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Optimal Impulse Control for Cash Management with Two Sources of Short-term Funds

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Abstract Many firms face a problem of managing cash balances to maximize the availability of cash for investment and to avoid the risk of insolvency. In this paper, we consider a cash management model in which the two types of funds are available when a manager adjusts cash level. We suppose that the rate of utilizing the two funds for the amount of adjustment is constant. Our objective is to minimize the expected discounted costs over an infinite horizon. We formulate this cash balance management problem as an impulse control problem and then derive an optimal cash management policy. We show the existence of an optimal policy and demonstrate it numerically.

Keywords Cash management; Impulse control; Stochastic model; Quasi-variational inequality

1 Introduction

Managing cash flow is an important decision point in the enterprise that provides the products and services. The cash flow is caused by the various types of cash inflow and expenditures. The time lag between the necessary expenses for production and the income from future sales rises uncertainty of the cash level. Such uncertainty may occurs in the parent company that uses a centralized system of cash management for the group of the subsidiaries.

The financial manager can increase or decrease the amount of cash by selling or buying short-term securities. The transfer cost occurs at the time of changing the cash level. While the manager dose not makes any changes of the cash level, there are holding and understock costs. A cash management problem is to find an optimal level of the cash balance in order to minimize the discounted expected total of these costs.

The cash management problem was first analyzed by Baumol [4] and Tobin [11]. They applied inventory control analysis to the management of cash balance because the cash balance is also an inventory level of the product. After that, there has been several papers which apply the optimal control technique of impulse control into the cash management problem. Constantinides and Richard [7], Harison et al. [9] showed the existence of an optimal control band policy in continuous time under the assumption that linear holding

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and penalty costs. Baccarin [1] also showed that there exists an optimal policy for the model of quadratic holding and penalty costs. Although the cash stock process of all the above models is one-dimensional Brownian motion, Bar-Ilan et al. [3] generalized this process to a superposition of a Brownian motion and a compound Poisson Process. Moreover, Baccarin [2] showed that it is possible to solve numerically a more realistic multidimensional cash management problem. This model allows the transaction costs and the holding and penalty costs to be nonlinear, and the cash balance fluctuation follows a homogeneous diffusion process. Buckley and Korn [5] consider the cash management in equity index tracking. They showed that there exists an optimal strategy such that it can achieve the best expected returns and maintain a low tracking error variance with small transaction costs.

Although previous papers have not specified the type of investment choices, many practical investment choices should be available. In this paper, we deal with the cash management model in which the two types of funds with different transaction costs are available whenever the manager adjusts the cash level. The first paper which deals with this type of problem seems to be Daellenbach [8]. He formulated this model by using dynamic programming formulation in discrete time. Perhaps the paper which is closest to ours in terms of the structure of cost function is Elton and Gruber [6]. However, the existence of the optimal policy remains unproved in it. We reformulate this problem in continuous time as an extension of Constantinides and Richard [7]. Furthermore, we show that there exists an optimal policy for infinite-horizon by using the impulse control. The policy that we propose is based on band policy, that is, when the cash level falls to d (rise to u), then it is adjusted up to level D (down to U), $d < D < U < u$. Our policy is differ from their model in the sense of each values of band changes depending on the amount of short-term debt outstanding at intervention time. We also illustrate the form of value function and optimal policy through the numerical examples.

The rest of this paper is organized as follows. We introduce notations and present the problem formulation in section 2. In section 3, we characterize the value function and control band policies. Finally, we conclude this paper in Section 4.

2 The Analysis of the Model

Let $(\Omega,\mathscr{F},P)$ be a complete probability space equipped with a filtration $\mathscr{F}_t$ satisfying the usual information structure, and w_t a one dimensional Brownian motion. Consider a manager who is in change of the cash management of the company. He/she wishes to control the stochastic cash level X_t. The cash level at time t is given by

$$\begin{cases} dX_t = \mu dt + \sigma dw_t \\ X_0 = x \end{cases} \tag{1}$$

where x is the initial cash level. X_t is a Brownian motion with drift μ and a diffusion parameter $\sigma > 0$. The manager can change this cash level by using two sources of funds at any time. Suppose that the sources of funds are short-term borrowing and marketable securities. Let B_t be the amount of short-term debt outstanding at time t.

A policy $v \in \mathscr{V}$ consists of the two sequences of stopping times $\{\tau_0, \tau_1, \cdots\}$ and random variables $\{\xi_0, \xi_1, \cdots\}$ such that

$$\begin{cases} P(0 = \tau_0 < \tau_1 < \cdots < \tau_i < \cdots \to \infty) = 1, \\ \xi_i \text{ is } \mathscr{F}_{\tau_i}\text{-measurable.} \end{cases} \tag{2}$$

where τ and ξ are the time of cash level to be changed and the size of control, respectively. When the cash level is changed from x to $x + \xi$, we suppose that the rate of utilizing the two funds for the amount of adjustment ξ is θ ($0 \leq \theta \leq 1$), that is, the amount of borrowing is $\theta\xi$ and the amount of securities is $(1-\theta)\xi$.

Given an impulse control v, the state of the system is the defined as

$$\begin{cases} dX_t^v = \mu dt + \sigma dw_t, \ \ \tau_i < t < \tau_{i+1}, \, i \geq 0, \\ X_{\tau_i}^v = X_{\tau_i^-}^v + \xi_i, \ \ i \geq 1, \\ dB_t^v = 0, \\ B_{\tau_i}^v = B_{\tau_{i-1}}^v + \theta\xi_i, \ \ i \geq 1, \\ X_0^v = x, \, B_0^v = b. \end{cases} \tag{3}$$

If $\theta|\xi_i| > B_t^v$ at the time of pay out the debts ($\xi_i < 0$), then we assume that the amount of difference $\theta|\xi| - B_t^v$ is used to buy securities. Then, there are transition costs such that

$$T(\xi_i) = \begin{cases} K_B^u + k_B^u \theta \xi_i & \text{if } \xi_i \geq 0 \ \text{(Debt extinguishment)} \\ K_B^d + k_B^d \theta |\xi_i| & \text{if } \xi_i < 0 \ \text{(Debt finance)} \\ K_S^u + k_S^u (1-\theta) \xi_i & \text{if } \xi_i \geq 0 \ \text{(Buying the security)} \\ K_S^d + k_S^d (1-\theta) |\xi_i| & \text{if } \xi_i < 0 \ \text{(Selling the security)} \end{cases} \tag{4}$$

where K and k are fixed cost and variable cost, respectively. The subscripts B and S indicate borrowings and securities, and the superscripts u (d) represent an increase (decrease) the cash level. Furthermore, $T(\xi)$ can be rewritten as follows;

$$T(\xi_i) = \begin{cases} K_1 + k_1(\theta)\xi_i, & \text{if } \xi_i \geq 0, \\ K_2 + k_2(\theta)\xi_i, & \text{if } \xi_i < 0, \end{cases} \tag{5}$$

where $K_1 = K_B^u + K_S^u$, $K_2 = K_B^d + K_S^d$, $k_1(\theta) = k_S^u + (k_B^u - k_S^u)\theta$ and $k_2(\theta) = k_S^d + (k_B^d - k_S^d)\theta$.

We assume that the holding and penalty cost rates are

$$C(B_{\tau_i}^v, X_t^v) = \begin{cases} -pX_t^v, & \text{if } X_t^v \leq 0, \\ h_1 X_t^v, & \text{if } 0 < X_t^v \leq B_{\tau_i}^v, \\ h_1 \max\{0, B_{\tau_i}^v\} + h_2(X_t^v - \max\{0, B_{\tau_i}^v\}), & \text{if } B_{\tau_i}^v < X_t^v, \end{cases} \tag{6}$$

where p is the penalty cost, h_1 is the interest rate on short-term debt and h_2 is the opportunity cost without marketable securities instead of cash. We also assume that the opportunity cost h_2 is less than the interest rate h_1, $h_1 > h_2$, since there are some costs based upon risks of securities.

Assumption. 1 We assume that the parameters must satisfy the following inequalities;

(a) $p-\rho\max\{k_B^u,k_S^u\}\geq h_1-h_2$
(b) $h_1-\rho\max\{k_B^u,k_S^u\}\geq 0$
(c) $h_2-\rho\max\{k_B^d,k_S^d\}\geq 0$

where ρ is a discount rate, $0<\rho<1$

We define the total discounted expected cost function for a given policy v as follows;

$$J_{b,x}(v)\equiv E_x^v\left[\int_0^\infty e^{-\rho s}C(B_s^v,X_s^v)ds+\sum_{i=1}^\infty e^{-\rho s}T(\xi_i)\,\middle|\, X_0^v=x,B_0^v=b\right]. \tag{7}$$

Then, the value function is as follows;

$$\Phi(b,x)=\inf_{v\in\mathscr{V}}J_{b,x}(v). \tag{8}$$

We consider the QVI (Quasi-Variational Inequality) problem to show the existence of an optimal policy that achieves the e infimum in equation (8) as well as to obtain a closed-form solution of value function. In order to obtain QVI, we follow the same approach as [1], [7].

If the manager needs volume of transaction ξ at time t, then the cash level jumps from x to $x+\xi$ and the amount of short-term debt outstanding jumps from b to $b+\xi$. The total cost of this case is given by

$$\inf_\xi\{T(\xi)+\Phi(b+\theta\xi,x+\xi))\}. \tag{9}$$

On the other hand, the manager dose not transact cash in the small interval, then the amount of short-term debt outstanding B_t does not change. Hence, the cost is similar to Constantinides and Richard [7]. Here, we define two operators L and M as follows;

$$L\phi(b,x) = -\rho\phi(b,x)+\mu\phi'(b,x)+\frac{1}{2}\sigma^2\phi''(b,x) \tag{10}$$

$$M\phi(b,x) = \inf_\xi\{T(\xi)+\phi(b+\theta\xi,x+\xi))\} \tag{11}$$

where $\phi'=\frac{\partial\phi}{\partial x}$ and $\phi''=\frac{\partial^2\phi}{\partial x^2}$. Then, the following relations are called QVI for problem (8);

$$L\phi+C\geq 0 \tag{12}$$

$$\phi\leq M\phi \tag{13}$$

$$(L\phi-C)(\phi-M\phi)=0 \tag{14}$$

The following theorem is given by Korn [10]. It guarantees that the solution of QVI is equal to the value function given by equation (8).

Theorem. 1 (Korn [10]) *If there exists a solution $\phi\in C^2$ that satisfies the growth conditions*

$$E_x^v\left[\int_0^\infty(e^{-\rho s}\sigma(X_s)\phi'(B_s,X_s))^2ds\right]<\infty, \tag{15}$$

$$\lim_{T\to\infty}E[e^{-rT}\phi(B_T,X_T)]=0, \tag{16}$$

for every process X_t corresponding to an admissible impulse control v, then we have

$$\Phi(b,x) \geq \phi(b,x) \tag{17}$$

for every $x \in \mathbb{R}$. Moreover, if the QVI-control corresponding to ϕ, that is, the impulse control v satisfying

(i) $(\tau_0, \xi_0) = (0,0)$
(i) $\tau_i := \inf\{t \geq \tau_{i-1} : \phi(B_{\tau_{i-1}}, X_{t^-}) = M\phi(B_{\tau_{i-1}}, X_{t^-})\}$
(ii) $\xi_i := \arg\min_\xi E[T(\xi) + \phi(B_{\tau_{i-1}} + \xi, X_{\tau_i^-} + \xi)]$

is admissible, then it is an optimal impulse control, and for every $x \in \mathbb{R}$

$$\Phi(b,x) = \phi(b,x). \tag{18}$$

3 Existence of Optimal Cash Management Policy

In this section, we present a solution of the QVI by assuming that the value function is continuous and twice differentiable. After we guess the optimal policy of the band type, we show that our solution satisfies the hypothesis of the theorem 1.

Let $\mathbf{p} := (d_b, D_b, U_b, u_b)$ be the parameters of a control band policy satisfying $d_b < D_b < U_b < u_b$. Then we suppose that the continuation region has the form of

$$D = \{(b,x) : d_b < x < u_b\}. \tag{19}$$

All of the parameters are expressed as a function of the amount of short-term debt outstanding B_t because the holding and penalty cost C depends on B_t. Recall that the changing of B_t is exclusive to the transaction time, these parameters are constant in the continuation region.

Our policy is as follows (see Figure 1);

1. Determine the values $\mathbf{p}$ based on initial value b of the cycle.
2. If the cash level reaches d_b or u_b, then we increase the cash level to D_b or decrease to U_b. And then, B_t changes from b to $b + \theta(D_b - x)$ or $b - \theta(x - U_b)$.
3. Repeat these procedure.

In D, inequation (12) holds as an equality, that is,

$$C(b,x) - \rho\phi(b,x) + \mu\phi'(b,x) + \frac{1}{2}\sigma^2\phi''(b,x) = 0. \tag{20}$$

The solution of this equation is as follows;

$$\phi(b,x) = \begin{cases} c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x} + \frac{h_2}{\rho}x + \frac{(h_1 - h_2)b}{\rho} + \frac{\mu}{\rho^2}h_2 & \text{for } \min\{b, u_b\} \leq x < u_b \\ c_3 e^{\lambda_1 x} + c_4 e^{\lambda_2 x} + \frac{h_1}{\rho}x + \frac{\mu}{\rho^2}h_1 & \text{for } 0 \leq x \leq \min\{b, u_b\} \\ c_5 e^{\lambda_1 x} + c_6 e^{\lambda_2 x} - \frac{p}{\rho}x - \frac{\mu}{\rho^2}p & \text{for } d_b < x \leq 0 \end{cases} \tag{21}$$

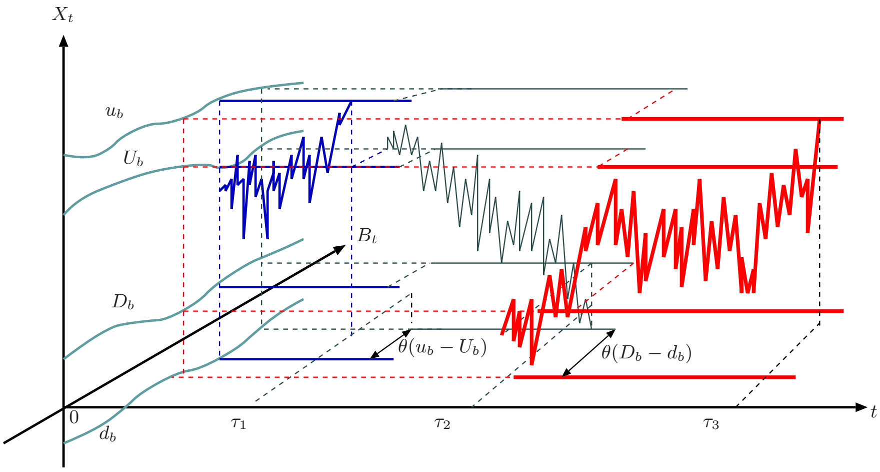

Figure 1: A Control Band Policy for Our Model.

where

$$\lambda_1 = -\frac{\mu}{\sigma^2} + \frac{1}{\sigma^2}\sqrt{\mu^2 + 2\rho\sigma^2}, \quad \lambda_2 = -\frac{\mu}{\sigma^2} - \frac{1}{\sigma^2}\sqrt{\mu^2 + 2\rho\sigma^2}. \tag{22}$$

Here, the matching conditions at the points 0 and b imply that $\phi(b,0^+) = \phi(b,0^-)$, $\phi'(b,0^+) = \phi'(b,0^-)$, $\phi(b,b^+) = \phi(b,b^-)$ and $\phi'(b,b^+) = \phi'(b,b^-)$ give

$$c_3 = c_1 + \frac{\lambda_2(h_1 - h_2)}{\rho\lambda_1(\lambda_1 - \lambda_2)} e^{-\lambda_1 b}, \tag{23}$$

$$c_4 = c_2 + \frac{\lambda_1(h_2 - h_1)}{\rho\lambda_2(\lambda_1 - \lambda_2)} e^{-\lambda_2 b}, \tag{24}$$

$$c_5 = c_3 - \frac{\lambda_2(h_1 + p)}{\rho\lambda_1(\lambda_1 - \lambda_2)}, \tag{25}$$

$$c_6 = c_4 + \frac{\lambda_1(h_1 + p)}{\rho\lambda_2(\lambda_1 - \lambda_2)}. \tag{26}$$

For $x \leq d_b$ and $x \geq u_b$, the cash level is changed and the inequation (13) holds as an equality. Then, the form of cost function ϕ is given by

$$\phi(b,x) = \begin{cases} \phi(b + \theta(D_b - x), D_b) + K_1 + k_1(\theta)(D_b - x) & x \leq d_b \\ \phi(b - \theta(x - U_b), U_b) + K_2 + k_2(\theta)(x - U_b) & x \geq u_b. \end{cases} \tag{27}$$

By equation (21), this equation can be rewritten as follows;

For $x \le d_b$,

$$\phi(b,x) = \begin{cases} \phi(b,d_b) + \left(k_1(\theta) + \frac{\theta(h_1-h_2)}{\rho}\right)(d_b - x), \quad \text{for } b \le D_b - \theta(D_b - x), \\ \phi(b,d_b) + k_1(\theta)(d_b - x) - \frac{h_1-h_2}{\rho} L_D(x,\theta) \\ \qquad + \frac{h_1-h_2}{\rho^2}\{\mu - \rho(b - D_b + \theta(D_b - d_b))\}, \\ \qquad \text{for } D_b - \theta(D_b - x) \le b \le D_b - \theta(D_b - d_b), \\ \phi(b,d_b) + k_1(\theta)(d_b - x) - \frac{h_1-h_2}{\rho}(L_D(x,\theta) - L_D(d_b,\theta)), \\ \qquad \text{for } D_b - \theta(D_b - d_b) \le b, \end{cases} \tag{28}$$

where

$$L_D(x,\theta) = \frac{1}{\lambda_1 - \lambda_2}\left\{\frac{\lambda_1}{\lambda_2} e^{-\lambda_2(b - D_b + \theta(D_b - x))} - \frac{\lambda_2}{\lambda_1} e^{-\lambda_1(b - D_b + \theta(D_b - x))}\right\}. \tag{29}$$

For $x \ge u_b$,

$$\phi(b,x) = \begin{cases} \phi(b,u_b) + \left(-k_2(\theta) + \frac{\theta(h_1-h_2)}{\rho}\right)(u_b - x), \quad \text{for } b \le U_b - \theta(U_b - u_b), \\ \phi(b,u_b) + k_2(\theta)(x - u_b) + \frac{h_1-h_2}{\rho} L_U(u_b,\theta) \\ \qquad - \frac{h_1-h_2}{\rho^2}\{\mu - \rho(b - U_b + \theta(U_b - x))\}, \\ \qquad \text{for } U_b - \theta(U_b - u_b) \le b \le U_b - \theta(U_b - x), \\ \phi(b,u_b) + k_2(\theta)(x - u_b) - \frac{h_1-h_2}{\rho}(L_U(x,\theta) - L_U(u_b,\theta)), \\ \qquad \text{for } U_b - \theta(U_b - x) \le b, \end{cases} \tag{30}$$

where

$$L_U(x,\theta) = \frac{1}{\lambda_1 - \lambda_2}\left\{\frac{\lambda_1}{\lambda_2} e^{-\lambda_2(b - U_b + \theta(U_b - x))} - \frac{\lambda_2}{\lambda_1} e^{-\lambda_1(b - U_b + \theta(U_b - x))}\right\}. \tag{31}$$

In order to find the values of $\mathbf{p}$, c_1 and c_2, we consider the value-matching and smooth-pasting condition with respect to d_b, D_b, U_b and u_b.

(V1) The value-matching condition of d_b

$$\begin{aligned} &\phi(b,d_b) - \phi(b,D_b) - K_1 - k_1(D_b - d_b) \\ &= \begin{cases} \frac{\theta(h_1-h_2)(D_b-d_b)}{\rho}, & b < D_b - \theta(D_b - d_b) \\ \frac{h_1-h_2}{\rho}\left(D_b - b + \frac{\mu}{\rho} - L_D(d_b,\theta)\right), & D_b - \theta(D_b - d_b) \le b < D_b \\ \frac{h_1-h_2}{\rho}(L_D(d_b,0) - L_D(d_b,\theta)), & D_b \le b \end{cases} \end{aligned} \tag{32}$$

(V2) The value-matching condition of u_b

$$\begin{aligned} &\phi(b,u_b) - \phi(b,U_b) - K_2 + k_2(U_b - u_b) \\ &= \begin{cases} \frac{\theta(h_1-h_2)(U_b-u_b)}{\rho}, & b < U_b \\ \frac{h_2-h_1}{\rho}\left(U_b - b + \frac{\mu}{\rho} - \theta(U_b - u_b) - L_U(u_b,0)\right), & U_b \le b < U_b + \theta(u_b - U_b) \\ \frac{h_1-h_2}{\rho}(L_U(u_b,0) - L_U(u_b,\theta)), & U_b + \theta(u_b - U_b) \le b \end{cases} \end{aligned} \tag{33}$$

(S1) The smooth-pasting condition of d_b

$$\phi^{'}(b,d_b)=\begin{cases}-k_1(\theta)-\frac{\theta(h_1-h_2)}{\rho}, & b<D_b-\theta(D_b-d_b)\\ -k_1(\theta)-\frac{\theta(h_1-h_2)}{\rho}l_D(d_b,\theta), & D_b-\theta(D_b-d_b)\le b\end{cases} \tag{34}$$

where

$$l_D(x,\theta)=\frac{\lambda_1 e^{-\lambda_2(b-D_b+\theta(D_b-x))}-\lambda_2 e^{-\lambda_1(b-D_b+\theta(D_b-x))}}{\lambda_1-\lambda_2}\ge 0. \tag{35}$$

Note that we have $\frac{\partial}{\partial x}L_D(x,\theta)=\theta l_D(x,\theta)$.

(S2) The smooth-pasting condition of D_b

$$\begin{aligned}&\phi^{'}(b,D_b)\\ &=\begin{cases}\phi^{'}(b,d_b), & b<D_b-\theta(D_b-d_b)\\ \phi^{'}(b,d_b)-\frac{h_1-h_2}{\rho}(1-l_D(d_b,\theta)), & D_b-\theta(D_b-d_b)\le b<D_b\\ \phi^{'}(b,d_b)+\frac{h_1-h_2}{\rho}(l_D(d_b,\theta)-l_D(d_b,0)), & D_b\le b\end{cases}\end{aligned} \tag{36}$$

(S3) The smooth-pasting condition of u_b

$$\phi^{'}(b,u_b)=\begin{cases}k_2(\theta)-\frac{\theta(h_1-h_2)}{\rho}, & b<U_b+\theta(u_b-U_b)\\ k_2(\theta)-\frac{\theta(h_1-h_2)}{\rho}l_U(u_b,\theta), & U_b+\theta(u_b-U_b)\le b\end{cases} \tag{37}$$

where

$$l_U(x,\theta)=\frac{\lambda_1 e^{-\lambda_2(b-U_b+\theta(U_b-x))}-\lambda_2 e^{-\lambda_1(b-U_b+\theta(U_b-x))}}{\lambda_1-\lambda_2}\ge 0. \tag{38}$$

Note that we have $\frac{\partial}{\partial x}L_U(x,\theta)=\theta l_U(x,\theta)$.

(S4) The smooth-pasting condition of U_b

$$\begin{aligned}&\phi^{'}(b,U_b)\\ &=\begin{cases}\phi^{'}(b,u_b), & b<U_b\\ \phi^{'}(b,u_b)+\frac{h_1-h_2}{\rho}(1-l_U(u_b,0)), & U_b\le b<U_b+\theta(u_b-U_b)\\ \phi^{'}(b,u_b)+\frac{h_1-h_2}{\rho}(l_U(u_b,\theta)-l_U(u_b,0)), & U+\theta(u_b-U_b)\le b\end{cases}\end{aligned} \tag{39}$$

Conditions (32) - (36) are classified into five groups according to relationship between b and $\mathbf{p}$. When we observe b at the beginning of a cycle, the parameters $\mathbf{p}$, c_1 and c_2 can be obtained by solving simultaneous equation for corresponding group. In the following theorem, we provide that the function ϕ of equations (21), (28) and (30) is the solution of QVI.

Theorem. 2 *Suppose that assumption 1 holds and that there exists parameters* $\mathbf{p}$ *and a continuous function* ϕ *which satisfies equations (21), (28), (30) and conditions (32) - (37). There exists an optimal policy to the cash management problem (8).*

4 Conclusion

In this paper, we have formulated the cash management model in which the two types of funds are available when the manager adjusts cash level. The existence of an optimal policy has been shown by using QVI approach under some conditions. For the future, we would like to show that the existence and uniqueness of the parameters for the optimal policy. Furthermore, we also would like to provide some numerical results recognizing some analytical properties of the optimal policy.

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