The Eighth International Symposium on Operations Research and Its Applications (ISORA'09)
Zhangjiajie, China, September 20–22, 2009

Shift Scheduling Problems in Nanzan University Entrance Examinations --- Their Formulation and Implementation

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Abstract We introduce two shift scheduling problems in the administration of the entrance examination of Nanzan University. The first is to make the shift of the student assistants of the entrance examination. We formulate the problem as a network flow problem and solve it as a linear programming problem using an optimization software. The software solves the problem in three seconds by the standard PC. The second is to make a shift of administrative staffs checking the number of answer sheets of the entrance examination. We formulate the problems as a 0-1 integer programming problem. We solve it using the same optimization software. It takes about ten seconds also by the standard PC.

1 Introduction

Nanzan University, a middle sized private Catholic university in Nagoya, started to utilize Operations Research methods to improve their daily operations in 2004. The first project was to reduce the cost of its school bus system. OR professors had engaged in the project and succeeded in reducing the budget of the school buses by almost two thirds.

The OR professors' activity in Nanzan University is recognized for the distinguished outcomes and received the Franz Edelman Finalist Award in 2005 from INFORMS (the Institute for Operations Research and the Management Sciences) [1]. After the success of the school bus project, Nanzan University formed an OR team called Project N in 2005. N is the initial of Nanzan University. The authors are members of the team.

The team has solved many problems for the administration of Nanzan University. The problem of stopping some academic journals subscribed in the library was the first challenge of Project N [2]. We also made the evacuation plan for the future coming earthquake, a shift scheduling system of a student presentations, a shift scheduling system of supervisors of the entrance examination, an automatic school bus assignment system, etc. These activity and their achievement were recognized for the good applications of OR, and Project N received the Best Practice Award in 2007 from the Operations Research Society of Japan.

The two shift scheduling problems of the entrance examinations are the recent results of Project N. Project N solved these problems in corporation with the examination office.

The first is to make a shift of the student assistants of the entrance examination. We formulated the problem as a network flow problem and solved it as a linear programming problem using an optimization software, What's Best!. The number of variables and the constraints are about ten thousand and twenty seven thousand respectively. The software solved the problem in three seconds by the standard PC. Before we implemented the problem, the administrative staff spent a day or two to make the shift. Also, it often included errors and the error made the entrance examination confused. Now they make the shift in ten minutes including the data handling and the checking the results. It does not include errors.

The second is to make a shift of administrative staffs checking the number of answer sheets of the entrance examination. We formulated the problems as a 0-1 integer programming problem. The number of the variables and the constraints are about seven thousand and five hundred respectively. We solved it using the same optimization software. It takes about ten seconds by the standard PC to solve the problem. It took almost one day for an administrative staff to make the shift before we implemented the program. The shift is made in ten minutes including the data handling and the check of the results now.

The two programs are used for the administration of the entrance examination in 2009. It helps the administrative staff of the entrance examination to do their job correctly and speedily.

In the following sections, we show the details of the shift scheduling problems. In section 2, we explain the details of the two shift scheduling problems. In section 3, we show the formulation of the problems we adopted. In section 4, we show samples of the results of our method. In section 5, we conclude our paper by summarizing the results.

2 Shift Scheduling Problems of Entrance Examination

Project N challenged the two shift scheduling problems of the entrance examination. The examination is carried out for six days in February every year. We explain each of the problems into the details.

P1) Make a shift of the student assistants

Nanzan University hires undergraduate students as assistants for the entrance examination. Their jobs are categorized into the followings:

1) Room manager

A student is assigned to each examination room. His jobs are to check the room before, between, and after the examination time, and to hand messages from headquarter of the examination to the supervisor of the room.

2) Floor manager

Two students are assigned to each floor. Each floor has at least one examination room. One student is responsible for the management of the floor, and the other is for the messenger between headquarter and the floor.

3) Guide

Students are assigned at the entrances of the campus and the corner points in the campus to guide the students taking the entrance examination. This is the cruelest job, because they have to stand outside for a long time in the cold weather in February.

4) Reserve

A few students are reserved in case that students assigned the other jobs are not able to do their jobs because of sickness or accidents.

We need to make the shift of the students for six days of the examination considering many constraints. The main constraints the shift should satisfy are the followings:

1) The students are assigned to at least three days.
2) Both female and male students are assigned as the floor manager or the room manager at the floor. They need to accompany the student taking the examination who uses the washroom during the examination time. A female assistant should accompany the female student and a male assistant should accompany the male student.

We divide the problem into two sub problems. The first problem is to assign the students to the category of the jobs of each examination day. It is formulated as a minimum cost flow problem and solved as a linear programming problem. We use What's best! to solve the problem. The second problem is to decide the students' jobs assigned to a category in each examination day. It is solved by a greedy method.

P2) Make a shift of administrative staffs checking the number of answer sheets

A main job of headquarter of the entrance examination is to check the number of the answer sheets. More than thirty administrative staffs are assigned to the job. A staff is assigned to one or two examination rooms. He/She is paired with the other staff who is also assigned to the other one or two examination rooms. The supervisors of the examination rooms send the answer sheets to the staffs, and the staffs count the number of the sheets to check that the number coincide with the number of students taking the examination in the room. Then the paired staffs exchange their answer sheets and check the number of the answer sheets again. Because it is important but boring job, we need to consider detailed constraints. The main constraints are as follows:

1) Novice staffs who have no experience of this job should be paired with the experienced staffs.
2) The total number of the answer sheets assigned to the staff in six days should be in the appropriate range to equally share the burden.

We formulate the problem as a 0-1 integer programming problem and find a solution satisfying the constraints. We use What's best! to solve the problem, too.

3 Problem formulation

As we mentioned in the previous section, the first sub problem of the problem P1 was formulated as a variation of the minimum cost flow problem. Before describing the formulation, we list up the parameters we used for the formulation.

I : Set of nodes of the network
I_M : Set of male students nodes
I_F : Set of female students nodes
s : Source node
t : Sink node
d_1, d_2, d_3 : dummy nodes
λ : Examination day, λ =1, 2, …, 6
γ : Job category (with male and female distinction)
μ : Job category (without male and female distinction)
$k_{\lambda\mu}$: Node for examination day λ , job category μ
l_λ : Node of examination day
E : Set of edges
∂V_i^+ : Set of edges starting from node i
∂V_i^- : Set of edges terminating at node i
p_{ij} : Capacity of the edge $(i, j) \in E$
c_{ij} : Unit cost of the edge$(i, j) \in E$
a_i : The maximum assignment days of student i, $i \in I_M \cup I_F$
$N_{\lambda\mu}$: Number of assistants needed on the examination day λ for the job category μ
$M_{\max}$: The total flow to the dummy nodes d_1, d_2 and d_3
x_{ij} : Flow from node i to node j, $(i, j) \in$ E

Using these notations, the problem is formulated as follows:

[P1] $$\min \sum_{(i,j)\in E} c_{ij} x_{ij} \tag{1}$$

s.t.

$$\sum_{(i,j)\in \partial V_i^+} x_{ij} = a_i, \quad i \in I_M \cup I_F \tag{2}$$

$$\sum_{j\in K_\lambda} x_{ij} \le 1, \quad i \in I_M \cup I_F \ , \ \forall\lambda \tag{3}$$

$$x_{k_{\lambda 1} l_\lambda} + x_{k_{\lambda 2} l_\lambda} = N_{\lambda 1}, \quad \forall\lambda \tag{4}$$

$$x_{k_{\lambda 3} l_\lambda} + x_{k_{\lambda 4} l_\lambda} = N_{\lambda 2}, \quad \forall\lambda \tag{5}$$

$$x_{k_{\lambda 5} l_\lambda} = N_{\lambda 3}, \quad \forall \lambda \tag{6}$$

$$x_{k_{\lambda 6} l_\lambda} = N_{\lambda 4}, \quad \forall \lambda \tag{7}$$

$$x_{k_{\lambda 7} l_\lambda} \geq N_{\lambda 5}, \quad \forall \lambda \tag{8}$$

$$x_{k_{\lambda 2} l_\lambda} + x_{k_{\lambda 4} l_\lambda} \geq N_{\lambda 2}, \quad \forall \lambda \tag{9}$$

$$x_{d_1 t} + x_{d_2 t} + x_{d_3 t} = M_{\max} \tag{10}$$

$$\sum_{(i,j)\in \partial V_i^+} x_{ij} = \sum_{(h,i)\in \partial V_i^-} x_{hi}, \quad i \in I - \{s,t\} \tag{11}$$

$$\sum_{(s,j)\in \partial V_s^+} x_{sj} = \sum_{(i,t)\in \partial V_t^-} x_{it}, \tag{12}$$

$$0 \leq x_{ij} \leq p_{ij} \quad , \quad (i,j) \in E \tag{13}$$

The constraints represent the following conditions:
(2): The maximum number of days assigned to the student *i*
(3): Utmost one category of the jobs is assigned for each day to a student *i*.
(4) – (8): Minimum number of assistants needed for each job category
(9): Female assistants should be assigned to each floor.
(10): Minimum number of assigned day,
(11), (12): flow constraints

We obtain the value of x_{ij} to solve [P1]. Using the x_{ij}, we can calculate the assignment. If x_{ij} of the edge connecting a female student node *i* and the node representing examination day λ and job category μ is equal to one, it means that the student is assigned to the job.

The problem P2 is formulated as a 0-1 integer problem. In P2, a solution satisfying the constraints is good enough for the practical meanings. We use the following parameters and the decision variables.

I : Set of staffs
I_1 : Set of novice staff
I_2 : Set of experienced staff
J : Set of days
K_j : Number of examination rooms used on day *j*
m : Minimum number of answer sheets the experienced staff counts
M: Maximum number of answer sheets the experienced staff counts
n : Minimum number of answer sheets the novice staff counts
N : Maximum number of answer sheets the novice staff counts
a_{ik} : Number of sheets of examination room *k* on day *j*
x_{ijk} : 0-1 variables, if staff *i* is assigned to examination room *k* on day *j*, the value is one, otherwise, zero.

We obtain the solution which satisfies the following constraints.

[P2]

$$\sum_{l=0}^{\left\lceil \frac{K_j-1}{2} \right\rceil} x_{ij,2l+1} = 0, \qquad i \in I_1, j \in J \tag{14}$$

$$m \le \sum_{j \in J} \sum_{k=1}^{K_j} a_{jk} x_{ijk} \le M, \qquad i \in I_2 \tag{15}$$

$$n \le \sum_{j \in J} \sum_{k=1}^{K_j} a_{jk} x_{ijk} \le N, \qquad i \in I_1 \tag{16}$$

$$\sum_{k=1}^{K_j} x_{ijk} = 1, \qquad i \in I, j \in J \tag{17}$$

$$\sum_{i \in I} x_{ijk} = 1, \qquad j \in J, k = 1, \cdots, K_j \tag{18}$$

$$x_{ijk} \in \{0,1\}, \qquad i \in I, j \in J, k = 1, \cdots, K_j$$

The constraints represent the following conditions:
(14): The novice staffs are assigned to the even numbered examination rooms.
(15): The total number of answer sheets the experienced staff counts is in the given range.
(16): The total number of answer sheets the novice staff counts is in the given range.
(17): Each staff should be assigned to an examination room everyday.
(18): Every examination room should be covered by one staff everyday.

4 Results

We implemented the system using the optimization software, What's best!. For [P1], the number of variables is about thirteen thousand and the number of constraints is about twenty seven thousand in the problem solved for the 2009 entrance examination in Nanzan University. The system solved the problem in several seconds by a standard PC. The entrance examination office was able to make the shift in 10 minutes in 2009. Table 1 is a part of the output of the system. The system output the table for every examination day. Also the system output a table representing the assigned job for each assistant. Table 2 is a part of the table. The system output the tables automatically and the examination office only output the tables and delivered them to the assistants.

For [P2], the number of the 0-1 variables is about seven thousand, and the number of constraints is about five hundred in the problem of the entrance examination in Nanzan University in 2009. We also use the software to make the system, and the system solved the problem in fifteen seconds by a standard PC. Table

3 is a part of the output of the system. The table represents the examination room assigned to each staff. It also includes the total number of answer sheets the staff should count, and the number of sheets the staff counts each examination day. The system output the table automatically, and the only thing the examination office should do is to check the shift.

Table 1: A part of the output of the shift scheduling system. Shadowed cells represent female assistants.

Day 1			Room Manager		Floor Manager		Messenger	
Floor	NO	Room	ID	Name	ID	Name	ID	Name
1	1	GA	2007HP111	w	2005FA008	b	2007JJ173	g
	2	GB	2007FB085	N				
	3	GC	2007MI215	JJJJ				
	4	GD	2007HC007	III				
	5	GE	2008FB058	tttt				
2	6	GF	2005HA053	C	2006EE058	t	2008HA017	q
	7	GG	2005JJ060	LLLL				
	8	GH	2005HC016	R				
	9	GI	2007PP259	NNNN				
3	10	DB1	2008EE045	G	2007EE161	aa	2008JJ297	M
4	11	D31	2006FB171	O	2008FA038	GG	2008JJ027	U
	12	D32	2008FB187	W				
	13	D33	2007PP144	PPPP				
	14	D34	2008FB157	NNN				

Table 2: A part of the output of the shift scheduling system. Table representing the job for each assistant to be assigned

ID	Name	Day 1	Day 2
2007EE111	a		
2005FA008	b	Floor Manager, Floor1	
2008FS026	c		
2007HP074	d	Reserve	Floor Manager, Floor1
2008EE192	e	Guide, Main Entrance	
2007JJ303	f		Guide, Main Entrance
2007JJ173	g	Messenger, Floor1	Floor Manager, Floor2
2008EE086	h		
2008HP064	i		Floor Manager, Floor3

5 Conclusions

We use these shift scheduling systems for the entrance examination in 2009 in Nanzan University. It reduces the time needed for the administration of the examination. We improve the systems in variety of directions. The systems are used in 2009, however, they are still prototypes of the systems. It means the systems are not so friendly to the users. For example, they only output the message “infeasible”, if no feasible solution exists. There is no clue to correct the data or relax the constraints for the administrative staff. We modify the system to output which constraints are so tight, or point out the possibility of the incorrect data. Also we consider connecting our systems to the data of the personnel office. It will automatically compute the wage for the assistants or the administrative staffs.

Table 3: A part of the output of the shift scheduling system

Name	Total#	Day	Room	#sheets
A	444	2009/2/7	D32	46
		2009/2/9	EB1,E11	124
		2009/2/10	E12,E21	88
		2009/2/11	H26,H31	48
		2009/2/12	K21,K22	88
		2009/2/13	KB1	50
B	490	2009/2/7	GG	84
		2009/2/9	GB	96
		2009/2/10	H34	44
		2009/2/11	HB1,HB2	105
		2009/2/12	K25,K37	89
		2009/2/13	MB12	72

References

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[2] A. Suzuki, M. Fushimi, and M. Nishio, “Selecting Journals of University Library to Stop Subscription by OR/MS Approach”, in Lecture Notes in Operations Research, Vol. 8, 2008, “Operations Research and Its Applications”, eds., Xiang-Sun Zhang et al., pp. 231-236.

[3] What’s best! is a registered trademark of LINDO Inc.