The Eighth International Symposium on Operations Research and Its Applications (ISORA'09)
Zhangjiajie, China, September 20–22, 2009

An Dynamic Lot-Sizing Model with Multi-Mode Shipments*

Yu-Zhong Zhang[1,†] Ru-Zhen Yan[1] Wei Gao[2]

[1]College of Operations Research and Management Science ,
Qufu Normal University, Rizhao, Shandong, China 276826
[2]College of Business, Sichuan University, Chengdu, Sichuan, China 610065

Abstract In this paper, we consider a two-echelon dynamic lot-sizing problem with multiple shipment modes between the warehouse and the distribution center. We present an optimal polynomial algorithm for this problem. The computational complexity of the algorithm is $O(T^5)$, where T is the length of the finite planning horizon.

Keywords Lot-sizing; Inventory replenishment; Dynamic programming; Multiple shipment modes

1 Introduction

The two-echelon dynamic lot-sizing problem has received extensive attention in the literature of supply chain management. Immense interest in the two-echelon dynamic lot-sizing problem is due to the fact that the problem arises in many practical situations, often as subproblems of some inventory planning problems.

For the purpose of providing a brief overview, we focus on dynamic lot-sizing problem because this line of work is most closely related to our problem. Wanger and Whitin[1] first present such a single item dynamic lot-sizing problem. Subsequently, many authors (Lee[2], Chan et al.[3] and Zhang et al.[4]) analyze the similar problems with multiple items or two echelons. Recently, Lee[5] considers the single item dynamic lot-sizing problem with container-based transportation costs, and develops a dynamic programming algorithm to solve this problem in $O(n^4)$ time, where n is the length of the finite planning horizon. Lippman[6] considers a single item lot-sizing model with container-based transportation costs and presents a dynamic programming based approach with a computational complexity of $O(T^3)$, where T is the length of the finite planning horizon. Jaruphongsa et al.[7] consider the two-echelon dynamic lot-sizing problem with two transportation modes, and develops a dynamic programming algorithm in $O(T^5)$ time. Ouyang et al.[8] consider a just-in-time techniques in the supply chain system, and the lead time is shorted by adding extra crashing costs to reduce the total cost in this system. However, to the best of our knowledge, there are no papers studying the two-echelon dynamic lot-sizing problem with multi-mode shipments. In our paper, we have extended

*This work was supported by the National Natural Science Foundation of China (Grant No. 10671108) and the Ph.D. Programs Foundation of Ministry of Education of China (Grant No. 20070446001)
†E-mail: yuzhongrz@163.com.

the model of Jaruphongsa et al.[7] to the two-echelon dynamic lot-sizing problem with m modes of transportation and develop an optimal polynomial algorithm to solve our model in $O(T^5)$ time.

The remainder of this paper is organized as follows. In section 2, we describe our model and some notations that will be used throughout the whole paper. In section 3, we present some optimality properties of this problem. In section 4, we give a exact algorithm with running time in $O(T^5)$. In section 5, we consider a special case. Finally, we draw some conclusions and give suggestions for future research in section 6.

2 Descriptions and notations

For fitting in with realistic circumstances, the problem of determining the optimal replenishment policy with two-echelon lot-sizing model is considered in this article. The two-echelon lot-sizing model is composed of a warehouse and a distribution center. The warehouse has a fixed set-up cost of an inbound shipment from suppliers, while the distribution center, in the process of an inbound shipment from warehouse, is satisfied by multiple delivery modes.

Consider a dynamic lot-sizing problem with T periods. For $1 \leq t \leq T$, we define the following notations which is used throughout this paper.

d_t : the demand in period t,

x_{it} : the amount dispatched to the distribution center by mode-i outbound shipment (dispatch) in period t, $i = 1, \ldots, m$,

y_t : the inbound shipment(replenishment) quantity at the warehouse in period t,

I_t : the on-hand inventory level at the distribution center at the end of period t,

I'_t : the on-hand inventory level at the warehouse at the end of period t,

h_t : the cost of holding one unit production at the distribution center in period t,

h'_t : the cost of holding one unit production at the warehouse in period t. Following a common assumption of the multi-echelon inventory literature, we assume that $h'_t \leq h_t$ for all t,

K : the fixed set-up cost of an replenishment at the warehouse.

The cost function of mode i in period t is given by

$$C(x_{it}) = S_i \cdot \delta(x_{it}) + \lceil x_{it}/w \rceil \cdot A_i + p_i \cdot x_{it}, i = 1, \ldots, m.$$

where S_i, A_i, p_i, w, respectively, represent the fixed set-up cost for mode-i dispatch, the fixed cost per container for mode-i dispatch, unit dispatch cost for a mode-i dispatch and the capacity of container for dispatch. Suppose that p_i is not increased, i.e., $p_1 \geq p_2 \geq \ldots \geq p_m$. Given the cost and demand information, the objective is to find the optimal policy to satisfy all customer demands at the minimum cost. Then, the mathematical programming formulation for our model can be given by

$$\begin{array}{ll} \min & \sum_{t=1}^{n} \left(K\delta(y_t) + \sum_{i=1}^{m} C(x_{it}) + h'_t I'_t + h_t I_t \right) \\ \text{s.t.} & I'_{t-1} + y_t - d_t = I'_t \\ & I_{t-1} + x_t - d_t = I_t \\ & I_0 = I_T = I'_0 = I'_T = 0 \\ & x_t \geq 0, y_t \geq 0, I_t \geq 0, I'_t \geq 0 \end{array} \tag{1}$$

where $\delta(y_t) = 1$ if $y_t > 0$; $\delta(y_t = 0)$ if $y_t = 0$.

In this paper, our aim is to minimize the sum of inventory replenishment cost, inventory holding cost, and transportation cost with multiple shipment modes. The first and the second constraints provide balance for inventory flow of the warehouse and distribution center from the previous period $(t-1)$ into the current period. The third constraint ensures that initial inventory I_0 and I'_0 are zero, and the ending inventory level I_T and I'_T are zero. The final constraint characterize the variables domains: x_t, y_t, I_t, I'_t are non-negative for $t = 1, \ldots, T$.

Definition 1.
Period t is called a replenishment period if $y_t > 0$, and it is called a dispatch period if $x_t > 0$. Note that $x_t = \sum_{i=1}^{m} x_{it}$.

Definition 2.
Period t is called a regular dispatch period if $x_t = nw$ for a positive integer n, and it is called a non-regular dispatch period if $x_t \neq nw$ for a positive integer n.

Definition 3.
Period t is called a warehouse regeneration point if $I'_t = 0$, and it is called a customer regeneration point if $I_t = 0$.

Definition 4.
For $1 \leq s \leq t \leq T$,

$d(s,t) = \sum_{i=s}^{t} d_i$, denotes the sum of demand from period s to t,

$h'(s,t) = \sum_{i=s}^{t} h'_i$, denotes the sum of holding cost at the warehouse from period s to t,

$h(s,t) = \sum_{i=s}^{t} h_i$, denotes the sum of holding cost at the distribution center from period s to t,

$H(s,t) = \sum_{i=s+1}^{t} h(s,i-1)d_i$, denotes the sum of holding cost at the distribution center when period s supply all demands from period s to t.

We also define $d(s,t) = 0, h(s,t) = 0, h'(s,t) = 0$ and $H(s,t) = 0$ if $s > t$.

3 Optimality properties

We now provide some optimal properties that will be used in the following. The four properties is presented in Lee et al.[9] also hold for our problem.

Property 3.1.
There exists an optimal solution such that $I'_{t-1} \cdot y_t = 0$

Property 3.2.
There exists an optimal solution such that an inbound replenishment is received only when an outbound dispatch is made, i.e., for a given t, $y_t > 0$ only if $x_t > 0$.

Property 3.3.
There exists an optimal solution such that $x_t > 0$ *if and only if* $I_{t-1} < d_t$*, for any* t*,* $1 \leq t \leq T$.

Property 3.4.
There exists an optimal solution such that $x - T > 0$ *if and only if* $I_{t-1} < d_t$*, for any* t*,* $1 \leq t \leq T$.

It can be easily shown that each warehouse replenishment provides stock for several consecutive dispatches to satisfy customer demands for some consecutive periods. The following theorems will also be used in the optimal algorithm.

Theorem 3.1.
There exists an optimal solution such that if t *is a dispatch period, then the freight is dispatched to distribution center by at most two modes of transportation in period* t.

Proof. We prove it by contradiction.

Suppose that there exists an optimal solution such that the distribution center is satisfied by three modes of shipment in dispatch period t. Let i, j and l, $i < j < l$, be the mode of shipment in period t. It is easy to show that we have $p_i \geqslant p_j \geqslant p_l$. We decrease x_{it} by $\min\{x_{it}, \lceil x_{lt}/w \rceil \cdot w - x_{lt}\}$čňand increase x_{lt} by the same amount. The number of cargos used by each mode is not increased. Thus, the dispatch cost is not increased. After the perturbation, we have $x_{it} = 0$ or $x_{lt} = nw$ for some positive integer n.

If $x_{it} = 0$, then we have a contradiction.

If $x_{lt} = nw = \lfloor x_t/w \rfloor \cdot w$, then we decrease x_{it} by x_{it} and increase x_{jt} by the same amount . It is easy to show that the dispatch cost is not increased. After the perturbation, $x_{it} = 0$. Obviously, this is a contradiction.

If $x_{lt} = nw \neq \lfloor x_t/w \rfloor \cdot w$, then $x_{it} + x_{jt} > w$. In this case, we consider the following possibilities:

(1) If $x_{it} = mw$ for some positive integer m and $A_i + p_i w \leqslant A_l + p_l w$, then we decrease x_{lt} by x_{lt} and increase x_{it} by the same amount. The dispatch cost is not increased. After the perturbation, $x_{lt} = 0$. Obviously, this is a contradiction. If $x_{it} = mw$ for some positive integer m and $A_i + p_i w > A_l + p_l w$, then we can decrease x_{it} by x_{it} and increase x_{lt} by the same amount. The dispatch cost is not increased. After the perturbation, $x_{it} = 0$. This is a contradiction.

(2) If $x_{it} \neq mw$ for some positive integer m, then we decrease x_{it} by $\min\{x_{it}, \lceil x_{jt}/w \rceil \cdot w - x_{jt}\}$ and increase x_{jt} by the same amount. It is easy to show that the dispatch cost is not increased. After the perturbation, we have $x_{it} = 0$ or $x_{jt} = qw$ for some positive integer q.

If $x_{it} = 0$, then we have a contradiction.

If $x_{jt} = qw$ and $A_j + p_j w \leqslant A_l + p_l w$, then we decrease x_{lt} by x_{lt} and increase x_{jt} by the same amount. The dispatch cost is not increased. After the perturbation, $x_{lt} = 0$. This is a contradiction.

If $x_{jt} = qw$ and $A_j + p_j w > A_l + p_l w$ then we decrease x_{jt} by x_{jt} and increase x_{lt} by the same amount. The dispatch cost is not increased. After the perturbation, $x_{jt} = 0$. This is a contradiction. □

Theorem 3.2.
There exists an optimal solution such that, for any $t = 1,\ldots,T$,

(1) $x_{1t} \in \{0, x_t, x_t - \lfloor x_t/w \rfloor \cdot w\}$,

(2) $x_{it} \in \{0, x_t, \lfloor x_t/w \rfloor \cdot w, x_t - \lfloor x_t/w \rfloor \cdot w\}$.

Proof. We prove it by contradiction.

Part(1): Suppose $x_{1t} \notin \theta$ in the optimal solution, where $\theta = \{0, x_t, x_t - \lfloor x_t/w \rfloor \cdot w\}$. We have $x_{kt} \geqslant 0\,(k > 1)$ by theorem 3.1. If $x_{kt} = 0$, then $x_{1t} = x_t$. This is a contradiction. If $x_{kt} > 0(k > 1)$, then $p_1 \geqslant p_k$. We decrease x_{1t} by $\min\{x_{1t}, \lceil x_{kt}/w \rceil \cdot w - x_{kt}\}$, and increase x_{kt} by the same amount. The dispatch cost is not increased. After the perturbation, we have $x_{1t} = 0$ or $x_{kt} = nw$ for some positive integer n.

If $x_{1t} = 0$, then we have a contradiction.

If $x_{kt} = nw = \lfloor x_t/w \rfloor \cdot w$, then $x_{1t} = x_t - \lfloor x_t/w \rfloor \cdot w$. This is a contradiction.

If $x_{kt} = nw \neq \lfloor x_t/w \rfloor \cdot w$ and $A_1 + p_1 w \leqslant A_k + p_k w$, then we decrease x_{kt} by x_{kt}, and increase x_{1t} by the same amount. The dispatch cost is not increased. After the perturbation $x_{kt} = 0$. This is a contradiction. If $x_{kt} = nw \neq \lfloor x_t/w \rfloor \cdot w$ and $A_1 + p_1 w > A_k + p_k w$, then we decrease x_{1t} by $\lfloor x_{1t}/w \rfloor$ and increase x_{kt} by the same amount. The dispatch cost is reduced. After the perturbation $x_{kt} = \lfloor x_{kt}/w \rfloor \cdot w$. This is a contradiction.

Part(2): Suppose $x_{it} \notin \varphi$ in the optimal solution, where $\varphi = \{0, x_t, \lfloor x_t/w \rfloor \cdot w, x_t - \lfloor x_t/w \rfloor \cdot w\}$. We have $x_{kt} \geqslant 0$ for some positive integer k by theorem 3.1. If $k = 1$, then this case have been proved in part(1). If $x_{kt} = 0$, then we have a contradiction. Suppose $x_{kt} > 0$ and $k \geqslant 2$ in the optimal solution. Without loss of generality, let $i < k$. Recalling that $p_k \leqslant p_i$, we can decrease x_{it} by $\min\{x_{it}, \lceil x_{kt}/w \rceil \cdot w - x_{kt}\}$ and increase x_{kt} by the same amount. The dispatch cost is not increased . After the perturbation, we have $x_{it} = 0$ or $x_{kt} = nw$ for some positive integer n.

If $x_{it} = 0$, then we have a contradiction.

If $x_{kt} = nw = \lfloor x_t/w \rfloor \cdot w$, then $x_{it} = \lfloor x_t/w \rfloor \cdot w$. This is a contradiction.

If $x_{kt} = nw \neq \lfloor x_t/w \rfloor \cdot w$ and $A_i + p_i w \leqslant A_k + p_k w$, then we can decrease x_{kt} by x_{kt} and increase x_{it} by the same amount. The dispatch cost is not increased. After the perturbation, $x_{kt} = 0$. This is a contradiction. If $x_{kt} = nw \neq \lfloor x_t/w \rfloor \cdot w$ and $A_i + p_i w > A_k + p_k w$, then we can decrease x_{it} by $\lfloor x_{it}/w \rfloor \cdot w$ and increase x_{kt} by the same amount. The dispatch cost is reduced. After the perturbation, we have $x_{kt} = \lfloor x_t/w \rfloor \cdot w$. Obviously, This is a contradiction. □

Theorem 3.3.
There exists an optimal solution such that, for any $t = 1,2,\ldots,T$, $x_t > 0$ *only if* $I_{t-1} < \min\{d_t, w\}$ *or* $d_t - w < I_{t-1} < d_t$.

Theorem 3.4.
There exists an optimal solution such that if $l-1$ and m are two consecutive customer regeneration points, then at most one period during l and m is a non-regular dispatch period.

It is not difficult to show that we can prove the theorem 3 and theorem 4 by the way of the theorem 2.

4 Optimal procedure

In this section, we present the optimal algorithm. First of all, we consider which modes will be used in period t. Without lost of generality, we suppose that the mode-i and the mode-j are used in period t, and the total dispatch quantity of the distribution center is x_t, $nw \le x_t < (n+1)w$. The value of i, j can be given by

$$(i,j) = \{(k,l) : \min\{c_k(nw) + c_l(x_t - nw)\}, k = 2,\dots,m, l = 1,\dots,m\}.$$

Suppose that period i is the replenishment period in the optimal replenishment policy. Hence, period i is also a dispatch period by property 3.2. Let $C_{ij}(a,b)$ be the minimum total cost of replenishing in period i to satisfy $d(i,j)$ with $I_{i-1} = a, I_j = b$ and $I'_{i-1} = I'_j = 0$. Hence, we have $0 \le a < d_i$ and $0 \le b < d_{j+1}$ by the property 3.3. Then, We analyze the possible value of a. Suppose that $l-1$ and m are two consecutive customer regeneration points such that $l \le i \le m$ in the optimal solution. Also let τ be the non-regular dispatch period. If $i = l$, then $a = 0$. If $l < i \le m$, then we consider the following possible cases.

Case 1: $i \le \tau$. In this case, all dispatches during periods $l,\dots,i-1$ are regular dispatches. Thus, theorem 3.4 implies that one of the following conditions is satisfied.

(1) $I_{i-1} < \min\{d_i, w\}$: For a given l, period i is considered as a potential non-regular dispatch period when $0 < a = I_{i-1} = I_1(l,i) = \lceil d(l,i-1)/w \rceil w - d(l,i-1) < d_i$.

(2) $d_i - w < I_{i-1} < d_i$: For a given l, period i is considered as a potential non-regular dispatch period when $a = I_{i-1} = I'_2(i,m) = \lfloor d(l,i)/w \rfloor w - d(l,i-1) \ge w$.

Case 2: $i > \tau$. In this case, all dispatches during periods $i,\dots,m$ are regular dispatches. Thus, theorem 3.4 implies that one of the following conditions is satisfied.

(1) $I_{i-1} < \min\{d_i, w\}$: For a given m, period m is considered as a potential non-regular dispatch period when $0 < a = I_{i-1} = I_2(i,m) = d(i,m) - \lfloor d(i,m)/w \rfloor w < d_i$.

(2) $d_i - w < I_{i-1} < d_i$: For a given l, period i is considered as a potential non-regular dispatch period when $0 < a = I_{i-1} = I'_2(i,m) = d(i,m) - \lceil d(i+1,m)/w \rceil w \ge w$.

Hence, for $2 \le i \le T$, the set of all possible values of a for a dispatch in period i can be defined by $E(i) = \{I_1(l,i) : 0 < I_1(l,i) < d_i, 1 \leqslant l < i\} \cup \{I'_1(l,i) : I'_1(l,i) \geqslant w, 1 \leqslant l < i\} \cup \{I_2(i,m) : 0 < I_2(i,m) < d_i, i \leqslant m \leqslant T\} \cup \{I'_2(i,m) : I'_2(i,m) \geqslant w.i \leqslant m \leqslant T\} \cup \{0\}$, where $E(1) = E(T+1) = 0$. Since $I_0 = I_T = 0$, we define $E(1) = E(T+1) = 0$.

Now, we analyze the possible values of b. If $j = T$, then $b = 0$. If $j < T$, then period $j+1$ must be a dispatch period with $I_j = b$. Similar to the above method ,we have $b \in E(j+1)$. Let $F_j(b)$ be the minimal total cost of satisfying $d_1,\dots,d_j$ with $b \in E(j+1), I_j = b$.

Algorithm 4.1.
For $1 \leqslant j \leqslant T$ and $b \in E(j+1)$, let:
$F_j(b) = \min\left\{F_{i-1}(a) + C_{ij}(a,b) : 1 \leqslant i \leqslant j \text{ and } a \in E(i)\right\}$, *where* $F_0(0) = 0$.

For each $s,t (1 \leqslant s \leqslant t \leqslant T)$, let $g_{st}(x,y)$ be the minimum total cost of satisfying $d(s,t)$ with $x \in E(s)$ and $y \in E(t+1)$ such that $I_{s-1} = x, I_t = y, I_k > 0 \, (s \leqslant k < t)$, $I'_t = 0, I'_{s-1} = d(s,t) - x + y, x \in E(s), y \in E(t+1)$. It is easy to show that there is not customer regeneration point during period s and $t-1$.

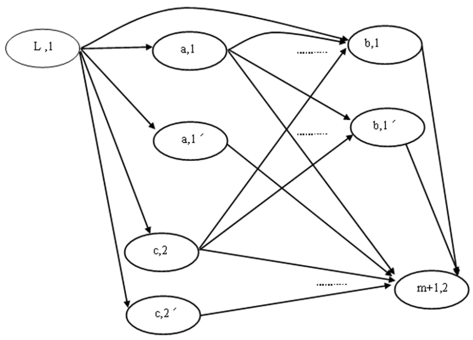

Figure 1: Network (l,m)

Algorithm 4.2.
For $1 \leqslant i < j \leqslant T$, $a \in E(i)$ *or* $b \in E(j+1)$, *let*

$$C_{ij}(a,b) = \min \begin{cases} K + g_{ij}(a,b) \\ \min\{g_{im}(a,0) + C_{m+1,j}(0,b) + h'(i,m)\cdot[d(m+1,j)+b] : i \leq m \leq j\} \end{cases}$$

Given the values of $g_{st}(x,y)$ for all $1 \leq s \leq t \leq T, x \in E(s)$ and $y \in E(t+1)$, we can compute the value of $C_{ij}(a,b)$ for all $1 \leq s \leq t \leq T, a \in E(i)$ and $b \in E(j+1)$ in $O\left(T^5\right)$.

Suppose that $l-1$ and m are the two consecutive customer regeneration points. In order to find the values of $g_{st}(x,y)$, a network (l,m) is generated for each pair of l and m, $1 \leq l \leq m \leq T$.

Nodes with a label of type$(t,1)$,$(t,1')$,$(t,2)$,$(t,2')$ denote $t \leqslant \tau$ and $0 < I_{t-1} < d_t$ with $l \leq t \leq m$, $t \leqslant \tau$ and $I_{t-1} \geqslant w$ with $l < t \leq m$, $t > \tau$ and $0 < I_{t-1} < d_t$ with $l < t \leq m+1$, $t > \tau$ and $I_{t-1} \geqslant w$ with $l < t \leq m+1$.

All nodes represent the potential dispatch periods, and each arc between these nodes represents a dispatch decision. Let $S(t,\bullet) = I_{t-1}$, and also let $(s,\bullet) \to (t,\bullet), l \leqslant s < t \leqslant m+1$, denotes the arc connecting node$(s,\bullet)$ and node $(t,\bullet)$. The arc $(s,\bullet) \to (t,\bullet)$ represents the dispatch amount of $d(s,t-1) - S(s,\bullet) + S(t,\bullet)$ in period s, and the total cost of each arc $(s,\bullet) \to (t,\bullet)$, $l \leqslant s < t \leqslant m+1$ is given by $P(d(s,t-1) - S(s,\bullet) + S(t,\bullet)) + h'(l,s-1)\cdot(d(s,t-1) - S(s,\bullet) + S(t,\bullet)) + H(s,t-1) + h(s,t-1)\cdot S(t,\bullet)$. Each network (l,m) is composed of $O(m-l)$ nodes and $O((m-l)^2)$ arcs. All arc costs can be computed in $O(T^4)$ time. Suppose that $Q((s,\bullet),(t,\bullet))$ is the cost of the shortest path from node$(s,\bullet)$ to node$(t,\bullet)$ with $l \leqslant s < t \leqslant m+1$. The $g_{st}(x,y)$ can be computed using the following algorithm.

Algorithm 4.3. *For each pair* $(s,\bullet)$ *and* $(t,\bullet)$, $l \leqslant s < t \leqslant m+1$, *let*

$$Q((s,\bullet),(t,\bullet)) = g_{s,t}((s,\bullet),(t,\bullet)) + h'(l,s-1)\cdot(d(s,t-1) - S(s,\bullet) + S(t,\bullet)).$$

$Q((s,\bullet),(t,\bullet))$, $1 \leq s < t \leq T$, can be computed using Floyd-Warshall Method [10], which runs in $O((m-l)^3)$time. Thus, the all values of $g_{st}(x,y)$ can be composed in $O(T^4)$ time, and each network(l,m) has $O((m-l)^2)$ pairs of nodes. Hence, the computational complexity of this problem is $O(T^5)$.

5 Special case

In this section, we consider a special case from many practical situations. For sufficient large x_{it}, the cost function of mode i in period t is given by

$$C(x_{it}) = S_i \cdot \delta(x_{it}) + (A_i/w + p_i) \cdot x_{it}, i = 1, \dots, m.$$

Obviously, the mode of shipment for the distribution center will be taken whose cost function contains minimum c', where $c' = A_i/w + p_i$.

6 Conclusions

This paper analyzes the two-echelon dynamic lot-sizing problem by considering multiple transportation modes, and considers the case that the dispatch quantity is sufficient large. The future work is to further study the problem that multiple items are delivered to multiple customers with multiple shipment modes.

References

[1] H.M.Wagner, T.M.Whitin. Dynamic Version of the Economic Lot-Size Model. *Management Science*. 5:89-96.1958.

[2] C.Y.Lee. A Solution to the Multiple Set-Up Problem with Dynamic Demand. *IIE Transaction*, 21:226-270.1989.

[3] L.M.A.Chan, A.Muriel, Z.J.Shen and D.Smichi-levi. On the Effectiveness of Zero-Inventory-Ordering Policies for the Economic Lot Sizing Model With Piecewise Linear Cost Structures. *Operations Research*. 50:1058-1067.2002.

[4] Y.Z.Zhang, J.T.Xu, Q.G.Bai. Dynamic Lot-Sizing Model with a Class of Multi-Breakpoint Discount Cost Structures. *Lecture Notes in Operations Research*. 6:264-269.2006.

[5] C.Y.Lee. Inventory Replenishment Model: Lot Sizing versus Just-In-Time Delivery. *Operation Research Letters*. 32:581-590.2004.

[6] S.A.Lippman. Optimal Inventory Policy with Multiple Set-Up Costs. *Management Science*. 16:118-138.1969.

[7] W.Jaruphongsa, S.Çetinkaya, C.Y.Lee. Outbound Shipment Mode Considerations for Integrated Inventory and Delivery Lot-Sizing Decisions. *Operations Research Letters*. 35:813-822.2007.

[8] L.Y.Ouyang, B.R.Chuang. Mixture Inventory Model Involving Variable Lead Time and Controllable Backorder Rate. *Computers* & *Industrial Engineering*. 40:339-348.2001.

[9] C.Y.Lee, S.Çetinkaya, W.Jaruphongsa. A Dynamic Model for Inventory Lot-Sizing and Outbound Shipment Scheduling at a Third Party Warehouse. *Operations Research*. 51:735-747.2003.

[10] E.L.Lawler. *Combinatorial Optimization:Networks and Matroids*.New York.Holt, Rinehart and Winston. 16:118-138.1976.