WAKE HOMING TORPEDO CONFRONTS WITH WARSHIP BASED ON GAME THEORY

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Abstract

The process of wake homing torpedo attacking the target warship is a typical antagonized chase & evade problem. We have developed a two person zero sum game model to describe this problem and used LINGO to find out the optimal strategies for each players based on linear programming. We divide the strategies of the WHT as well as the target into several typical situation and construct the reward matrix of the game. The results show that there is no saddle point in common situation. Both sides of the game can receive the optimal results if the optimal mixed strategies are followed. Compared with traditional torpedo attacking method, this paper has revealed more possibilities for the submarine torpedo attacking results under sophisticated sea battle environment.

1 Introduction

Wake homing torpedo (WHT) is becoming a significant underwater weapon against the warships. As the great progress has been made on the acoustic countermeasure towards the threat of the acoustic homing torpedo, the attacking results of the traditional torpedo have been decreased to some extent. Wake homing torpedo tracks warship by detecting the wake of the warship and entering the wake. Compared with the methods of countermeasure of the acoustic homing torpedo, there is no optimal method to avoid the tracking of the wake homing torpedo.
Recently, various methods have been proposed to confront the threat of the wake homing torpedo. One of these methods is maneuvering----turning the warship rapidly to avoid the tracking of the torpedo[1]. By means of maneuvering, the warship changes the shape of the wake from the normal style to an abnormal style which is more difficult for the wake homing torpedo to run after the target along the twisting ship wake. In order to escape from the torpedo's tracking effectively, the warship has to decide when and how to turn before responsing to the threat of the wake homing torpedo.

However, most researches put focuses on the wake guidance trajectory. Qian Dong and Han Xiao[1] introduces France DCN's work on the simulation of the surface vessel ship's wake and the countermeasures to the wake homing torpedo and proposes that hard-kill is the main method to the wake homing torpedo. Zhao Xiangtao and Sun Xuwen[2] established a simulation model of the maneuvering surface vessel wake and conducted the trajectory logic of the wake homing torpedo. Dong Chunpeng and Shi Xiaolong[3] researches on the guidance method of trip beams wake homing and trajectory logic of the three phases of torpedo's tracking process. Though some interesting results have been achieved through those works, but most of the studies stress the WHT's trajectory under certain tactical environment, for example, the attacking target maneuvers only once. So these results may be valid on some situation, but not valid to others.

In this paper, a game model of WHT attacking maneuvering warship under more complicated tactical environment has been introduced to show the comprehensive results of the game. Several parameters are considered to change the attacking tactic of the WHT, at the same time, the target surface vessel may chose different escaping strategies. As a result, some interesting conclusions could be drawn from the simulation.

2 The process of WHT attacking the Maneuvering Surface Ship

Game theory is a useful tool which is capable of dealing with the competition or confronting problems taking place in the area of economic, society and military. The process of WHT attacking the surface vessel is a typical game problem with two players. In a single game, either player may have several different strategies to choose. Either player tries his best to get the most optimal strategy in order to make the best profits.

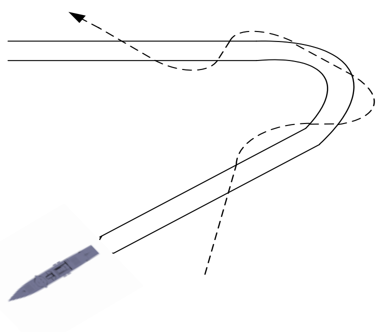

Figure 1 The process of the WHT tracking the target ship

While the thorough description of the wake homing torpedo attacking process has been presented before, the following discusses the fundamentals that would be used in this paper. Figure 1. shows the process of the WHT tracking the target ship. To simplify the problem, we suppose that the submarine has finished the TMA process and decided to attack the target with WHT. Before launching the torpedo, torpedo firing parameters including anticipated wake entering angle and entering range must be set. These parameters which grantee the capture of the target ship are calculated by the Command and Control System(CCS) of the torpedo launching station. Under perfect condition, which means there's no random errors occurring both in torpedo and target, the WHT would enter the wake at the exactly angle and range set by the CCS.

The wake entering range is defined as the distance between the WHT entering point and the attacking target. It affects the target tracking range of WHT, while the wake entering angle effects the quality of the trajectory. If the wake entering range is longer than normal, the tracking range of the torpedo will be longer too. On the condition of normal combating process when the target speeds up to escape the tracking of the torpedo, this range would be extended to some extent that the torpedo is unable to catch up with the target.

Another main factor which plays an important role on the results of chasing-escaping game between WHT and the target warship is wake entering angle. The wake entering angle is the angle between the target course and the torpedo course just at the moment when WHT enters the target wake. For the purpose of automatic adapting to the target wake, the wake entering angle, the angle a in Fig1, must be limited from a minimum value to a maximum value.

On the other hand, the target warship could take some strategy to escape from the tracking of the WHT. One of the actions is turning away rapidly as soon as the torpedo is found. When the target takes this action, the turning angle and velocity of the escaping strategy should be determined.

In the above situation, the WHT is a decision maker which involves the attacking side. The target warship is another decision side, and the action chosen by each side affects the results of the WHT attacking process. For example, if the target warship chooses the big speed and big swerving angle and the WHT chooses far entering range and big entering angle, the hitting probability of the WHT would be different with the result of game when the target takes low speed and small swerving angle. The situation constructs a two-person zero-sum game.

3 Method: The Game Theory model of WHT Attacking the Maneuvering Surface Warship

In a typical two-person zero-sum game, there are two players. One is called row player and the other called column player. In the problem described above, we suppose the row player is WHT and the column player is target warship. The rewards of the two-person zero-sum game is the hitting probability of the WHT. The higher the hitting probability, the more rewards the WHT receives, and on the contrary, the less the target warship will be.

3.1 The strategies of the WHT

On the opinion of the WHT, it will try to select a strategy that makes the most rewards he can receive. As we have analyzed above, the strategies of the WHT include two factors. One factor is the wake entering range(WER) which could be described into three levels. The first level has the near distance from the tail of the target warship. The second level has the medium distance and the third level has the farthest distance. Another factor is wake entering angle(WEA) that also could be divided by three levels as small, medium and large. Then from the point of WHT, the strategies set includes 3*3=9 kinds of situations listed in the Table 1.

Table 1. The Strategies of WHT

WER	Near			Middle			Far		
WEA	Small	Medium	Big	Small	Medium	Big	Small	Medium	Big
strategies	1	2	3	4	5	6	7	8	9

3.2 The strategies of the target warship

On the opinion of the target warship, since the escaping velocity and turning angle's values could be selected between a wide range, the combination of these two factors is infinite. Just like the way we take for the strategies of WHT, the escaping velocity could be divided by two levels, which could be called as low velocity and high velocity. The turning angle is also classified as three sets. By combining the three sets of escaping velocity and three sets of turning angle of the target warship, the total strategies of the target warship is 2*3 = 6 listed in Table 2

Table 2. The Strategies of the Target Warship

Velocity	low			high		
Turning Angle	Small	Medium	Big	Small	Medium	Big
strategies	1	2	3	4	5	6

3.3 The reward of the WHT under particular situation

From the above description, the WHT, called the row player in this game, has 9 strategies to choose. And the target warship, called the column player, has 6 strategies to choose. As the row player chooses the *i*th strategy from the 9 strategies as well as the column player chooses the *j*th strategy from the 6 strategies, then the row player receives a reward of a_{ij} and the row player losses an amount of a_{ij}. In this paper, the reward is the hitting probability of the WHT against the target warship. As the basic assumption of the two-person zero-sum game theory, each player chooses a strategy that enables him to hold the best reward he can. We must decide how much the reward of the row player gains in each possible situation of the attacking-escaping game.

Since the value of hitting probability of the WHT against the target warship can't be calculated analytically, or derived through statistical data from the real attacking process, it is attained according to the computer simulation method. Many factors could determine the attacking process of the WHT to the warship. For example, launching parameters, torpedo cruiser errors, target motion parameters and measure errors can affect the hitting probability synthetically. Fortunately, these could be taken into consideration when simulation model is constructed. Table 3 gives one of the simulation results of the WHT against the warship under 9*6 kinds situation

Table 3 A simulation results of WHT against the warship

Hitting probability		The target warship strategies					
		1	2	3	4	5	6
The WHT strategies	1	0.91	0.87	0.78	0.93	0.82	0.65
	2	0.92	0.78	0.65	0.87	0.84	0.62
	3	0.86	0.72	0.58	0.75	0.66	0.48
	4	0.93	0.92	0.78	0.95	0.81	0.69
	5	0.89	0.79	0.65	0.87	0.77	0.62
	6	0.75	0.60	0.58	0.75	0.82	0.68
	7	0.82	0.92	0.75	0.70	0.58	0.65
	8	0.78	0.78	0.65	0.83	0.78	0.82
	9	0.75	0.72	0.63	0.74	0.65	0.86

4 Result: Optical strategy of the Two-Person Zero-Sum Game

Warship does not have a saddle point because the largest number of all row minimums 0.65 which is not equal to the smallest number of all column maximum 0.93. So the saddle point condition can't be satisfied. There is no equilibrium point in this game which means neither player can benefit from a fixed strategy without considering another player's strategy. In this case, a randomized or mixed strategies can be assumed to each player.

Take the WHT as an example, we can define:

x_i = probability that the WHT chooses the *i*th strategy, $i = 1,2,...,9$.

Obviously,

$x_i \geq 0$;

$$\sum_{i=1}^{9} x_i = 1;$$

v = maximum expected reward of the WHT;

Linear programming can be used to find the value of the v which is in correspondence with the optimal mixed strategies of the row player. We can see that the row player's optimal strategy can be gotten by solving the following LP:

Max z = v

S.t.

$0.91*x_1+0.92*x_2+0.86*x_3+0.93*x_4+0.89*x_5+0.75*x_6+0.82*x_7+0.78*x_8+0.75*x_9>=v;$

$0.87*x_1+0.78*x_2+0.72*x_3+0.92*x_4+0.79*x_5+0.60*x_6+0.72*x_7+0.78*x_8+0.72*x_9>=v;$

$0.78*x_1+0.65*x_2+0.58*x_3+0.77*x_4+0.85*x_5+0.58*x_6+0.75*x_7+0.65*x_8+0.63*x_9>=v;$

$0.83*x_1+0.87*x_2+0.75*x_3+0.95*x_4+0.87*x_5+0.75*x_6+0.70*x_7+0.83*x_8+0.74*x_9>=v;$

$0.72*x_1+0.84*x_2+0.66*x_3+0.71*x_4+0.77*x_5+0.82*x_6+0.58*x_7+0.78*x_8+0.65*x_9>=v;$

$0.65*x_1+0.62*x_2+0.48*x_3+0.79*x_4+0.63*x_5+0.68*x_6+0.65*x_7+0.82*x_8+0.66*x_9>=v;$

$$\sum_{i=1}^{9} x_i = 1;$$

$x_i \geq 0$;

$i = 1,2,...,9$.

LINGO model can be used to solve the optimal reward of the row player. In this case, the optimal mixed strategies of the WHT is:

$x = \{0, 0, 0, 0.352, 0.30, 0, 0, 0.348, 0\}$;

The expected largest expected reward v = 0.752.

Similarly, the optimal mixed strategies of column player(the target warship) is:

y = {0.23,0,0,0,0.77,0},The expected smallest loss v = 0.752 which is the same as the row player.

5 Conclusions

Our work pays much attention to the wake homing torpedo attacking the target warship under complicated sea battle environment. We use Game theory to analyze the optimal strategy for both the WHT and the target warship. The reward matrix is 9*6 which can be solved by LINGO. If the row player(the WHT) chooses the optimal strategies, x = {0,0,0,0.352,0.30,0,0,0.348,0}, an expected maximum reward is 0.752. And at the same time, the column player(the target warship) must follow his optimal strategies, y = {0.23,0,0,0,0.77,0}, to attain the smallest loss which is equal to 0.752. No player can benefit from the alternation of his optimal strategies during the Game.

To sum up, the WHT must follow a mixed probabilities when it attacks the warship in the particular case given in this paper. If the WHT departs from this optimal strategy, the expected reward will reduced by the target warship choosing his optimal strategy. By comparison, in traditional torpedo attacking, a dominating strategy is always considered to be the optimal strategy. Based on our findings from this work, we have found the optimal strategies for the WHT as well as for the target warship.

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