ij0}$,then let $k = k+1$, go to step 2, where λ is a given precision;
Step 4: Output c_{ijk} $f_{ijk}(x)$,x_0,x_q, λ, stop.
The entire road network can be optimized by Algorithm 1.

2.3 Fermat Point Adding Optimization

It is well-known that intersections, T-shape crossings are very common in transportation, which play important roles. Therefore, it is necessary to explore the basic road network programming with intersections,that is, to add virtual points in the road network. Here, all newly added virtual points are regarded as new transportation demand point whose transportation demand volume are zero. The adjacency matrix is still valid. The main idea and strategy mentioned before are valid, too.
Therefore, we put the emphasis on the determination of the numbers and locations of all virtual points. As mentioned before, to basic road network programming, all existing trunk roads are taken for granted. Naturally, these trunk roads divide the planning area into many small parts. If necessary, further division can be carried out to form smaller parts for the research.
According to the mathematical theory, a dot is called the Fermat point if it has the minimal sum of distances to all apexes in a polygon[8]. Note that the road network construction cost, land expropriation cost, transport cost have close relationships with the length of roads, then the total cost can be reduced if the total road length is reduced by adding Fermat points. Usually, Fermat points are easily calculated in a triangle, a quadrangle. To more complicated graphs, least square method is more efficient.
Now, suppose the whole planning area is divided into n small parts $S_p,(p = 1,...,n)$ and each is divided into m smaller parts $s_{pq}, q = 1,...,m$ according to landform, distances between demand points, etc. Meanwhile $s_{p1} \cup s_{p2} ... \cup s_{pm} = S_p, p = 1,..,n$ and$s_{pi} \cap s_{pj} = \emptyset, i, j = 1,...,m, i \neq j, p = 1,..,n$. In each $s_{pq}, q = 1,...,m, p = 1,...,n$, there are w demand points($w = 1,2,3....$), the coordinates of which are $(x_{ki}, y_{ki}), k = 1,...,m, i = 1,...w$, the coordinates of the Fermat point are$(x_k^*, y_k^*), k = 1,...,m$. Algorithm 2:
step 1: $k = 1$, set precision ε;
step 2: Solve min $f = \sum_{i=1}^{w} \sqrt{[(x_{ki} - x_k^*)^2 + (y_{ki} - y_k^*)^2]}$ until $f < \varepsilon$;
step 3: Output (x_k^*, y_k^*), go to step 4;
step 4: If $k < m$, then let $k = k+1$, go to step 2, else, stop.
Thus, the number and the locations of all Fermat points (virtual points) can be determined.
Remark: check the connectivity of the graph at last and do proper amendment if necessary.

3 Example and Evaluation

3.1 Example

In order to compare the effectiveness of the strategy above, we utilize it to the basic road network programming of Dacheng Country, Langfang City, Hebei Province.
Dacheng Country, south to Beijing, with a total area of $904km^2$, 0.47 million of population, locates on a soil deposit plain land in Hebei Province. In Dacheng, there are 1 state highway, 5 provincial highways and 10 transportation demand points. Before research, careful investigation has been done to obtain the specific transportation demand

volume data (including freight and passenger) and their main destinations[9−11]. Generally, the main destinations of all demand points are Beijing, Tianjin, Shanghai, other cities in Hebei,etc.

Code	Demand Point	Demand Volume(ton)
K1	Pingshu Town	282146
K2	Wang Village	66980
K3	Dashang Hamlet	837132
K4	Nanzhaofu Town	105917
K5	Liuge Village	1029246
K6	Quan Village	469246
K7	Litan Town	66387
K8	Beiwei Town	109949
K9	Daguang'an Town	146349
K10	Zang Hamlet	146073

Table 3.1 Code, name and demand volume of demand points in Dacheng Country

The proposed method is applied to Dacheng Country with the following result:

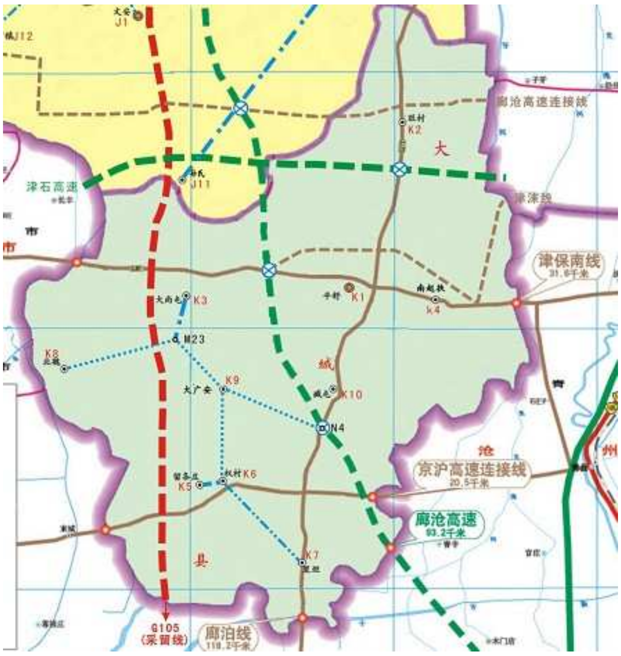

Figure 1: Basic road network of Dacheng Country

In detail, the planning basic road network contains 6.36 kilometers of second rank road, 7.155 kilometers of the third rank and 24.804 kilometers of the fourth. Besides, all the planning roads are straight lines since the landform of Dacheng is plain with fairly plain surface relief and no large rivers.

3.2 Evaluation Index Calculation

In order to testify the feasibility and effectiveness of the proposed methodology, relevant road network evaluation is required. First, we denote the existing basic road network of Dacheng Country as the Project 1, and our network planning as Project 2. Obviously, the existing basic road network is the result of certain traditional programming method. Therefore, the comparison of Project 1 and 2 can be regarded as the comparison of the old and new methods. The selected evaluation indices contain the following: road network construction cost, land expropriation cost, transportation cost and maintenance cost. Actually, to all road users, trunk roads and basic roads are undividable. So, both of the two road networks must be taken into consideration in the evaluation[12−14].

3.2.1 Construction Cost Calculation

Total lengths of the entire road networks including the existing and the planning ones for Project 1 and 2 are 422.9km and 253.4km. Among which, the lengths of the basic road network are 207.8km and 38.3km, respectively. Then, the construction cost of two projects is as follows:

Rank / Project	Highway	1st rank	2nd rank	3rd rank	4th rank	Total
Project 1	3371680	1046820	596676	156136	46191.13	5217503.1
Project 2	3371680	1046820	625506	11448	10541.7	5065995.7

Table 3.2 Construction cost for entire road network (unit: thousand yuan)

where the cost for basic road network is showed in Table 3.3:

Rank / Project	Highway	1st rank	2nd rank	3rd rank	4th rank	Total
Project 1	0	0	930	156136	46191.13	211657.1
Project 2	0	0	38160	11448	10541.7	60149.7

Table 3.3 Construction cost for basic roads (unit: thousand yuan)

3.2.2 Land Expropriation Cost Calculation

The function below is utilized in calculating the total land expropriation cost:
Cost= existing trunk road land expropriation cost + future trunk road land expropriation cost + basic road land expropriation cost of Project i (i=1,2). The results are showed in Table 3.4. Definitely, the proposed programming demands much less cropland area.

Project / Cost	Project 1	Project 2
Cost for trunk road	2564139.8	1946645.2
Cost for basic road	722380.0	104885.4
Total cost	3286519.8	2051530.6

Table 3.4 Land expropriation cost (unit: thousand yuan)

3.2.3 Transportation Cost and Maintenance Cost Calculation

Suppose all the transport demand volume predicted in Table 3.1 can be accomplished by both Project 1 and 2. Referring to the existing price system and possible development tendency, the transportation costs in the coming 10 years are calculated and outlined in Table 3.5:

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	total
Project 1	79169.5	77225.7	75329.6	73480.1	71676	69916.2	68199.5	66525.1	64891.7	63298.5	790874
Project 2	28262.4	27568.5	26891.6	26231.4	25587.4	24959.1	24346.3	23748.6	23165.5	22596.7	282331.3

Table 3.5 Transportation costs for the coming 10 years (unit: thousand yuan)

Besides, maintenance costs are also considered here, which are based on the knowing of the maintenance plan.

Cost \ Project	Project 1	Project 2
Cost for trunk road	69653.5	57432.7
Cost for basic road	15536	3315.2
Total cost	85169.5	60747.9

Table 3.6 Maintenance cost for year 2011-2020 (unit: thousand yuan)

3.3 Evaluation Result

After the calculations above, evaluation results can be obtained easily:

Cost \ Project	Project 1	Project 2
Construction Cost	5217503.1	5065995.7
Land Expropriation Cost	3286519.8	2051530.6
Transportation Cost	790874	282331.3
Maintenance Cost	85169.5	60747.9
Total	9380066.4	7460605.5

Table 3.7 Total cost for year 2011-2020 (unit: thousand yuan)

From the discussion above, the following conclusions can be obtained easily:
(1) From Table 3.7, the total cost of Project 1 is about 9.38 billion yuan and it of Project 2 is about 7.46 billion yuan. That is, the proposed programming plan can save about 1.92 billion yuan altogether. Among which, the government can save about 1.41 billion yuan on the network construction, land expropriation and long-term maintenance.To all road users, 0.41 billion yuan can be reduced due to the cost saving on transportation. Therefore, both the government and the common people can be benefited from the proposed method.
(2) The decreasing on land expropriation area can save more cropland either for crop growing or other purposes. To a nation with a large population and relatively less cropland, this is of great significance.
(3) All the transportation tasks can be accomplished very well. It's undoubtedly that the proposed basic road network plan has much less roads compared with the existing one. But all the possible transportation demand are achieved in our example with less transportation cost. Meanwhile, all the ten demand points are connected as showed in Figure 1, no one left isolated. Moreover, the road network transportation efficiency can be kept in a reasonable interval: $[0.6, 0.8]$ during our theoretical exploration and example discussion.

4 Conclusions

In this paper, a new road network programming model and strategy are proposed. First, the concept of basic road network has been drawn into. Then a new mathematical model and method on solving the basic road network programming were proposed. Last, an example of Dacheng Country and relevant evaluation were outlined, which testified the feasibility and efficiency of the proposed model and method. From the evaluation, we can draw the conclusion that the proposed model and method can solving the road network programming more effectively by offering an money saving and high efficiency solution to the basic road network programming. In the future study, we'd like to apply the proposed method to more problems with more transportation demand points and more complex landform.

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