STATISTICAL DATA ANALYSES OF RECENT NATIONAL ELECTIONS IN JAPAN

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Abstract

This paper investigates the recent national elections in Japan quantitatively by applying various statistical methods and mathematical models. Firstly, we examine the regional characteristics that affect voter turnouts and the efficiency of the political parties in Japan. We find that the high income and most urbanized prefectures are categorized in the same cluster regarding voter turnouts. The voting efficiency according to prefectures and different parties also shows certain patterns. In addition, the relationship between the vote share and the seat share for all political parties was examined by applying mathematical models with both polynomial and exponential functions. Actual data obtained from Japan's recent national elections for both the House of Councilors and the House of Representatives from 2005 to 2014 have been used. To make election more representative Japan's reform to introduce proportional representation ensures linear predictability of seat shares by vote shares. However, our analysis found that the existing plurality system of election also has substantial degree of predictability. This predictability can be approximated by an exponential function, which in earlier studies is done by using polynomial functions. We conclude that the coexistence of the both, gives Japanese national elections a better representation of people's choice.

1 Introduction

One of the fundamental issues of democracies to reflect the popular will through elections has been contested in many countries. The contradiction between proportional representation and popularity election gradually is becoming wider. In the popularity elections, a proportional reflection of people's choice is far from reality (Taagepera, 1986). To bring forth the more representative elections, in 1994 Japan introduced a new policy as a transition from plurality known as popularity election to proportional representation. However, advocates of plurality system argue that proportional representation systems vary from ideal proportion, mainly due to different district magnitudes (i.e. number of seats per districts) (Taagepera, 1986). This contradiction evolved many votes-to-seats conversion rules for the plurality systems of election. The so-called cubic law in election results is the most famous one among these conversion rules. Subsequently many other researchers either reformulated and renovated the cubic law or devised other predictive equations.

In the wake of recent changes in Japanese election system, where both proportional representation and plurality voting co-exist, one can raise a question that whether the proportional representation is more suitable than the newly developed mixed election procedure. To answer this question, like many other researchers found, it is imperative to come up with a predictive equation for the plurality system of voting in Japan which is compatible with the proportional representation system.

Henceforth, the purpose of this paper is to develop and test a compatible predictive equation for the plurality system in Japanese elections with proper curve sitting based on seat shares and vote shares. In several sections of this paper, we designed and verified different predictive equations. The next section discusses on political and election system including the recent changes in election systems of Japan. Section 3 focuses on the methodology employed to analyze the data for the study. It is followed by the data analysis part in section 4. The data analyses are divided into three subsections. Section 4.1 shows the relationship between regional characteristics and voter turnout in the elections. In section 4.2 we devised various predictive equations for plurality system of election and in section 4.3 we test the (exponential function) predictability of plurality system against the (linear function) predictability of proportional representation system by using 'voting efficiency' test. Finally, section 5 summarizes all the findings from the analyses and draws an overall conclusion.

2 Political System and National Election System in Japan

2.1 Changes in National election system

The new election system introduced in Japan in 1994 is likely to change the organization of the contesting political parties. The new system is likely to yield less bureaucratic and more internally democratic political parties

(Christensen, 1996). The change was a transition from plurality to proportional representation. For instance, earlier under the plurality system of representation, it was less likely that smaller political parties could get a decisive mandate. In contrast, under the new system, with a mix of plurality and proportional representation, smaller parties (with fewer campaign resources) will be incentivized by the increased likelihood of winning a mandate. The barrier to entry for smaller parties is lowered, if the mandate is decided on the basis of the percentage share of support, and not on the majority support. Therefore, in the new system, the share of support matters more. For example, if a smaller political party can win 3 percent of the nation-wide votes, it is now able to get at least 3 percent of the proportional seats. The effects of such change might be observable in terms of campaign funding via an altered incentive structure for deploying campaign resources. With the lower barriers to entry, even an independent candidate may choose to run.

The national-level outcome of an election has some effects on the happiness of the electorates. In Japan, the supporters of winning political parties are found to be elated after the national outcome is announced, as determined by Kinari et al. (2015) for the election held in the year 2009. Such happiness was not linked to any material benefits obtained from aligning with the winning political party, but rather, emanated from the implied fulfillment of policies promised in the election manifestos of the winner. In addition, using a counterfactual simulation of electoral outcomes in Japan, Baker & Scheiner (2007) simulated party adaptation the new system and identified which systemic feature has helped the Liberal Democratic Party's dominance, historically. In a review paper, Horiuchi (2009) indicated that at the sub-national level, certain important changes are taking place that are influenced by changes in national politics, and commented that such changes are worth examining in order to deepen our understanding of the permeating process of national politics on sub-national policies. We are not aware of any attempt to analyze the interrelation among vote share (VS), seat share (SS), pass votes (PV), and fail votes (FV). The research studies we have referred to above is primarily focused on the consequences of the new election system, voting behavior, voter turnouts, and party analysis. Therefore, to address this gap in the literature, we attempt to examine the prefecture-wise voter turnouts and estimate the effect of regional characteristics on the voter turnouts. At the same time, we attempt to find the interrelation between vote share (VS) and seat share (SS) pass votes (PV) and fail votes (FV) using data of the most recent national elections of Japan in 2005-2015, as well as determining the voting-efficiency. We find a functional relation between vote share and seat share valid for the most recent national elections in Japan.

2.2 Political systems

Japan has a democratic election system. The national government of Japan is comprises of the legislative, judicial, and executive branches. The Emperor of Japan, constitutionally, is symbolic to the unity of the country. As the Head of State, the Emperor appoints the Prime Minister and the Chief Justice of the Supreme Court. The Prime Minister holds the executive powers to form his cabinet of ministers from the Members of Parliament. The Diet (parliament of Japan) consists of lower and upper houses. Since a member of the lower house has a term of four years, the election of the 480 members of the lower house takes place in every four years. Of these, 300 seats are determined by direct votes (based on plurality), and the remaining 180 seats are allocated to political parties based on their percentage share of the proportional votes. In the upper house, out of 242 seats, the election for 121 seats takes place every three years. The tenure of the members of the upper house is six years. Out of 121 seats that are up for election triennially, direct votes are cast for 73 seats and decided based on plurality. The remaining 48 seats are allocated among political parties based on their percentage share of the proportional votes. The upper house cannot be dissolved, but the lower house can be dissolved by the Prime Minister if a no-confidence motion is passed by a majority. The most recent election was held in December 2014 for the lower house. The next election for the lower house will be in 2018. The upper house election was held in 2013, and the next election will be held in 2016. The Diet decides on legislative matters on the basis of a simple majority or in certain cases by a two-third majority. The lower house is more powerful than the upper house.

2.3 National election systems

Three kinds of elections are held in Japan, namely (a) for the lower house, (b) for the upper house (c) and for the local bodies, such as cities or prefectures. The Constitution of Japan confers certain rights to citizens of Japan, and these are (a) universal suffrage, (b) equality of votes, (c) secrecy of the ballot, and (d) representative democracy. The two election committees that conduct elections in Japan are (a) the Administrative Level Election Committee and (2) the Central Election Committee. The Central Election Committee conducts all national level elections in Japan.

The voters and the candidates must fulfil certain criteria to take part in national elections. To cast a vote, a citizen of Japan must be aged 20 years or above and reside in an electorate jurisdiction for at least three months. The minimum age for a candidate is 25 years for the lower house and 30 years for the upper house. Each candidate must deposit 3 million yen to contest an election for a single-seat constituency. The deposit is double (6 million yen) for a proportional seat.

Each voter has the right to cast two votes, one for the single-seat constituency and the other for the proportional seat. Each political party draws a candidate list for the proportional seats. Proportional seats are allocated to the parties on the basis of their proportional share of the votes following the D'Hondt method. In this system a candidate has a chance to be elected, even if (s)he cannot garner a pass vote in a single-seat constituency, but (s)he is included on the list of proportional seat of a political party, which might receive a greater proportional seat share.

2.4 Results of recent national elections

From 2005 to 2014 there were a total of seven national elections held in Japan. Among the seven elections, the 2005, 2009, 2012 and 2014 elections were for the House of

Table 1: Seat share of the winning party of the HR and HC for recent national elections

Year	Winning Party	Number of seats	
		Constituency	Proportional
House of Representatives			
2005	Liberal Democratic Party	219	77
2009	Democratic Party	221	87
2012	Liberal Democratic Party	237	57
2014	Liberal Democratic Party	223	68
House of Councilors			
2007	Democratic Party	40	20
2010	Democratic Party	39	12
2013	Liberal Democratic Party	47	18

Source: Ministry of Internal Affairs and Communications

Representatives (HR) or, in other words, Lower House elections and the 2007, 2010, and 2013 elections were for the House of Councillors (HC) or Upper House elections. The number of seats in both the Upper House and the Lower House election of the winning party is given in Table 1.

proportional system. Therefore, voter turnouts for single seat constituencies and proportional seats show a strong correlation. Table 2 shows the correlations coefficient between the constituency and proportional voter turnouts. In recent national elections, all the coefficients are very

Table 2: Correlation coefficients between constituency and proportional voter turnouts

		Constituency			Proportional		
		2007	2010	2013	2007	2010	2013
Constituency	2007	1					
	2010	0.888	1				
	2013	0.732	0.662	1			
Proportional	2007	0.999	0.888	0.732	1		
	2010	0.887	0.999	0.661	0.888	1	
	2013	0.691	0.637	0.976	0.691	0.637	1

3 Methodology

This paper employs various statistical and mathematical tools to analyze the results of recent national elections in Japan. To conclude on demographic characteristics, it uses correlation and cluster analysis. Then, to formulate the predictive equation for the plurality system of voting we use multiple regression analysis. This paper uses ordinary least square (OLS) method to estimate the regression coefficients and to fit quadratic, cubic and exponential equations. The data for this study has been collected from the website of Ministry of Internal Affairs and Communications and other websites. Some of the national dailies (Ashahi Shimbun etc.) also published these data after the respective elections.

4 Statistical Data Analyses on Recent National Elections

4.1 Voting ratio analyses

The recent elections demonstrate that there has been a gradual decline in the voter turnouts in all the national elections. However, in any election when voters decide to vote they vote for both the constituency and the close to 1, which means that if the voters cast their votes for constituencies they also cast votes for the proportional seats of candidates. Regarding the non-diagonal elements, we see that the correlation coefficient between 2007 and 2010 is rather higher than the others because both cases correlate to the same election result in which the Democratic Party won. The Democratic Party, especially in 2010, became the dominant party over the Liberal Democratic Party. The correlation coefficients between 2007 & 2013 and 2010 & 2013 are relatively lower because, in these elections, the opposition party won.

According to the voter turnouts in three recent elections held in 2007, 2010 and 2013 for the House of Councilors, the prefectures are distributed among four clusters. By applying the K-means and Hierarchical method for our cluster analysis, we found almost similar results for the clustering of 47 prefectures shown in Table 3. The high income and densely populated prefectures tend to cluster in a single group. We find that these clusters are represented by the major characteristics for the statistics shown in Table 4. From Table 3, we find that in cluster I, the prefectures are the most urbanized, and the next (II) group consists of relatively urban prefectures without a large designated capital city. The third and fourth groups are relatively

Table 3: Clustering of prefectures (regions) according to voter turnouts in recent national elections for the House of Councilors.

Cluster	Hierarchical method	K-Means Method
I	Saitama, Chiba, Tokyo, Kanagawa, Kyoto, Osaka, Hyogo, Kochi, Fukuoka, Okinawa	Saitama, Chiba, Tokyo, Kanagawa, Kyoto, Osaka, Hyogo, Kochi, Fukuoka, Okinawa
II	Hokkaido, Iwate, Miyagi, Yamanashi, Nagano, Aichi, Mie, Nara, Fukushima, Niigata, Shizuoka, Shiga	Hokkaido, Iwate, Miyagi, Yamanashi, Nagano, Aichi, Mie, Nara
III	Aomori, Yamagata, Ibaraki, Tochigi, Gifu, Wakayama, Tottori, Okayama, Hiroshima, Tokushima, Ehime, Nagasaki, Kumamoto, Oita, Akita, Toyama, Kagawa, Saga, Miyazaki	Aomori, Yamagata, Ibaraki, Tochigi, Gifu, Wakayama, Tottori, Okayama, Hiroshima, Tokushima, Ehime, Nagasaki, Kumamoto, Oita, Fukushima, Niigata, Shizuoka, Shiga
IV	Gunma, Ishikawa, Fukui, Shimane, Yamaguchi, Kagoshima	Gunma, Ishikawa, Fukui, Shimane, Yamaguchi, Kagoshima, Akita, Toyama, Kagawa, Saga, Miyazaki

remote prefectures, with a conservative population that is interested in politics and elections.

The clusters show that as the degree of urbanization declines the average turnout increases in prefectures. Table 4 demonstrates, cluster-I which is the group of most urbanized prefectures, has the lowest turnout, whereas the average turnout increases gradually in cluster II, cluster III, and cluster IV. This clearly indicates that people living in less urbanized prefectures are more likely to vote in elections. In addition, less urbanized prefectures are more conservative than urbanized prefectures. In such prefectures, peoples are more interested in politics compared to those in urbanized prefectures.

Table 4: Voter turnout characteristics among different clusters

	I	II	III	IV
Mean	28.07	32.36	33.90	38.68
Standard Deviation	3.81	4.86	5.45	7.57
Maximum	34.82	45.97	46.87	49.35
Minimum	21.93	21.94	25.11	27.84
Median	29.01	33.37	32.83	38.96

Above analyses draw us to a conclusion that changes in voter turnout is a mere effect of demographic characteristics of the prefectures. The introduction of new election system has no vivid indication of increasing the voting turn out. Therefore, based on the voting turnout rations, it is very difficult to infer that new election system has a cutting edge advantage over simple plurality system of election. On the contrary, we did not find any strong evidence that can help to conclude that the plurality system is better than proportional representation. The result is rather ambiguous.

However, one obvious advantage of proportional representation over plurality system is its linear predictability. Whereas the predictability of plurality system is non-linear. It can be reiterated that the purpose of this paper is to device a predictive equation which is compatible to the newly introduced proportional system of representation. The following section discusses how plurality system can predict seat shares based on vote shares.

4.2 Relation between vote share and seat share

(*a*) In case of democracies, election results are the key to representing the peoples' preferences for a particular party or candidate. According to Taagepera (1986), the extent to which elections reflect the popular will has become a critical issue for democracies. Many mathematicians and social scientists have taken an interests in how the winning candidates'/parties' share of seats can be approximated as a mathematical function of the peoples' preferences represented by their votes obtained in particular elections. Since the middle of the 20th century, eminent mathematicians and scientists have developed various mathematical functions to approximate the number of seats by people's preferences in the form of popular votes in the elections. In the history of political science, such a mathematical function was first proposed in 1909 when the Rt. Hon. James Parker Smith determined that the proportion of seats won by the victorious party varied as the cube of the proportion of the votes cast for that party to the country as a whole. Kendall and Stuart (1950) claimed that Smith termed this phenomenon as MacMahon's Law after P. A. MacMahon, an eminent mathematician of that time. Afterward, the law became more popularly known as the cubic law of election results. The law states that in a two-party contest between, say, Whites and Blacks, let p0 be the proportion of votes cast for the winning party, say, White, over the whole electoral area. Then if the number of seats won by White and Black is W and B respectively

$$\frac{W}{B} \geq \frac{p_0^3}{q_0^3} \quad (1)$$

where q0 is defined as 1 – p0. Kendall and Stuart (1950) applied this law to data from two general elections of the United Kingdom held in 1935 and 1945. More surprisingly and remarkably there is a striking agreement with the result using New Zealand election data of 1949.

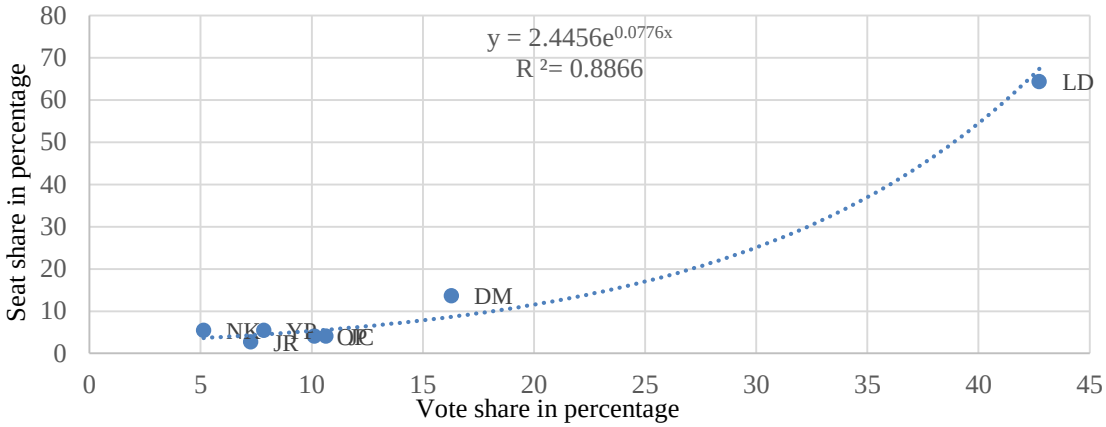


Figure 1: Relation between vote share and seat share of HC, 2013

Due to the narrow or specific applicability of the cubic law to Anglo-Saxon elections, the law loses its generality over elections held around the globe. In 1986, Taagepera introduced the most general form of the cubic law. Compared to the cubic law that is appropriate for single district plurality parliamentary elections like those of Great Britain, Taagepera generalized the mathematical function for multi-seat election districts cases such as those in the United States. Subsequently, in 1969, Theil suggested the following mathematical equation for multi-seat election districts.

$$s_K = v_K{}^n / \sum v_i{}^n \qquad (2)$$

where s_K and v_K are one particular party's votes and seat shares, respectively, n is a constant, and the summation is over the vote shares of all parties. This equation expresses the seat share of one specific party, K, in terms of the vote shares of K and all other parties. When dividing equation (2) by the analogous equation for party L, the summation term cancels out, and one obtains

$$s_K / s_L = v_K{}^n / v_K{}^n = (v_K / v_L)^n \qquad (3)$$

This form would include, as special cases, the cube law (for n = 3) and perfect proportional representation (for n =1). Equation (2), in turn, can be derived from equation (3), which means that the two forms are mathematically equivalent.

The cube law and its later generalizations clearly indicate a mathematical approximation of the polynomial function (more specifically 3rd-degree polynomials) in the seat-vote relationship. In this paper, we approximated the 2nd and third order polynomial functions for the seat-vote relationship in recent Japanese parliamentary elections. Unlike the United Kingdom or the United States, Japan's electoral system consists of both plurality and proportional representation in parliamentary elections. To deduce the seat-vote relationship in Japanese national elections we only consider the plurality system of elections in this country. It should be noted that all these constituencies (seats) are single electoral district seats.

Japan is a multi-party democracy with numerous parties taking part in the general elections. Despite the multi-party nature, except in a few elections, the Liberal Democratic Party (LDP) has won all the elections after the 2nd World War. It is obvious that in many elections, the LDP alone dominates the election obtaining a lion's share of the total number of seats, whereas, all the other parties marginally manage to survive. Nonetheless, in some of the elections, the LDP has faced serious challenges from the other

Table 5: Vote share (VS) and seat share (SS) in percentage of HC

Name of Party	2007		2010		2013	
	VS	SS	VS	SS	VS	SS
Liberal Democratic Party	31.35	31.51	33.38	53.42	42.74	64.38
New Komeito	5.96	2.74			5.13	5.48
Democratic Party	40.45	54.79	38.97	38.36	16.29	13.70
Japan Restoration					7.25	2.74
Japanese Communist Party					10.64	4.11
People's New Party	1.87	1.37				
Your Party			10.24	4.11	7.84	5.48
Other Parties	20.38	9.59	17.42	4.11	10.11	4.11

Note: In the case of the blank cells, the votes that the parties obtained in the elections without gaining any seat are included in 'other parties'.

Table 6: Vote share (VS) and seat share (SS) in percentage of (HR)

Name of Party	2005		2009		2012		2014	
	VS	SS	VS	SS	VS	SS	VS	SS
Liberal Democratic Party	47.8	73.00	38.68	21.33	43.01	79.00	48.10	75.59
Democratic Party	36.40	17.33	47.43	73.67	22.81	9.00	22.51	12.88
Japan Restoration					11.64	4.67		
New Komeito	1.40	2.67			1.49	3.00	1.45	3.05
Your Party			0.87	0.67	4.71	1.33		
Tomorrow Party of Japan					5.02	0.67		
Social Democratic Party			1.95	1.00	0.76	0.33		
New Party DAICHI					0.53	0.33		
People's New Party	0.60	0.67	1.04	1.00				
New Party Nippon			0.31	0.33				
Japan Innovation Party							8.16	3.73
PPR	0.70	0.67						
Other parties	13.10	5.67	9.72	2.00	10.04	1.67	19.79	4.74

Note. In the case of the blank cells, the votes that the parties obtained in the elections without gaining any seats are included in 'other parties'.

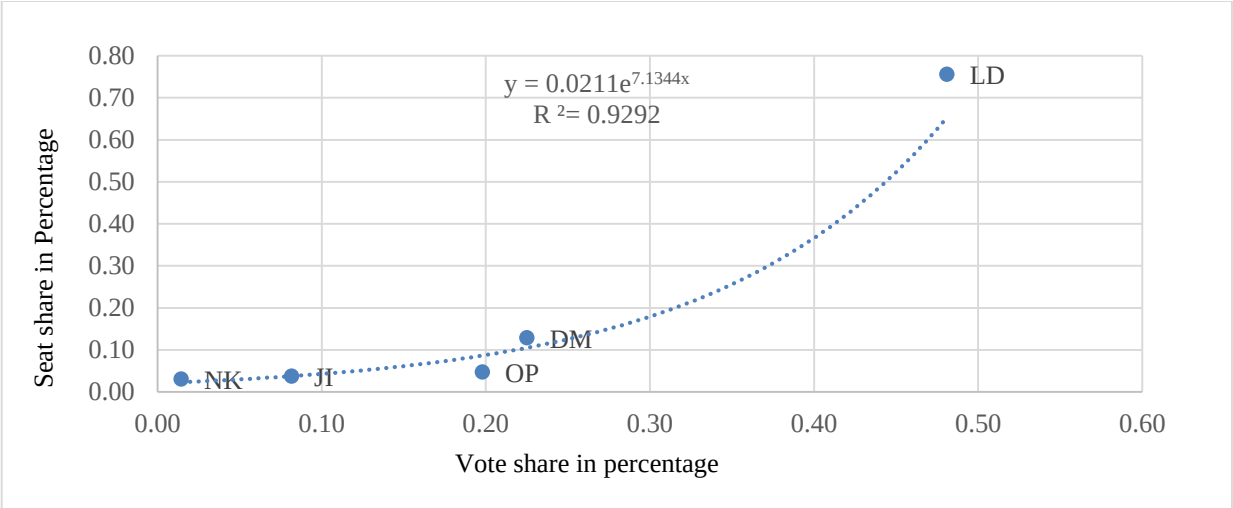
opposition parties, and even in some cases, the LDP has even suffered a defeat. Considering these circumstances, we can think of two different cases: firstly when only a single party dominates the election result, and when two parties dominate the election, that is, the victorious party faces severe challenges from the major opposition party.

We summarized the recent national elections results of the House of Councillors (HC) and the House of Representatives (HR) by vote share (VS) and seat share (SS), which are presented in Table 5 and Table 6.

(*b*) We apply two types of functions, the polynomial and the exponential, in order to find an appropriate function to express both approximately and accurately the relationship between vote share (v_i/v) and seat share (s_i/s). Firstly, we apply the polynomial function for the share of votes – share of seats relationship. The relationship is given in equation (4)

$$\frac{s_i}{s} = a\,(\frac{v_i}{v})^b \tag{4}$$

where s_i is the number of total seats gained by the i^{th} party, s is the total number of seats in the election, v_i is the total votes obtained by the i^{th} party, and v is the total votes cast in the elections. a and b are the parameters or any constant that needs to be estimated from the given election data. In these models, we exclude the intercept terms assuming that in case of zero votes obtained by a party, there should be no



Note: LD=Liberal Democratic Party, DM=Democratic Party, NK=New Komeito, JI=Japanese Innovation Party, and OP=Other Parties

Figure 2: Relation between vote share and seat share of HR, 2014

Table 7: Coefficients of regression between seat shares and vote shares in the cubic form

Dependent Variable: Seat Shares						
Regressors	Year 2009	Year 2012	Year 2014	Year 2007	Year 2010	Year 2013
Votes share	0.541	0.501	0.251	0.482	-1.285	0.431
p-value	0.002	0.015	0.487	0.161	0.003	0.073
t-value	4.408	2.938	0.734	1.544	-3.850	1.979
Vote Share(square)	-9.297	-4.376	-0.292	0.016	24.289	1.856
p-value	0.000	0.011	0.914	0.995	0.000	0.441
t-value	-14.391	-3.103	-0.112	0.006	7.959	0.799
Vote Share(cube)	24.106	17.393	6.323	5.287	-47.248	1.551
p-value	0.000	0.000	0.163	0.293	0.000	0.727
t-value	26.548	7.100	1.560	1.125	-7.640	0.359
R-Square	0.999	0.997	0.995	0.988	0.987	0.995

seat won by it. That implies that the curve passes through the origin. It is also reasonable to assume that the curve does not go beyond the horizontal axis to show a negative number of seats for any value of vote shares. We expect that the fitted line shows a smooth and increasing rise as the vote share increases providing a convex shape of the function.

We estimated the parameters of the second and third-degree polynomial models by using the Ordinary Least Square (OLS) estimation method. Although our sample sizes are very small, we find that for all cases, the goodness of fit for the mathematical model is very high such that R^2 is larger than 0.98. OLS estimators are the simplest and best representative for our models. Both the cubic and quadratic models have been estimated for six general elections. The parameter estimates are presented in Table 7 and Table 8 along with the respective p-values and t-values.

contradicts our assumptions. All of these curves do not show a gradual increase rather at some point with the increase of vote shares the graphs tend to decline for a while and then rise again. In case of the 2009 and 2010 elections, the decline is so severe that the graphs go beyond the horizontal axis, which gives a negative outcome for seat share as vote share increases. In the 2012 data though, the graph does not cross the horizontal axis, but the declining portion is easily noticeable.

On the other hand, plotting the graphs of the quadratic functions of the results of the 2014 House of Representatives election and the 2007 and 2013 House of Councillors elections, one can easily observe that the graphs show a gradual increase unlike the cubic function approximations. However, among these elections in 2007, the victorious party faces challenges from the main opposition party. Some researchers may find this similar to

Table 8: Coefficients of regression between seat shares and vote shares in the quadratic form

	Dependent Variable: Seat Shares					
Regressors	Year 2009	Year 2012	Year 2014	Year 2007	Year 2010	Year 2013
Vote Share	-1.998	-0.573	-0.245	0.208	0.692	0.359
p-value	0.017	0.010	0.113	0.321	0.214	0.001
t-value	-2.923	-3.108	-1.778	1.049	1.320	4.440
Vote Share(square)	7.215	5.524	3.765	2.760	1.418	2.685
p-value	0.001	0.000	0.000	0.001	0.358	0.000
t-value	4.639	11.274	11.711	5.014	0.960	12.660
R-Square	0.953	0.985	0.993	0.986	0.915	0.995

Table 7 shows that the year 2009 and 2012 House of Representatives elections and the 2010 House of Councilors election results support the cubic function of our model. The coefficient of the vote share cube is significant at the 1 percent level, and the p-values are much closer to zero. In the other three election results (2014, 2007, 2013), the coefficients of the square of vote share are highly significant. Table 8 shows that the p-value is almost zero for the vote share (square) in these election results.

Plotting the graphs of the cubic function for the results of the 2009 and, 2012 House of Representatives elections and the 2010 House of Councilors election, it is observed that the movement of the curve is unconventional, which

a single party dominated result as well because the victorious party's seat share is 55 percent, whereas the main opposition obtained only 32 percent of the seats. Except for this definitional difference in the 2007 election result, the other two results clearly indicate single party domination by the victorious party. Therefore, we cannot find enough evidence to support that in two-party dominated election results, quadratic functions provide a better approximation.

(c) Bearing the above analyses in mind we formulate another hypothesis regarding whether an exponential function of vote shares can approximate the seat shares of the contesting parties in an election. We use the same data set of Japanese recent general elections including both the

Table 9: Estimated parameters and R^2 of the exponential model for election results in all the House of Representatives and House of Councilors elections from 2005 to 2014

	Dependent Variable: log (Seat Share)						
	2005	2009	2012	2014	2007	2010	2013
log *p*	-4.52727	-5.05711	-5.06532	-3.85834	-4.29946	-4.42811	-3.71089
p value	0.000	0.000	0.000	0.001	0.000	0.023	0.000
t value	-13.07	-26.97	-15.85	-13.25	-31.77	-6.42	-15.99
p	0.0108	0.0064	0.0063	0.0211	0.0136	0.0119	0.0245
q	8.58	9.71	11.54	7.13	9.51	9.74	7.76
p value	0.003	0.000	0.000	0.008	0.000	0.060	0.002
t value	6.22	12.14	6.21	6.27	17.42	3.89	6.25
R-square	0.9063	0.9606	0.8462	0.9292	0.9063	0.9063	0.9063

House of Representatives and the House of Councilors. However, the approximation of an exponential function can violate one of our important assumptions that zero vote share yields zero seat shares. As the logarithm of zero is undefined, the dependent variable seat share always has to take some positive values. In many cases, contesting parties yield zero seats in an election though they may gain some vote share. In contrast with exponential functions, the polynomial functions provide realistic results when the party gets zero vote shares or zero seat shares in an election.

To overcome the problem in the approximation, we pooled all the parties acquiring zero seats and independent candidates as Other Parties. We pooled the parties in such a way that every party or group of parties in the Other Parties label has some positive seat shares in the election. In this paper, we approximated all the elections results beginning from the 2005 general election until the latest 2014 House of Representatives elections. We introduce a new model to approximate seat share by vote share which is given in equation (5).

$$\frac{s_i}{s} = \mathrm{p}\exp\left(q\frac{v_i}{v}\right) \qquad (5)$$

where s_i is the number of seats won by the i^{th} party, s is the total number of seats in the election, v_i is the total votes obtained by the i^{th} party and v is the total votes cast in the election. p and q are the constant parameters that need to be estimated from the election results.

From the above model, it is evident that if the i^{th} party obtains zero votes, it can still secure some positive seat share, which is not realistic. However, our estimation of the parameters gives some values for 'a' which can be approximated as zero. In case of polynomial functions without a constant intercept term, it can be explained that with a zero vote share a party acquires no seats, implying that the seat share is also zero. We find that for all, the goodness of fit for the mathematical model is very high such that R^2 is larger than 0.84. These impractical results from these exponential models are examined by estimating all the constant terms in the exponential equations for all the elections after 2005. Our study uncovers that all the constants are closed to zero or can be approximated by zero in real life phenomena.

Table 9 shows that the constant term (log a) is significant in all election years. In addition, the exponential coefficients (b) are also statistically significant at the 1 percent level except for the year 2010. However, at the 10 percent level, this coefficient is also significant. According to this table, the exponential function produces a statistically significant approximation for the election results. Moreover, the highest value for '*a*' has been estimated to be 0.02. In case the vote share equals to zero, we can approximate $0.02 \approx 0$.

Plotting the graphs for the exponential function of the vote share to approximate seat share, one can see that the graphs show a gradual and smooth increase of seat share with the increase of vote share. However, the increase in vote share results in a steep increase in the seat share as the vote share increases further. Furthermore, this increase is faster than that of the cubic or quadratic functions. Therefore, at the extreme ends when the vote shares approach higher than 50 percent, the seat share approaches to 1 more quickly than in the polynomial functions. However, Table 10 shows that in recent elections in Japan, the vote share of the winning party never crosses the 50 percent mark. It can be assumed that when there are many parties competing in an election, the winning party is very unlikely to achieve more than 50 percent of the total votes. Considering this phenomenon, the exponential function is a better approximation for Japan's multi-party elections.

In the exponential approximations, R^2s are found to be quite satisfactory implying that the fit is very good, although the R^2 obtained by quadratic or cubic functions are often very high. Table 9 shows that R^2s are larger than 85 percent, except in the 2014 election.

4.3 Pass votes and fail votes

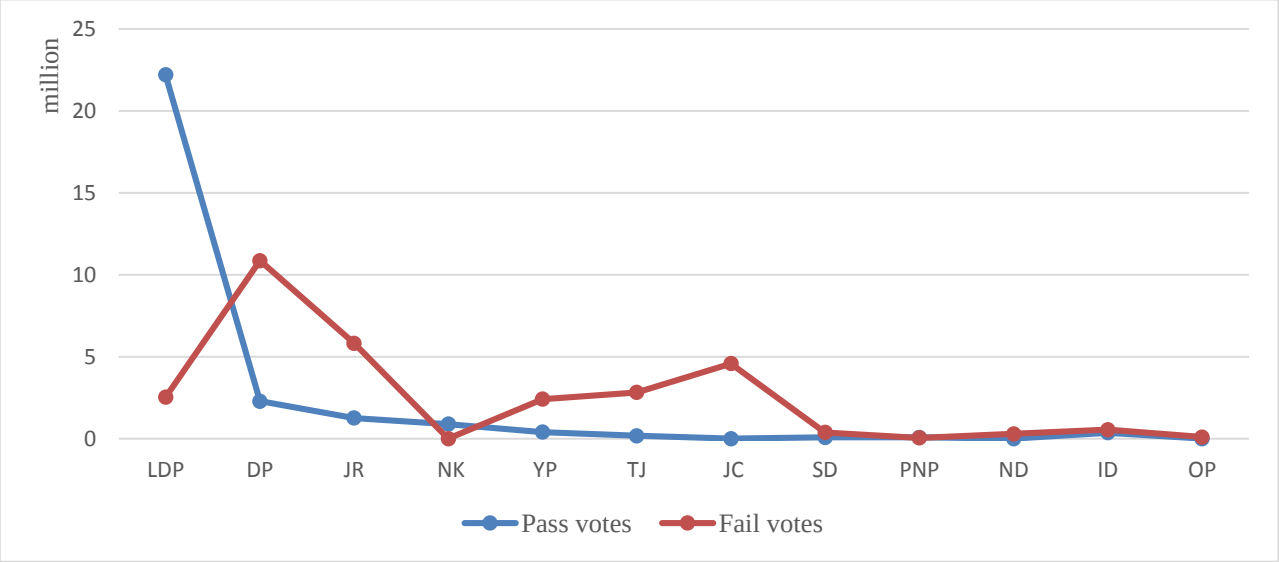
Despite the compatible (exponential) predictability of the plurality system, the proportional representation system has stronger (linear) predictability than the former. We test the predictability of the two systems by using "voting efficiency' or 'pass-votes and fail-votes ratio' test. This

Table 10: Vote shares and seat shares of the winning parties in recent general elections in Japan.

Year	Political party	Vote share	Seat share
2005	Liberal Democratic Party	0.48	0.73
2009	Democratic Party	0.47	0.74
2012	Liberal Democratic Party	0.43	0.79
2014	Liberal Democratic Party	0.48	0.76
2007	Liberal Democratic Party	0.31	0.32
2010	Democratic Party	0.39	0.38
2013	Liberal Democratic Party	0.43	0.64

section compares the voting efficiency of contesting parties in both types of elections.

Table 11 shows the pass votes and fail votes for the national elections of the House of Representatives and the House of



Note: LDP=Liberal Democratic Party, DP=Democratic Party, JR=Japan Restoration, NK=New Komeito, YP=Your Party, TJ=Tomorrow Party Japan, JC=Japan Communist Part, SD=Social Democratic Party, PNP=People's New Party, ND=New Party DAICHI, ID=Independent, OP=Other Parties

Figure 3: Relation between pass votes and fail votes by party HR, 2012

We define pass votes (PV) as votes which made the candidate pass or win the election and fail votes (FV) as those which could not make the candidate pass and thus are considered as wasted or dead votes. PV and FV are considered to be measures of "efficiency" and "inefficiency" respectively.

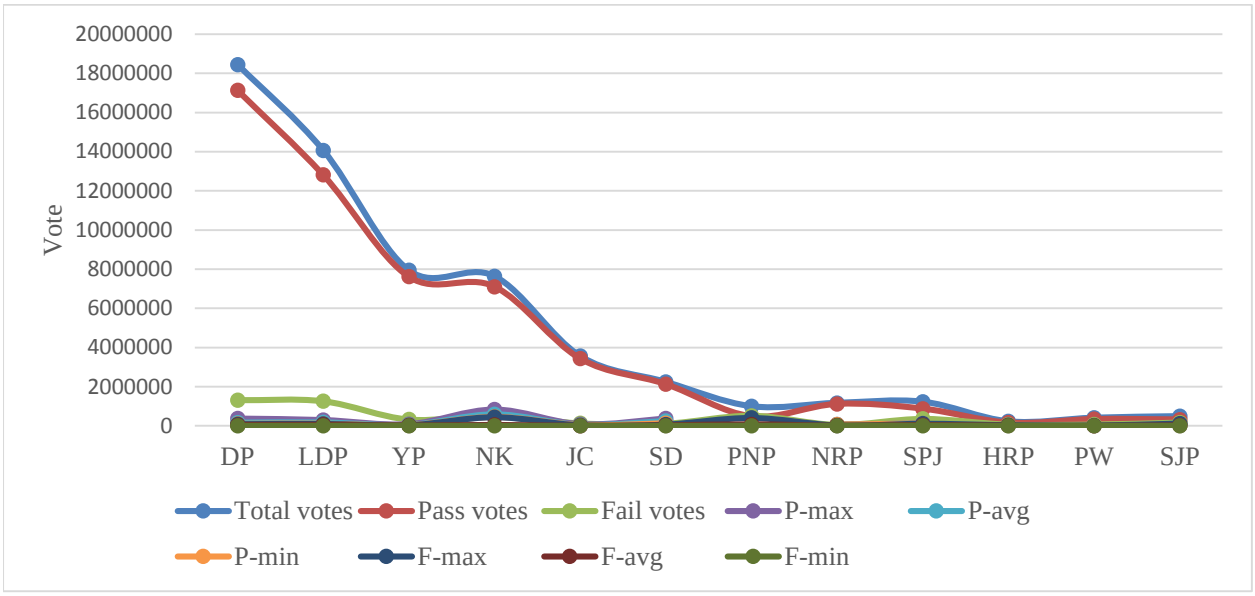
$$\text{PFR} = \frac{\text{Pass votes}}{\text{Fail votes}} = \frac{\text{PV}}{\text{FV}}$$

The pass fail ratio (PFR) for each political party i can be defined as follows:

$$\text{PFR}_i = \frac{\text{Pass votes for party}_\text{i}}{\text{Total votes for party}_i} = \frac{\text{PV}_i}{\text{TV}_i}$$

Councillors from the year 2007 to 2014. It is clearly shown that the pass votes are slightly greater (more than 50%) than the fail votes, except in the 2012 election, where the fail votes are greater than the pass votes. The House of Councilors elections have more pass votes compared to the election for the House of Representatives.

Table 12 shows the party-wise voting efficiency obtained by the contesting parties in the 2012 House of Representatives election. The New Komeito party (NK) is reported as 100 percent efficient among all the parties, gaining all the seats they contested for. NK is followed by the Liberal Democratic Party, which has usually been in electoral alliance with NK. Compared to the overall efficiency, only three parties gained more pass votes than fail votes in this election. The third party that obtained more pass votes is the People's New Party. However, the main



Note: DP=Democratic Party, LDP=Liberal Democratic Party, YP=Your Party, NK=New Komeito, JC=Japan Communist Part, SD=Social Democratic Party, PNP=People's New Party, NRP= New Renaissance Party, SPJ=Sunrise Party Japan, HRP=Happiness Realization Party, PW=Party for Woman, SJP=Sunrise Party Japan, P-Max=Pass votes Maximum, P-avg=Pass vote average, P-min=Pass vote minimum, F-max=Fail votes maximum, F-avg=Fail votes average, F-min=Fail votes minimum

Figure 4: Relation between pass votes and fail votes by party, HC (proportional), 2010

opposition party in this election managed to obtain only 17 percent pass votes which is clearly indicated in the election result.

votes exist in the proportional system, and that is the highest for the LDP and the Democratic Party.

Table 12 shows that under the plurality system of election

Table 11: Pass votes and fail vote's percentage of national elections

Types of election	Year	PV	FV
House of Representatives	2009	52	48
	2012	48	52
	2014	51	49
House of Councilors	2007	66	34
	2010	60	40
	2013	66	34

Figure 3 shows the party efficiency for the election in 2012, where the New Komeito party had the most voting efficiency (100%), followed by the Liberal Democratic Party (90%). This may be because the New Komeito Party is more organized and developed a coalition with the Liberal Democratic Party. That is why the New Komeito had no fail or dead votes. Having more pass votes reflects that a party has more efficient votes, whereas having more fail votes reflects less efficiency. Thus, the efficiency of a political party depends on the ratio of its pass votes and total votes. A greater share of pass votes for a political party tends to more efficiency of a political party.

Figure 4 shows that in the proportional system, the Japanese Communist Party (96.32) and Your Party (95.91) have more efficient proportional votes compare to the top two political parties, the Democratic Party (92.89) and the LDP (91.07). The New Komeito Party obtained maximum individual fail votes, and LDP got the highest average pass votes in the proportional system. Pass votes seem to be relatively rational compared to the total votes but still fail only two parties have 90 percent or more voting efficiency. On the other hand under the proportional representation system almost all the parties have more than 90 percent voting efficiency, which leads us to infer that proportional representation has a stronger predictability which is linear.

5 Summary and Conclusion

5.1 Summary of Results

The study shows that the pattern of voter turnouts largely depends on the regional characteristics of the prefectures in Japan. The most urbanized prefectures tend to have lower voter turnouts than that of less urbanized prefectures. A cluster analysis shows that as the degree of urbanization increases the voter turnouts in elections declines gradually. The decrease in voter turnouts is attributed to the fact that peoples in less urbanized prefectures are more conservative and are more interested in politics and elections than those in other region. Therefore, introduction of the new system

Table 12: Measuring efficiency for party, HR, 2012

Party	Pass vote	Fail vote	Total	Efficiency
Liberal Democratic Party	22192427	2529702	24722129	89.77
Democratic Party	2277029	10857966	13134995	17.34
Japan Restoration	1255463	5816773	7072236	17.75
New Komeito	885881	0	885881	100.00
Your Party	400171	2406283	2806454	14.26
Tomorrow Party of Japan	169135	2818021	2987156	5.66
Japanese Communist Party	0	4575040	4575040	0.00
Social Democratic Party	73498	378264	451762	16.27
People's New Party	70320	46865	117185	60.01
New Party DAICHI	0	289826	289826	0.00
Other Parties	0	95139	95139	0.00
Independent	359546	539333	898879	40.00

of election has failed to bring in more voting turnout compared to the existing plurality election system, because it is a mere regional effect rather than an effect of new election system.

To test the predictability of plurality voting over proportional representation we approximate polynomial and exponential functions to the result of recent elections. Analyzing these recent election results, it can be determined that an exponential model approximates seat-shares better than any polynomial approximation as a function of vote shares obtained by the parties. Unlike polynomial functions, the exponential model has several limitations. However, in real life scenarios we can overcome those limitations by approximating realistic parameter values.

Despite a compatible predictive power of the plurality system of the election (through exponential approximation), the predictability power of proportional representation is stronger (because of the linear approximation). By using the voting efficiency test or pass votes and fail votes ratio (PFR) test, we can establish this claim. In 2012 House of Representative election, under the plurality system only two parties (the winning coalition) received 90 percent or more efficient votes. On the contrary in the same election, under proportional representation, almost all the parties received 90 percent or more effective votes. So the linear approximation of proportional representation yields higher predictability than that of the plurality system.

5.2 Scope of further studies

In this paper, to formulate the predictive equation for plurality system, we fit the exponential model only to recent Japanese general election results. However, it is quite evident that there is scope to generalize this model to other democratic elections of other countries across the world. This study indicates that the exponential model can be used to predict seat shares for the contesting parties in an election where many parties take part in the election process. In addition, this model is applicable to single seats electoral districts. For democracies like the US, where only two parties contest the election, this model, may not work well.

Moreover, as the sample size is usually small in these cases due to the limited number of contesting parties in an election, further statistical manipulation is needed to improve prediction. In our future work, we will attempt to develop better estimation methods allowing for more accurate prediction.

5.3 Conclusion

This paper has shown that plurality system of elections has the compatible predictability of seat-shares as a function of vote shares. By dint of this predictability, one can claim that plurality system of voting to a large extent has considerable degree of representation of people's choice in the election results, which can be explained by a mathematical model (in this case exponential model). Nonetheless, the linear predictability (also a mathematical approximation) of proportional representation system of election has stronger predictive power over the plurality system of election. Despite the stronger predictive power, in practice proportional representation system deviate from the ideal proportional system of representation. As a result, we conclude both the systems should be regarded as complementary rather substitute for each other. The new Japanese election system that comprises both the systems ensures improved representation of popular choices in the election results.

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