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Gateway Location Models

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Abstract In this paper, we consider the design of airline network in Asia where optimal locations of the gateways and local hubs are established. We report some results using the demand data reported by IATA in 2005. The results show interesting features of gateway locations in Asia.

Keywords Hub-and-Spoke Network; Gateway Location

1 Introduction

In this decade, huge airports have been established in Asia such as Kuala Lumpur International Airport (Kuala Lumpur, 1998), Chek Lap Kok Airport (Hong Kong, 1998), Pudong International Airport (Shanghai, 1999), Incheon International Airport (Incheon, 2001), Guangzhou Baiyun International Airport (Guangzhou, 2004), Suvarnabhumi Airport (Bangkok, 2006) and so on. These airports compete fiercely with one another to serve themselves as an international hub airport in Asia.

In Europe, Frankfurt, Paris-Charles de Gaulle Airport and London Heathrow Airport have been known as the three major hubs. Some other airports are recently planning a major expansion of the airports to participate in the competition to serve as an international hub airport. In addition, recent rapid increase in demand for both passengers and freight is making the competition in Europe even more intense.

Hub-and-spoke networks have become commonly used by many air carriers to capture widespread demand efficiently since the United States Airline Deregulation Act commenced in 1978. Using hub networks enables to reduce transportation costs by enjoying the economies of scale from consolidated flows. Studies on various hub location models have been attracting a great deal of attention since O'Kelly [5] proposed a quadratic programming problem formulation. The literature on hub location research is summarized in Alumur and Kara [1] and Campbell et al. [3].

Although various hub location models have been studied so far, most of them are considered based on the US domestic market. However, international market is also important when establishing hub network in a region made up of many countries (e.g., Asia and Europe). In such region, traditional hub-and spoke models based on domestic market may not be suitable. Specifically, recent rapid growth of international passenger demand between Asia and other regions (e.g., North America and Europe) indicates importance

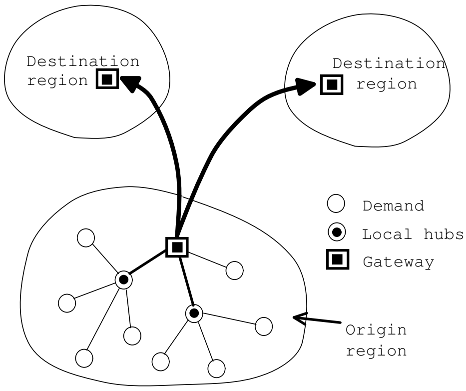


Figure 1: Gateway location model ($p = 2, q = 1$)

to consider international demand between different regions for the design of airline network in Asia. Given the importance of huge market in Asia, we present a gateway location model that has different network structure compared to those of traditional hub-and-spoke model to find optimal gateways and local hubs in a given region. There are many factors behind gateway selection, however, we focus on strategic model to find geographically optimal locations without taking into account factors such as airport capacities, airline's base and so on.

The remainder of this paper is organized as follows. In Section 2, we present a gateway location model that locates given number of gateways and local hubs in a region. In Section 3, we formulate the model as a mixed integer programming problem and briefly mention the solution approach. In Section 4, we show computational results using real aviation data reported by IATA (International Air Transport Association) in 2005. In Section 5, we give concluding remarks and mention some future work.

2 Model Description

In the gateway location model, a given number of gateways and local hubs are selected among candidates in a given region. The local hubs play a role as a flow consolidating point within the region and also a connecting point between local area and one of the gateways, while the gateways are huge hubs in the region that connect to other regions (e.g., Asia and North America).

This problem can be regarded as a facility location problem that has two levels where each of the levels corresponds to the location of gateways and local hubs. Berman et al. [2] provide a facility location model with two levels that correspond to the facility and transfer points. They also proposed heuristic solution methods to solve the problems. Narula et al. [4] presents improved formulations for similar problems. Sasaki et al. [6] further improve the formulations with smaller number of variables and constraints. In this paper, we formulate the gateway location model based on the formulation presented in [6] and obtain exact optimal locations of the gateway and hubs.

Figure 1 shows an example with two local hubs and one gateway. Customers within the origin region are routed via one of the local hubs and one of the gateways to travel outside the region. The customers are also allowed to be routed via only one of the gateways. Thus, all customers can travel from their origins to the destinations either using one-stop (only the gateway) or two-stop (one local hub and the gateway) route. Although the final destination of each customer may be different, we consider transportation to the gateway in the destination regions without taking account transportation with in the destination region.

3 Formulation

We employ the following notation:

N: the set of demand nodes.
w_i: the demand of node $i \in N$.
d_{ij}: the great circle distance between nodes $i \in N$ and $j \in N$.
f_i: great circle distance between demand node $i \in N$ and the destination.
p: the number of local hubs.
q: the number of gateways.
α: discount factor between local hubs and gateways $(0 < \alpha < 1)$.
β: discount factor between gateways and the destinations $(0 < \beta < \alpha)$.
M: the total sum of demands ($\sum_{i \in N} w_i$).

We assume that $0 < \beta < \alpha < 1$ because flow volume between gateways and the destinations is larger than those between local hubs and gateways, which leads smaller unit transportation cost on arcs connecting the gateways and the destination.

Decision variables are as follows:

x_j: 0-1 variable that takes 1 if node $j \in N$ is selected as a local hub or gateway, 0 otherwise.
y_k: 0-1 variable that takes 1 if node $k \in N$ is selected as a gateway, 0 otherwise.
φ_{ij}: flow volume between nodes $i \in N$ and $j \in N$.
ψ_{jk}: flow volume between node $j \in N$ and gateway $k \in N$.

The gateway location model is formulated as follows.

$$\min \quad \sum_{i \in N} \sum_{j \in N} d_{ij} \varphi_{ij} + \alpha \sum_{j \in N} \sum_{k \in N} d_{jk} \psi_{jk} + \beta M \sum_{k \in N} f_k y_k$$

$$\text{s.t.} \quad \sum_{i \in N} \varphi_{ij} = \sum_{k \in N} \psi_{jk}, \qquad j \in N, \qquad (1)$$

$$\sum_{j \in N} \varphi_{ij} = w_i, \qquad i \in N, \qquad (2)$$

$$\varphi_{ij} \le w_i x_j, \qquad i, j \in N, \qquad (3)$$

$$\sum_{k \in N} \psi_{jk} \le M x_j, \qquad j \in N, \qquad (4)$$

$$\sum_{j \in N} \psi_{jk} \le M y_k, \qquad k \in N, \qquad (5)$$

$$y_j \le x_j, \qquad j \in N, \tag{6}$$

$$\sum_{j \in N} x_j = p + q, \tag{7}$$

$$\sum_{k \in N} y_k = q, \tag{8}$$

$$\varphi_{ij} \ge 0, \qquad i, j \in N,$$

$$\psi_{jk} \ge 0, \qquad j, k \in N,$$

$$x_j \in \{0, 1\}, \qquad j \in N,$$

$$y_k \in \{0, 1\}, \qquad k \in N.$$

The objective is to minimize the total transportation cost, which is assumed to be proportional to travel distance. The objective function includes three summations which express the total transportation cost between nodes and local hubs or gateways, between local hubs and gateways, between gateways and the destination, respectively. Constraints (1) represent that the total flow out of and into hub $j \in N$ are the same. Constraints (2) represent that the total flow out of non-hub node $i \in N$ is the same as the demand at node $i \in N$, so all demand is transported. Constraints (3)-(5) ensure that non-hub nodes are not used as transfer points. Constraints (6) ensure that gateways are selected from among hubs. Constraints (7) and (8) ensure that exactly $p+q$ hubs are selected and q out of $p+q$ hubs are gateways.

4 Results

This section reports computational results using air passenger traffic data reported by IATA in 2005. The IATA data includes the top 3000 OD (origin and destination) pairs between the cities in Asia and in North America, and the top 3000 OD pairs between the cities in Asia and in Europe.

The test data set is generated from the IATA data as follows. First, 131 airports in the following 17 countries and areas within Asia are selected as demand nodes: Brunei(1), Cambodia(2), China(47), Hong Kong(1), Indonesia(4), Japan(16), Laos(1), Macau(1), Malaysia(5), Mongolia(1), Myanmar(1), Philippines(19), Singapore(1), Korea(7), Taiwan(2), Thailand(16), Vietnam(6).

We assume that all demand nodes are hub candidates. Note that the numbers in the parentheses show the number of demand nodes in each country.

As for the destination region, we consider North America and Europe because they are major destinations according to the international flight demand in Asia. We utilize three different scenarios to reflect the different situations in pattern of demand. Each of the scenarios assumes all demand goes to North America, all demand goes to Europe, and half of the demand goes to North America while the other half goes to Europe, respectively. The last scenario is based on the fact that the number of passengers that went to North America is almost the same as that went to Europe in 2005.

To calculate the distance between nodes in the origin and the destination region, we do not utilize the fixed location of the gateway such as Chicago or Detroit in the destination region. Instead, we define the distance between node $i \in N$ and the gateway in the destination region as the average great circle distance between node $i \in N$ and each

Table 1: Optimal results with $(p,q) = (3,1)$

Destination	α	β	Gateway	Local hubs		
North America	0.8	0.6	Niigata	Hong Kong	Seoul	Taipei
	0.8	0.4	Taegu	Hong Kong	Tokyo	Taipei
	0.8	0.2	Chinju	Hong Kong	Tokyo	Taipei
	0.6	0.4	Tokyo	Hong Kong	Seoul	Taipei
	0.6	0.2	Seoul	Hong Kong	Tokyo	Taipei
	0.4	0.2	Seoul	Hong Kong	Tokyo	Taipei
Europe	0.8	0.6	Lanzhou	Bangkok	Hong Kong	Seoul
	0.8	0.4	Dayong	Bangkok	Osaka(Itami)	Singapore
	0.8	0.2	Guangzhou	Bangkok	Johor Bahru	Osaka
	0.6	0.4	Xianyang	Bangkok	Hong Kong	Tokyo
	0.6	0.2	Changsha	Bangkok	Tokyo	Singapore
	0.4	0.2	Wuhan	Bangkok	Tokyo	Singapore
North America & Europe	0.8	0.6	Dalian	Bangkok	Hong Kong	Tokyo
	0.8	0.4	Qingdao	Bangkok	Hong Kong	Tokyo
	0.8	0.2	Shanghai	Bangkok	Tokyo	Singapore
	0.6	0.4	Weihai	Bangkok	Hong Kong	Tokyo
	0.6	0.2	Shanghai	Bangkok	Tokyo	Singapore
	0.4	0.2	Qingdao	Bangkok	Hong Kong	Tokyo

airport in the destination region. For example, the distance between Tokyo-Narita and the gateway in North America is the average great circle distance between Tokyo-Narita and each airport in North America. More precisely, we define f_i for all $i \in N$ as follows:

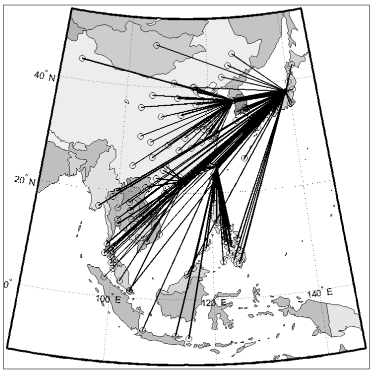
$$f_i = \frac{1}{|K|} \sum_{k \in K} d_{ik},$$

where K is the set of airports in the destination region and d_{ik} is the great circle distance between node $i \in N$ and $k \in K$. For the demand at node $i \in N$, we used the sum of the demand originating and terminating at the node. For example, the demand between Tokyo-Narita and North America is the sum of the demand between Tokyo-Narita and each airport in North America.

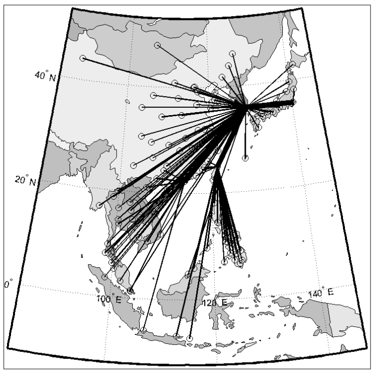
To generate distance data set, we selected 223 airports in the US, Canada and Mexico as major destinations in North America. We also selected 141 airports from 36 countries as major destinations in Europe. The airports in isolate islands such as Hawaii are not included. For airports in Russia, only those located 40 degrees of east longitude and to the west are selected.

If the gateway locations are given, the gateway location model can be regarded as a p-median problem. Thus, we can obtain an optimal solution of the gateway location model by solving p-median problems for all possible combination of gateway locations [6]. The problems were solved using ILOG CPLEX10.0 on a DELL DIMENSION 8300 with a 3.19 GHz Intel Pentium 4 processor operated under Windows XP Professional with 2.0 GB RAM memory. The cpu time ranges from 100 to 170 seconds for one example.

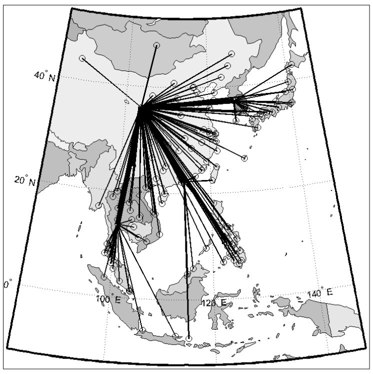
Table 1 shows optimal locations of the gateway and local hubs for 18 examples with $(p,q) = (3,1)$, three different scenarios and six combinations of (α,β): $(\alpha,\beta) = (0.8,$



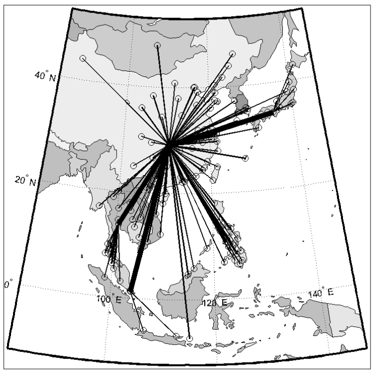
(a) North America: $(\alpha,\beta) = (0.8, 0.6)$



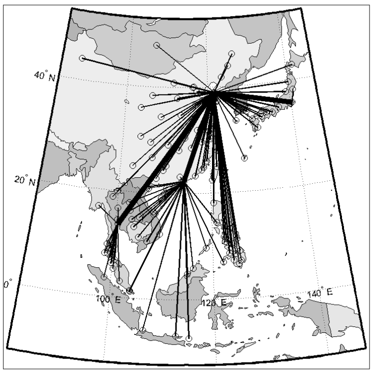
(b) North America: $(\alpha,\beta) = (0.8, 0.4)$



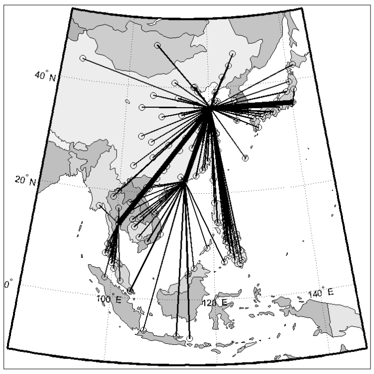
(c) Europe: $(\alpha,\beta) = (0.8, 0.6)$



(d) Europe: $(\alpha,\beta) = (0.8, 0.4)$



(e) North America & Europe: $(\alpha,\beta) = (0.8, 0.6)$



(f) North America & Europe: $(\alpha,\beta) = (0.8, 0.4)$

Figure 2: Results with $(p,q) = (3,1)$

0.6), (0.8, 0.4), (0.8, 0.2), (0.6, 0.4), (0.6, 0.2), (0.4, 0.2). Local hubs use major cities while the gateway uses some small cities such as Niigata (Japan), Chinju (Korea) and Dayong (China). These cities may have geographic advantages due to large flows on the arc between the gateway and the destination.

Figure 2 shows optimal networks for six of 18 examples with two different combinations of (α,β): (0.8,0.6) and (0.8,0.4), and three scenarios. Thick lines show arcs that connect local hubs and the gateway. Figures 2(a)-2(d) show optimal gateway locations move toward the destination with bigger value of β so as to reduce the transportation cost between the gateway and the destination. On the other hand, Figures 2(e)-2(f) show more stable results.

Figure 3 shows how optimal gateway locations change with different values of (α,β). One interesting observation from this figure is that there is a diagonal pattern, which indicates that the ratio of α and β may have an important meaning.

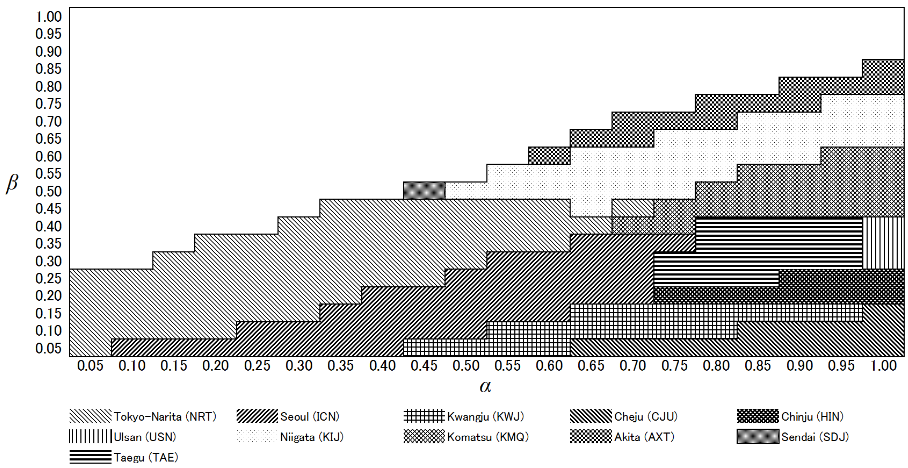


Figure 3: Optimal gateways (Destination: North America)

5 Conclusions

This paper presents a model and results for gateway and local hub location in a given region. This model is more appropriate than traditional hub-and-spoke network to some regions such as Asia and Europe. Althought the model and the solution method are primitive, some interesting features are obtained. Future research includes extensions of the model by taking into account airport capacities and local demand within the region as well as international demand; exploring optimal locations changes with different values of (α,β) in detail; developing better solution approach to solve larger problems with $q \geq 2$.

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References

[1] S. Alumur and B. Kara: Network hub location problems: The state of the art. *European Journal of Operational Research*, **12** (2008) 1–21.

[2] O. Berman, Z. Drezner and G.O. Wesolowsky: The facility and transfer points location problem. *International Transactions in Operational Research*, **12** (2005) 387–402.

[3] J.F. Campbell, A.T. Ernst and M. Krishnamoorthy: Hub location problems, in Z. Drezner and H.W. Hamacher (Eds.), Facility Location - Applications and Theory, Springer, Berlin, 2002, 373–408.

[4] S.C. Narula and U.I. Ogbu: An hierarchical location-allocation problem. *Omega International Journal of Management Science*, **7** (1979) 137–143.

[5] M. E. O'Kelly: A quadratic integer program for the location of interacting hub facilities, *European Journal of Operational Research*, **32** (1987) 393–404.

[6] M. Sasaki, T. Furuta and A. Suzuki: Exact optimal solutions of the minisum facility and transfer points location problems on a network, *International Transactions in Operational Research* , **15** (2008), 295–306.