A MODEL AND METHOD FOR SUPPLY CHAIN MANAGEMENT PROBLEMS

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Abstract

In this paper, a mathematical model on supply chain management problem is given. To present optimal decision for the problem, we propose a new type of algorithm, and the global convergence of algorithm is also established under milder conditions. Furthermore, we prove that the method has R-linear convergence rate with the underlying mapping being P-uniform function and Lipschitz continuous.

1 Introduction

With the development of computer networks, the globalization and networking process for manufacturing have been gaining rapid development. With the emergence of new manufacturing models such as Hypothesized Manufacture Dynamic Alliance, the new management models which adapt to that are more urgent. The supply chain management concept appears under this new environment of competition. In recent years, both in the application and academic research, supply chain management has been becoming a hot topic of modern logistics research, and so provoked strong research interest of many scholars. The research area involves the model, analysis and computation for supply chain management, related to manufacturing, transportation, logistics, retail, and sales([1, 2, 3]). Nagurney et al. ([4]) gave variational inequality models of supply chain network equilibrium consisting of three tiers of decision-makers on the network, and then established that the governing equilibrium conditions which reflected the optimality conditions of the decision-makers consisting of manufacturers, retailers, and consumers along with the market equilibrium conditions. Dong et al. ([5]) established a finite dimensional variational inequality supply chain network model which contained the manufacturers, demand suppliers and retailers. In 2005, Nagurney et al.([6]) established a supply chain network model of finite dimensional variation inequality which contained the supply side and demand side. Zhang ([7]) gave a non-linear complementary model for supply chain network equilibrium. Although the models of supply chain management under certain conditions were given in the above-mentioned documents, but the conditions of models tenable and algorithms convergence were too harsh. To this end, we reformulate the supply chain management problem as a nonlinear complementarity model in the paper. To solve this model, many effective methods have been proposed for solving it ([8, 9, 10, 11]). The basic idea of these methods is to reformulate the problem as an unconstrained or simply constrained optimization problem (see [8, 9, 11]), and use the Newton-type or the trust region algorithms to solve it. Although the corresponding convergence and convergence rate were established, but, most of these algorithms require the Jacobian matrix is nonsingular at the solutions or require there exists strict complementary solution, and require the mapping function is monotonic at the same time.This motivates us to consider a new method for solving this model under weaker conditions.

This paper is organized as follows. In Section 2, we give the supply chain management problem and the nonlinear complementarity problem model. A new type of algorithm for solving this model is proposed in Section 3. In Section 4, we establish the global convergence of the algorithm under milder conditions, and also prove that the method has R-linear convergence rate with the underlying mapping being P-uniform function and Lipschitz continuous. The paper concludes with Section 5, in which we summarize our results and present suggestions for future research.

2 The Supply Chain Management Problem and Model

In this section, we give the supply chain management problem and a nonlinear complementarity model. We first consider a two-story supply chain problem with m manufacturers and n vendors (see Figure 1):

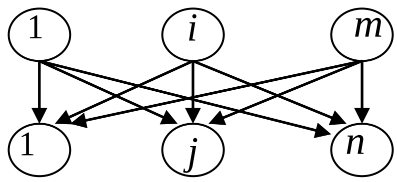

Figure 1

Let q_i denote the amount of production of manufacturer i ($i=1,\cdots,m.$), Denote $q:=(q_1,\cdots,q_m)^T\in R_+^m$. Suppose that the production costs of manufacturer i is f_i, the amount of goods of manufacturer i supply to vendor j is q_{ij}, the transaction costs between them is c_{ij}, where c_{ij} including transportation costs. Denote

$$Q := (q_{ij}, i = 1, \cdots, m, j = 1, \cdots, n) \in R_+^{mn}.$$

Assume the cost of production f_i is a function of Q, transaction cost c_{ij} is a function of q_{ij}, and

$$q_i = \sum_{j=1}^{n} q_{ij}, \quad i = 1, \cdots, m.$$

The total income of manufacturer i is the commodity price multiplied by the amount of commodity sold, the total expenditure is the sum of production cost and transaction cost. Let ρ_{ij}^* denote the price of the goods of manufacturer i sell to vendor j, then, the revenue maximization problem of manufacturer i can be expressed as

$$(PM_i) \quad \max\{\sum_{j=1}^{n} \rho_{ij}^* q_{ij} - f_i(Q) - \sum_{j=1}^{n} c_{ij}(q_{ij})\}$$

$$s.t \quad q_{ij} \geq 0, j = 1, \cdots, n.$$

Suppose that the competitions of manufacturers are non-cooperative, the production cost functions f_i and the transaction cost functions c_{ij} are continuously differentiable convex functions, then, the optimization problem (PM_i) is a convex programming. Assume that $Q^* = (q_{ij}^*, \forall i, j)$ is the optimal solution of (PM_i), $i = 1, \cdots, m$, then, Q^* is a " equilibrium solution" of manufacturer, and Q^* satisfies the following KKT condition:

$$\begin{cases} \nabla f_i(Q^*) + \sum_{j=1}^{n} \nabla c_{ij}(q_{ij}^*) - \sum_{j=1}^{n} \rho_{ij}^* e_j = (\lambda_{i1}, \cdots, \lambda_{in})^T \geq 0 \\ q_{ij}^* \geq 0, \lambda_{ij} q_{ij}^* = 0, j = 1, \cdots, n \end{cases} \quad (1)$$

where $\lambda_{ij}, j = 1, \cdots, n$ is Lagrange multiplier, e_j is unit vector, its j-th component is 1, and the other components are 0. Formula (1) is equivalent to

$$\begin{cases} \frac{\partial f_i(Q^*)}{\partial q_j} + \frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}} - \rho_{ij}^* \geq 0, q_{ij}^* \geq 0, j = 1, \cdots, n, \\ (\frac{\partial f_i(Q^*)}{\partial q_j} + \frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}} - \rho_{ij}^*) q_{ij}^* = 0, j = 1, \cdots, n. \end{cases} \quad (2)$$

Let $f : R^{nm} \to R^{nm}$, and

$$f(Q) = (f_{ij}(Q), i = 1, \cdots, m, j = 1, \cdots, n),$$

where

$$f_{ij}(Q) := \frac{\partial f_i(Q^*)}{\partial q_j} + \frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}} - \rho_{ij}^*, \forall i, j. \quad (3)$$

For the convenience of the following description, the (2) is abbreviated as

$$f(Q) \geq 0, Q \geq 0, Q^T f(Q) = 0. \quad (4)$$

We use X^* to denotes the solution of (4), and assume that X^* is not empty.

3 Algorithm

In this section, we would establish a new-type method to solve (4), we first need the definition of projection operator and some relate properties ([12]).

Definition 1. Assume that $Z \subset R^n$ is a non-empty closed convex set, for any $x \in R^n$, the orthogonal projection of x onto Z, i.e., $\arg\min\{\| x - y \| | y \in Z\}$, is denoted by $P_Z(x)$.

Lemma 1. For $u \in R^n, v \in Z$, we have

(i) $\langle P_z(u) - u, v - P_z(u) \rangle \geq 0$,

(ii) $\| P_z(u) - P_z(v) \| \leq \| u - v \|$.

For (4), we call

$$R(Q, \rho) = \min\{Q, \rho f(Q)\} = Q - P_{R_+^{mn}}\{Q - \rho f(Q)\}$$

is the projection residual, where ρ is positive constant. There is an important relationship between the projection residual and the solution of (4)([13]).

Lemma 2. The Q^* is a solution of (4) if and only if $R(Q^*, \rho) = 0$, where ρ is positive constant.

Now, we give a new algorithm for solving (4) .

Algorithm 1

Step 1. Give $Q^0 \in R_+^{mn}$, let $0 < \sigma < 1$, $\rho_{-1} = 1$, $0 < \varphi < 2$, $k := 0$.

Step 2. For $Q^k \in R_+^{mn}$, letting

$$Q^{k-1}(\rho_{k-1}) = P_{R_+^{mn}}\{Q^k - \rho_{k-1} f(Q^k)\}.$$

If $R(x^k, \rho_{k-1}) = Q^k - Q^{k-1}(\rho_{k-1}) = 0$, then, stop. Otherwise, letting $\rho_k = \gamma^{m_k}$, where m_k is the smallest non-negative integer m which satisfies the following inequality

$$\rho_k \| f(Q^k) - f(Q^k(\rho_k)) \| \leq \sigma \| R(Q^k, \rho_k) \|, \quad (5)$$

where

$$Q^k(\rho_k) = P_{R_+^{mn}}\{Q^k - \rho_k f(Q^k)\}. \quad (6)$$

Step 3. Let $Q^{k+1} = P_{R_+^{mn}}[Q^k + \varphi \alpha_k d_k]$, go to step 2, where

$$d_k = -\{R(Q^k, \rho_k) - \rho_k(f(Q^k) - f(Q^k(\rho_k)))\},$$

$$\alpha_k = (1 - \sigma) \| R(Q^k, \rho_k) \|^2 / \| d_k \|^2.$$

Remark 1. The search direction given by Algorithm 1 is different from the search direction given by Noor([15]).

$$-\{\eta_k R(u^k, \rho) + \eta_k T(u^k) + \rho T(v^k)\}$$

In this following, the rationality of step rule (5) is given.

Proposition 1. Assume that $f(Q)$ is continuously differentiable, if Q^k is not the solution of (4), then, for any $\sigma \in (0,1)$, there exists $\hat{\rho}(Q^k) \in (0,1]$, such that, for any $\rho \in (0, \hat{\rho}(Q^k)]$, we have

$$\rho \| f(Q^k) - f(Q^k(\rho)) \| \leq \sigma \| R(Q^k, \rho) \|,$$

where $Q^k \in R_+^{mn}$, $Q^k(\rho)$ were defined by (6).

Proof. Assume that the conclusion is false. Then, there exists $\sigma \in (0,1)$, for any $0 < \hat{\rho} \leq 1$, there exists $0 < \rho \leq \hat{\rho}$ such that

$$\rho \| f(Q^k) - f(Q^k(\rho)) \| > \sigma \| R(Q^k, \rho) \|. \quad (7)$$

Let $\hat{\rho}$ converges to 0. Then, one has ρ converges to 0. For any $\varepsilon > 0$, let $\delta = \varepsilon$, if

$$\|Q^k - Q^k(\rho)\| = \|Q^k - P_{R_+^{mn}}\{Q^k - \rho f(Q^k)\}\|$$
$$= \|\min\{Q^k, \rho f(Q^k)\}\| \le \delta,$$

then

$$\begin{aligned} &\| f(Q^k) - f(Q^k(\rho))\| \\ &= \| f(Q^k) - f(P_{R_+^{mn}}\{Q^k - \rho f(Q^k)\})\| \\ &\le c_1 \| Q^k - P_{R_+^{mn}}\{Q^k - \rho f(Q^k)\}\| \\ &= c_1 \| \min\{Q^k, \rho f(Q^k)\}\| \le c_1 \varepsilon, \end{aligned} \quad (8)$$

where the first inequality is based on that $f(Q)$ is continuously differentiable on the closed interval $[Q^k, Q^k(\hat{\rho}(Q^k))]$, $c_1 > 0$ is a constant, the last inequality is based on the continuity of $f(Q)$ at the point Q^k. By (7) and (8), we have

$$\sigma \| R(Q^k, \rho)\| < \rho \| f(Q^k) - f(Q^k(\rho))\| \le \rho c_1 \varepsilon.$$

Combining this with lemma 2, we know that $Q^k \in X^*$, this is contradictory with that Q^k is not the solution of (4).

Remark 2. From the proof of Proposition 1, it is easy to see that the conclusion still holds if we replace the condition " $f(Q)$ is continuously differentiable" by " $f(Q)$ is locally Lipschitz continuous".

4 Global Convergence Results of Algorithm

In this section, we would set up the global convergence and global linear convergence rate of the algorithm 1 under milder conditions. To this end, we need the following assumption and some technical lemmas for our subsequent analysis.

Assumption 1. For the function $f(Q)$ defined in (4), we assume that

$$\left\langle f(Q), Q - Q^* \right\rangle \ge 0, \forall Q \ge 0,$$

where $Q^* \in X^*$.

Obviously, if the function $f(Q)$ is pseudo-monotone, then, $f(Q)$ satisfies Assumption 1. Thus, Assumption 1 is weaker than the pseudo-monotone condition, is weaker than the strong pseudo-monotone, and is also weaker than monotonous ([14]).

Lemma 3. Assume that Assumption 1 holds and $f(Q)$ is continuously differentiable. Then, for any $Q^* \in X^*$, we have $\langle Q^k - Q^*, -d_k\rangle \ge (1-\sigma)\| R(Q^k, \rho_k)\|^2$.

Proof. According to the iterative process of algorithm 1, for any positive integer, we have $Q^k, Q^k(\rho_k) \in R_+^{mn}$. By $Q^* \in X^*$ and Lemma 1, we have

$$\left\langle [Q^k - \rho_k f(Q^k)] - Q^k(\rho_k),\ Q^k(\rho_k) - Q^* \right\rangle \ge 0.$$

Combines with the definition of $R(Q,\rho)$ again, we have

$$\langle R(Q^k, \rho_k) - \rho_k f(Q^k), Q^k - Q^* - R(Q^k, \rho_k)\rangle \ge 0,$$

thus, one has

$$\langle R(Q^k, \rho_k), Q^k - Q^*\rangle - \| R(Q^k, \rho_k)\|^2$$
$$-\langle \rho_k f(Q^k), Q^k - Q^*\rangle + \langle \rho_k f(Q^k), R(Q^k, \rho_k)\rangle \ge 0.$$

That is,

$$\left\langle R(Q^k, \rho_k) - \rho_k f(Q^k),\ Q^k - Q^* \right\rangle$$
$$\ge \| R(Q^k,\ \rho_k)\|^2 - \left\langle \rho_k f(Q^k),\ R(Q^k, \rho_k)\right\rangle \quad (9)$$

In addition, according to Assumption 1, we have

$$\langle f(Q^k(\rho_k)), Q^k(\rho_k) - Q^*\rangle \ge 0, \quad (10)$$

Combining this with the definition of $Q^k(\rho_k)$ in algorithm 1, we get $Q^k(\rho_k) = Q^k - R(Q^k, \rho_k)$. From (10), we have

$$0 \le \langle Q^k(\rho_k) - Q^*, f(Q^k(\rho_k))\rangle = \langle Q^k - R(Q^k, \rho_k) - Q^*, f(Q^k(\rho_k))\rangle$$
$$= \langle Q^k - Q^*, f(Q^k(\rho_k))\rangle - \langle R(Q^k, \rho_k), f(Q^k(\rho_k))\rangle,$$

that is

$$\langle Q^k - Q^*, f(Q^k(\rho_k))\rangle \ge \langle R(Q^k, \rho_k), f(Q^k(\rho_k))\rangle. \quad (11)$$

By the definition of d_k in Algorithm 1, we get

$$\begin{aligned} &\langle Q^k - Q^*, -d_k\rangle \\ &= \langle Q^k - Q^*, R(Q^k, \rho_k) + \rho_k\{ f(Q^k(\rho_k)) - f(Q^k)\}\rangle \\ &= \langle Q^k - Q^*, R(Q^k, \rho_k) - \rho_k f(Q^k)\rangle + \langle Q^k - Q^*, \rho_k f(Q^k(\rho_k))\rangle \\ &\ge \| R(Q^k, \rho_k)\|^2 - \langle \rho_k f(Q^k), R(Q^k, \rho_k)\rangle \\ &\quad + \langle \rho_k R(Q^k, \rho_k), f(Q^k(\rho_k))\rangle \\ &= \| R(Q^k, \rho_k)\|^2 - \rho_k \langle f(Q^k) - f(Q^k(\rho_k)), R(Q^k, \rho_k)\rangle \\ &\ge \| R(Q^k, \rho_k)\|^2 - \rho_k \| f(Q^k) - f(Q^k(\rho_k))\| \ \| R(Q^k, \rho_k)\| \\ &\ge (1-\sigma)\| R(Q^k, \rho_k)\|^2. \end{aligned}$$

where the first inequality is based on (9) and (11), and the second inequality is based on the Cauchy-Schwarz inequality, and the third inequality is based on (5).

Lemma 4. Assume that Assumption 1 holds and $f(Q)$ is continuously differentiable. Then, the sequence $\{\alpha_k\}$ and $\{\rho_k\}$ which defined by the Algorithm 1 have uniformly positive lower bounds, respectively.

Proof. Firstly, we prove $\{\alpha_k\}$ have uniformly positive lower bound.

By the expressions of d_k defined in Algorithm 1 and (5), we obtain

$$\| d_k\|^2 \le 2\| R(Q^k, \rho_k)\|^2 + 2\rho_k^2 \| f(Q^k) - f(Q^k(\rho_k))\|^2$$
$$\le 2(1+\sigma^2)\| R(Q^k, \rho_k)\|^2.$$

From the expressions of α_k defined in Algorithm 2.1, we have

$$\alpha_k = (1-\sigma)\| R(Q^k, \rho_k)\|^2 / \| d_k\|^2 \ge \frac{1-\sigma}{2(1+\sigma^2)}. \quad (12)$$

Secondly, we prove ρ_k have uniformly positive lower bound.

Using the step rule of Algorithm 1, we get

$$\sigma \| R(Q^k, \rho)\| < \rho \| f(Q^k) - f(Q^k(\rho))\|$$
$$\le \rho c_2 \| Q^k - Q^k(\rho)\| = \rho c_2 \| R(x^k, \rho)\|,$$

where the second inequality is based on that $f(Q)$ is continuously differentiable in closed interval $[Q^k, Q^k(1))]$, $c_2 > 0$ is a constant, that is, $\rho > \sigma / c_2$. Thus,

$$\rho_k \ge \min\{1, \sigma / c_2\}.$$

Lemma 5. Assume that Assumption 1 holds and $f(Q)$ is continuously differentiable. Then, the sequence $\{Q^k\}$ which defined by the Algorithm 1 is bounded.

Proof. Suppose that $Q^* \in X^*$, then

$$
\begin{aligned}
&\| Q^{k+1} - Q^*\|^2 \\
&= \| P_K[Q^k + \varphi\alpha_k d_k] - P_K[Q^*]\|^2 \\
&\le \| Q^k - Q^* + \varphi\alpha_k d_k\|^2 \\
&= \| Q^k - Q^*\|^2 + 2\varphi\alpha_k (Q^k - Q^*)^{\cdot} d_k + \varphi^2\alpha_k^2\| d_k\|^2 \\
&\le \| Q^k - Q^*\|^2 - 2\varphi\alpha_k (1-\sigma)\| R(Q^k,\rho_k)\|^2 + \varphi^2\alpha_k^2\| d_k\|^2 \\
&= \| Q^k - Q^*\|^2 - 2\varphi\alpha_k (1-\sigma)\| R(Q^k,\rho_k)\|^2 \\
&\quad + \varphi^2\alpha_k (1-\sigma)\| R(Q^k,\rho_k)\|^2 \\
&= \| Q^k - Q^*\|^2 - \varphi\alpha_k [2(1-\sigma) - \varphi(1-\sigma)]\| R(Q^k,\rho_k)\|^2 \\
&\le \| Q^k - Q^*\|^2 - \frac{(1-\sigma)^2}{2(1+\sigma^2)}\varphi(2-\varphi)\| R(Q^k,\rho_k)\|^2.
\end{aligned}
\tag{13}
$$

where the second inequality is based on Lemma 3, the third equality is based on the expression of α_k defined by Algorithm 1, and the last inequality is based on (12). Combining (13) and the definitions of φ,σ in the Algorithm 1, we obtain that the sequence $\{\| Q^k - Q^*\|\}$ is non-negative decreasing. So, $\{\| Q^k - Q^*\|\}$ is bounded, and then, $\{Q^k\}$ is bounded.

Theorem 1. Assume that Assumption 1 holds and $f(Q)$ is continuously differentiable. Then, the sequence $\{Q^k\}$ which defined by the algorithm 1 is global convergence to the solution of (4).

Proof. By (13), we have $\{\| Q^k - Q^*\|\}$ is non-negative decreasing. Thus, $\{\| Q^k - Q^*\|\}$ is convergence. Combining this with (13), one has

$$\sum_{k=0}^{\infty}\|R(Q^k,\rho_k)\|^2 \le \infty,$$

that is, $\lim_{k\to\infty}\| R(Q^k,\rho_k)\| = 0.$ Therefore, every accumulation point $\overline{Q}$ of $\{Q^k\}$ is the solution of (4). Letting $Q^* = \overline{Q}$ in (13), then $\{\| Q^k - \overline{Q}\|\}$ is convergence, and $\{Q^k\}$ is global convergence to $\overline{Q}$.

In order to establish the global linear convergence rate of Algorithm 1, we give the following definition and lemmas which is easy to prove.

Definition 2. We call $f(Q)$ is $P-$ uniform function, if for any $u,v \in R^{mn}$, $\exists\beta > 0$, such that

$$\max_{1\le i\le mn}\{[f(u) - f(v)]_i[u-v]_i\} \ge \beta\| u - v\|^2.$$

Lemma 6. Suppose $a,b\in R, a^*,b^*\in R_+$. If $a^*b^* = 0$, then,

$$(a - a^* - \min\{a,b\})(b - b^* - \min\{a,b\}) \le 0.$$

Lemma7. If $f(Q)$ is $P-$ uniform function and Lipschitz continuous, then, for $Q^k \in R_+^{\ mn}$ and $\rho_k > 0$, we have $\| Q^k - Q^*\| \le (\beta\rho_k)^{-1}(L_1 + \rho_k L_2) R(Q^k,\rho_k).$

Proof. By Lemma 6, for any $Q^k \in R_+^{mn},\quad Q^* \in X^*$, we get

$$
\begin{aligned}
&0 \ge [Q_i^k - Q_i^* - \min\{Q_i^k, \rho_k f_i(Q^k)\}] \\
&\quad \cdot[\rho_k f_i(Q^k) - \rho_k f_i(Q^*) - \min\{Q_i^k, \rho_k f_i(Q^k)\}] \\
&\quad = \rho_k (Q_i^k - Q_i^*)(f_i(Q^k) - f_i(Q^*)) + [\min\{Q_i^k,\rho_k f_i(Q^k)\}]^2 \\
&\quad\quad - \min\{Q_i^k,\rho_k f_i(Q^k)\}[(Q_i^k - Q_i^*) + \rho_k (f_i(Q^k) - f_i(Q^*))] \\
&\quad \ge \rho_k (Q_i^k - Q_i^*)(f_i(Q^k) - f_i(Q^*)) \\
&\quad\quad - \min\{Q_i^k,\rho_k f_i(Q^k)\}[(Q_i^k - Q_i^*) + \rho_k (f_i(Q^k) - f_i(Q^*))],
\end{aligned}
$$

that is,

$$
\begin{aligned}
&\rho_k^{-1}\min\{Q_i^k,\rho_k f_i(Q^k)\}[(Q_i^k - Q_i^*) + \rho_k (f_i(Q^k) - f_i(Q^*))] \\
&\quad \ge (Q_i^k - Q_i^*)(f_i(Q^k) - f_i(Q^*)).
\end{aligned}
$$

Due to that $f(Q)$ is $P-$ uniform function and Lipschitz continuous, we have

$$
\begin{aligned}
&\beta\| Q^k - Q^*\|^2 \\
&\le \max_{1\le i\le mn}(Q_i^k - Q_i^*)(f_i(Q) - f_i(Q^*)) \\
&\le \rho_k^{-1}\|\min\{Q^k,\rho_k f(Q^k)\}\|[L_1\| Q^k - Q^*\| + \rho_k\| f(Q^k) - f(Q^*)\|] \\
&\le \rho_k^{-1}\|\min\{Q^k,\rho_k f(Q^k)\}\|[L_1\| Q^k - Q^*\| + \rho_k L_2\| Q^k - Q^*\|] \\
&= \rho_k^{-1}(L_1 + \rho_k L_2)\ \|\min\{Q^k,\rho_k f(Q^k)\}\|\ \ \| Q^k - Q^*\|.
\end{aligned}
$$

That is,

$$
\begin{aligned}
\| Q^k - Q^*\| &\le (\beta\rho_k)^{-1}(L_1 + \rho_k L_2)\ \|\min\{Q^k,\rho_k f(Q^k)\}\| \\
&= (\beta\rho_k)^{-1}(L_1 + \rho_k L_2) R(Q^k,\rho_k).
\end{aligned}
$$

Lemma 8. Suppose that $f(Q)$ is $P-$ uniform function and Lipschitz continuous. If (4) has the solution. Then, the solution is unique.

Proof. Suppose that Q_1^*, Q_2^* is any two solutions of (4), then, there exists $\beta > 0$, such that

$$
\begin{aligned}
&0 \le \beta\| Q_1^* - Q_2^*\|^2 \\
&\le \max_{1\le i\le mn}\{[(Q_1^*)_i - (Q_2^*)_i][f_i(Q_1^*) - f_i(Q_2^*)]\} \\
&= \max_{1\le i\le mn}\{(Q_1^*)_i f_i(Q_1^*) - (Q_1^*)_i f_i(Q_2^*) - (Q_2^*)_i f_i(Q_1^*) + (Q_2^*)_i f_i(Q_2^*)\} \\
&= -\min_{1\le i\le mn}\{(Q_1^*)_i f_i(Q_2^*) + (Q_2^*)_i f_i(Q_1^*)\} \le 0.
\end{aligned}
$$

Thus, the solution of (4) is unique.

Theorem 2. If Assumption 1 holds, and $f(Q)$ is $P-$ uniform function and Lipschitz continuous. Then, sequence $\{Q^k\}$ generated by the Algorithm 1 is global $R-$ linear convergence to the solution of (4).

Proof. By (13) and Lemmas 7-8, we get

$$
\begin{aligned}
&\| Q^{k+1} - Q^*\|^2 \\
&\le \| Q^k - Q^*\|^2 - \frac{(1-\sigma)^2}{2(1+\sigma^2)}\varphi(2-\varphi)\| R(Q^k,\rho_k)\|^2 \\
&\le \| Q^k - Q^*\|^2 - \frac{(1-\sigma)^2}{2(1+\sigma^2)}\varphi(2-\varphi)(\beta\rho_k)(L_1 + \rho_k L_2)^{-1}\| Q^k - Q^*\|^2 \\
&\le \left\{1 - \frac{(1-\sigma)^2}{2(1+\sigma^2)}\varphi(2-\varphi)(\beta\rho_k)(L_1 + \rho_k L_2)^{-1}\right\}\| Q^k - Q^*\|^2 \\
&\le \left\{1 - \frac{(1-\sigma)^2}{2(1+\sigma^2)}\varphi(2-\varphi)(\beta\min\{1,\sigma/c_2\})(L_1 + L_2)^{-1}\right\}\| Q^k - Q^*\|^2 .
\end{aligned}
$$

That is,

$$
\begin{aligned}
\| Q^{k+1} - Q^*\| &\le \sqrt{1-\lambda}\| Q^k - Q^*\| \\
&\le \left(\sqrt{1-\lambda}\right)^2\| Q^{k-1} - Q^*\| \\
&\le \cdots \le \left(\sqrt{1-\lambda}\right)^{k+1}\| Q^0 - Q^*\|.
\end{aligned}
$$

Select the appropriate parameter σ,φ such that

$$0 < \lambda := \frac{(1-\sigma)^2}{2(1+\sigma^2)}\varphi(2-\varphi)(\beta\min\{1,\sigma/c_2\})(L_1 + L_2)^{-1} < 1,$$

then, we have $0 < 1-\lambda < 1$. Thus, the sequence $\{Q^k\}$ generated by the Algorithm 1 is global $R-$ linear convergence to the solution of (4).

5 Conclusion

In this paper, we study a nonlinear complementarity model of the supply chain management problem. We propose a new type of algorithm for solving this model, and established the global convergence of the algorithm under milder conditions. In addition, its global R-linear convergence rate also proved with the underlying mapping being P-uniform function and Lipschitz continuous. In particular, our conditions required for the convergence of the proposed algorithm are weaker than those proposed in [9, 11]. However, the conditions under which is the convergence of the algorithm is still too strong, it would be interesting to investigate whether there exists new algorithm for the problem is strictly weaker than those existing ones. These will be our further research directions.

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