Hopf bifurcation and Turing instability in a modified Leslie-Gower prey-predator model

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***Abstract*—In this paper, we study a modified Leslie-Gower prey-predator model in the presence of nonlinear harvesting in prey subject to the Neumann boundary condition. Our results reveal the conditions on the parameters so that the periodic solution exist surrounding the interior equilibrium. Furthermore, the direction of Hopf bifurcation and the stability of bifurcated periodic solutions are investigated. For the model with the Neumann boundary condition, Turing instability of the interior equilibrium solution is studied. In particular, Turing instability region regarding the parameters is established. Numerical simulations are carried out to demonstrate the results obtained.**

Keywords*—Hopf bifurcation; Turing instability; Leslie-Gower prey-predator model

I. INTRODUCTION

Deterministic nonlinear mathematical models (ODE models) are widely used to understand the dynamics of interacting populations since Lotka [1] and Volterra proposed the well-known predator-prey model. They usually display similar dynamical behaviors. Due to its universal existence and importance, predator-prey models have been proposed ([2],[3],[4],[5]). Dai and Tang [6] considered a predator-prey model in which two ecologically interacting species are harvested independently with constant rates. Li and Xiao [7] proposed a Leslie-Gower prey-predator model with Holling-type III functional response for its bifurcation analysis. Aziz-Alaoui and Daher Okiye[8] studied the following two-dimensional system of autonomous differential equation model for a prey-predator system which incorporates a modified version of Leslie-Gower and Holling-type II functional response:

$$\begin{cases} \dot{x}_1 = rx_1(1-\dfrac{x_1}{K}) - \dfrac{a_1x_1x_2}{n_1+x_1} \\ \dot{x}_2 = sx_2(1-\dfrac{a_2x_2}{n_2+x_1}) \end{cases} \quad (1)$$

The positive initial conditions $x_1(0) > 0, x_2(0) > 0$. where, $x_1(t)$ and $x_2(t)$ are the prey and predator population densities respectively.s measures the growth rate of the predator species, r is intrinsic growth rate and K is environmental carrying capacity for the prey species respectively. a_1 is the maximum value of the per capita reduction rate of prey, $n_i, i = 1, 2$ measures the extent to which the environment provides protection to prey and predator respectively, and sa_2 is the maximum value of the per capita reduction rate of predator.

Biological resources in the prey-predator system are most likely to be harvested so that R.P.Gupta and Peeyush Chandra[9] introduced a Model with prey harvesting. The Michaelis-Menten type harvesting is more realistic from biological and economic points of view.The system of differential equations follows:

$$\begin{cases} \dot{x}_1 = rx_1(1-\dfrac{x_1}{K}) - \dfrac{a_1x_1x_2}{n+x_1} - \dfrac{qEx_1}{m_1E+m_2x_1} \\ \dot{x}_2 = sx_2(1-\dfrac{a_2x_2}{n+x_1}) \end{cases} \quad (2)$$

where q is the catchability coefficient, E is the effort applied to harvest the prey species, m_1 and m_2 are suitable constants.The rest of the parameters have similar meanings as for the model (1).

On the other hand, under some ecological settings, diffusion should be thought of as dispersal of population density and often be considered as a stabilizing process, thus it is the diffusion of a homogenous stable steady state that results in a reaction-diffusion system's spatial patterning. We pay attention to the fact that diffusion-driven instability can appear in the predator-prey system.

The purpose of this paper is to investigate the effect of diffusion on a modified Leslie-Gower prey-predator model. In section 2 we discuss the asymptotic behavior of the interior equilibrium and the existence of Hopf bifurcation of (2); In section 3, we investigate the direction of Hopf bifurcation and the stability of bifurcated periodic solutions. In section 4, Turing instability of (2) is considered. Finally, some numerical simulations are performed to illustrate our analytical results in section 5.

II. STABILITY AND HOPF BIFURCATION

To investigate the dynamics of system (2), we shall consider the following non-dimensional scheme:

$$x_1 = Kx, a_1x_2 = Ky, rt = t, \alpha = \frac{1}{r}, c = \frac{m_1E}{m_2K},$$
$$\beta = \frac{a_2}{a_1}, m = \frac{n}{K}, h = \frac{qE}{rm_2K}, \rho = \frac{s}{r}.$$

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We obtain the following system of differential equations:

$$\begin{cases} \dot{x} = x(1 - x - \dfrac{\alpha y}{m+x} - \dfrac{h}{c+x}) \\ \dot{y} = \rho y(1 - \dfrac{\beta y}{m+x}) \end{cases} \quad (3)$$

It is known from[9] that solutions of (3) are nonnegative with the initial conditions: $x(0) > 0, y(0) > 0$. In addition, except the origin $E_0 = (0,0), E_1 = (0, \frac{m}{\beta})$, if $h > c$ then $E_L = (x_L, 0)$ and $E_H = (x_H, 0)$ both the equilibrium points exist whenever $c < 1$ and $(1-c)^2 > 4(h-c)$ while if $h < c$ then E_H only exists.

$$x_L = \frac{1 - c - \sqrt{(1-c)^2 - 4(h-c)}}{2}$$
$$x_H = \frac{1 - c + \sqrt{(1-c)^2 - 4(h-c)}}{2}$$

The stability of trivial equilibria have been proved in[9]. From the biological point of view, however, it is more interesting to study the dynamical behaviors of the interior equilibrium point E_1^* and E_2^*. The interior equilibria are $E_1^* = (x_1^*, y_1^*)$ and $E_2^* = (x_2^*, y_2^*)$ where x_1^*, x_2^* are the roots of the quadratic equation:

$$x^2 + (\frac{\alpha}{\beta} + c - 1)x + c(\frac{\alpha}{\beta} + \frac{h}{c} - 1) = 0$$

$$x_1^* = \frac{1}{2}(1 - c - \frac{\alpha}{\beta}) - \frac{1}{2}\sqrt{(1 - c - \frac{\alpha}{\beta})^2 - 4c(\frac{\alpha}{\beta} + \frac{h}{c} - 1)}$$
$$x_2^* = \frac{1}{2}(1 - c - \frac{\alpha}{\beta}) + \frac{1}{2}\sqrt{(1 - c - \frac{\alpha}{\beta})^2 - 4c(\frac{\alpha}{\beta} + \frac{h}{c} - 1)}$$

together with $y_1^* = \dfrac{m + x_1^*}{\beta}$ and $y_2^* = \dfrac{m + x_2^*}{\beta}$.

The two distinct interior equilibrium points E_1^* and E_2^* exist whenever $\frac{\alpha}{\beta} + c < 1$ and $(1 - c - \frac{\alpha}{\beta})^2 - 4c(\frac{\alpha}{\beta} + \frac{h}{c} - 1) > 0$.

If $\frac{\alpha}{\beta} + c < 1$ and $(1 - c - \frac{\alpha}{\beta})^2 - 4c(\frac{\alpha}{\beta} + \frac{h}{c} - 1) = 0$, then exist $E^* = E_1^* = E_2^* = \frac{1}{2}(1 - c - \frac{\alpha}{\beta})$.

It is easy to get that Jacobian matrix of (3) is

$$J = \begin{pmatrix} x(-1 + \dfrac{\alpha y}{(m+x)^2} + \dfrac{h}{(c+x)^2}) & -\dfrac{\alpha x}{m+x} \\ \dfrac{\rho \beta y^2}{(m+x)^2} & \dfrac{\rho \beta y}{(m+x)} \end{pmatrix}$$

Then we suppose that $E^*(x^*, y^*) = E_1^*(x_1^*, y_1^*)$ We can rewrite J^* at the point of (x^*, y^*) as following:

$$J^* = \begin{pmatrix} x^*(-1 + \dfrac{\alpha y^*}{(m+x^*)^2} + \dfrac{h}{(c+x^*)^2}) & -\dfrac{\alpha x^*}{m+x} \\ \dfrac{\rho}{\beta} & -\rho \end{pmatrix}$$

For the sake of convenience, let $s_0 = -1 + \dfrac{\alpha y^*}{(m+x^*)^2} + \dfrac{h}{(c+x^*)^2}$, $s = \dfrac{\rho}{x^*}$. In the following, we use s as the parameter.Thus

$$\begin{aligned} \mathbf{trace}J^* &= x^*(-1 + \frac{\alpha y^*}{(m+x^*)^2} + \frac{h}{(c+x^*)^2}) - \rho \\ &= x^* s_0 - x^* s \end{aligned}$$

and

$$\mathbf{det}J^* = x^{*2} s(s_0 + \frac{\alpha}{\beta(m+x^*)})$$

Therefore, the characteristic equation of the linearized system of (3) at the interior equilibrium $E^*(x^*, y^*)$ is

$$\lambda^2 - \mathbf{trace}J^* \lambda + \mathbf{det}J^* = 0 \quad (4)$$

Obviously, (4) has only two roots and they can be expressed as

$$\lambda_{1,2} = \frac{\mathbf{trace}J^* \pm \sqrt{(\mathbf{trace}J^*)^2 - 4\mathbf{det}J^*}}{2}$$

when $\mathbf{trace}J^* < 0$ and $\mathbf{det}J^* > 0$,that is $s > s_0$, two roots of (3) have negative real parts and the interior equilibrium E^* is asymptotically stable.

Now, we study whether there exists periodic solutions of (3) about the interior equilibrium E^* as s passes through the value s_0. We note that $\mathbf{det}J^* > 0$ and it is easy to see that (4) has a pair of purely imaginary roots $\pm i\sqrt{-4\mathbf{det}J^*}$ when $s = s_0$. Therefore, according to Hopf bifurcation theorem, (3) can bifurcate a small amplitude nonconstant periodic solution from the interior equilibrium E^* when s crosses through s_0 if the transversality condition is satisfied.

In the following, by regarding s as the bifurcation parameter, we verify the transversality condition. Let $\lambda = p + qi$ $(p, q \in R)$ denotes one of the roots of (11) when $|s - s_0|$ is small sufficiently and $\lambda = \sqrt{\mathbf{det}J^*} i$ when $s = s_0$.

Substituting $\lambda = p + qi$ into (11) and separating real and imaginary parts, we have

$$p^2 - q^2 - p\mathbf{trace}J^* + \mathbf{det}J^* = 0 \quad (5)$$
$$2pq - q\mathbf{trace}J^* = 0 \quad (6)$$

Differentiating (5) with respect to s and noticing the fact that $p = 0$ when $s = s_0$, we get

$$\left[\frac{dp}{ds}\right]_{s=s_0} = -\frac{x^*}{2} < 0 \quad (7)$$

This shows that the transversality condition holds.Thus, (3) will undergo a Hopf bifurcation about the interior equilibrium E^*, as s passes through the value s_0.

Theorem 2.1: Suppose that the condition $\frac{\alpha}{\beta} + c < 1$ *and* $(1 - c - \frac{\alpha}{\beta})^2 - 4c(\frac{\alpha}{\beta} + \frac{h}{c} - 1) > 0$ *is satisfied.*

1) *The interior equilibrium E^* of (3) is asymptotically stable when $s > s_0$ and unstable when $s < s_0$.*
2) *(3) can undergo a Hopf bifurcation at the interior equilibrium E^* when $s = s_0$.*

We can suppose that the ecological system can keep dynamic balance without any species died out when the parameter s meets the conditions $s < s_0$.

III. Stability of bifurcated periodic solutions

In the previous section, we have obtained the conditions under which a family of periodic solutions bifurcate from the interior equilibrium E^* of (3) when the s crosses through the critical value s_0 . In this section, we shall study the direction of Hopf bifurcation and stability of bifurcated periodic solutions arising through Hopf bifurcation.

We translate the equilibrium E^* to the origin by the translation $\tilde{x} = x - x^*$, $\tilde{y} = y - y^*$. For the sake of convenience, we still denote $\tilde{x}$ and $\tilde{y}$ by x and y, respectively. Thus, the local system (3) becomes

$$\begin{cases} \dot{x} &= (x+x^*)(1-x-x^*-\dfrac{\alpha(y+y^*)}{m+x+x^*} \\ & \quad -\dfrac{h}{c+x+x^*}) \\ \dot{y} &= \rho(y+y^*)(1-\dfrac{\beta(y+y^*)}{m+x+x^*}) \end{cases} \tag{8}$$

Rewrite system (8) to

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = J^* \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} f(x,y,s) \\ g(x,y,s) \end{pmatrix}$$

where

$$\begin{aligned} f(x,y,s) &:= a_3x^2 + a_4xy + a_5y^2 + a_6x^3 + \cdots \\ g(x,y,s) &:= b_3x^2 + b_4xy + b_5y^2 + b_6x^3 + \cdots \end{aligned}$$

and

$$\begin{aligned} a_3 &= \frac{\alpha y}{(m+x^*)^2} + \frac{h}{(c+x^*)^2} \\ &\quad + x^*(-\frac{\alpha y^*}{(m+x^*)^3} - \frac{h}{c+x^*)^3}) \\ a_4 &= -\frac{a}{m+x^*} + \frac{ax^*}{(m+x^*)^2},\ a_5 = 0 \\ a_6 &= \frac{\alpha x^* y^*}{(m+x^*)^4} + \frac{hx^*}{(x^*+c)^4} \\ &\quad -\frac{\alpha y^*}{(m+x^*)^3} - \frac{h}{(c+x^*)^3} \\ b_3 &= -\frac{s\beta x^*}{m+x^*},\ b_4 = \frac{sx^*}{m+x^*} \\ b_5 &= -\frac{s\beta x^*}{m+x^*},\ b_6 = -\frac{sx^*}{(m+x^*)^2} \end{aligned}$$

The Jacobian matrix of (8) at the $E^* = (0,0)$ is

$$J^* = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}$$

We set matrix

$$B = \begin{pmatrix} 1 & 0 \\ M & N \end{pmatrix}$$

where

$$M = \frac{p(s) - a_1}{a_2}, N = -\frac{q(s)}{a_2}$$

By the transformation

$$\begin{pmatrix} x \\ y \end{pmatrix} = B \begin{pmatrix} u \\ v \end{pmatrix}$$

(8) becomes

$$\begin{cases} \dfrac{du}{dt} = p(s)u - q(s)v + f'(u,v,s) \\ \dfrac{dv}{dt} = q(s)u + p(s)v + g'(u,v,s) \end{cases} \tag{9}$$

where

$$\begin{aligned} f'(u,v,s) &= (a_3 - 1 + a_4M + a_5M^2)u^2 \\ &\quad + (a_4N + 2MNa_5)uv + a_5N^2v^2 + a_6u^3 + \cdots \\ g'(u,v,s) &= -\frac{M}{N}f'(u,v,s) + \frac{1}{N}g^1(u,v,s) + \cdots \\ g^1(u,v,s) &= (b_3 + b_4M + b_5M^2)u^2 + (b_4N + 2MNb_5)uv \\ &\quad + b_5N^2v^2 + b_6u^3 + \cdots \end{aligned}$$

Rewrite (9) in the following polar coordinates form

$$\begin{cases} \dot{r} = \tilde{p}(s)r + \tilde{a}(s)r^3 + \cdots \\ \dot{\theta} = \tilde{q}(s) + \tilde{b}(s)r^2 + \cdots \end{cases} \tag{10}$$

Then the Taylor expansion of (10) at $s = s_0$ yields

$$\begin{cases} \dot{r} = \tilde{p}(s_0)(s-s_0)r + \tilde{a}(s_0)r^3 + \cdots \\ \dot{\theta} = q(s_0) + \tilde{q}(s_0)(s-s_0) + \tilde{b}(s_0)r^2 + \cdots \end{cases} \tag{11}$$

To determine the stability of Hopf bifurcation periodic solution, we need to calculate the sign of the coefficient $\tilde{a}(s_0)$, which is given by

$$\begin{aligned} \tilde{a}(s_0) &= \frac{1}{16}[f'_{uuu} + f'_{uvv} + g'_{uuv} + g'_{vvv}]|_{(0,0,s_0)} \\ &\quad + \frac{1}{16q(s_0)}[f'_{uv}(f'_{uu} + f'_{vv}) - g'_{uv}(g'_{uu} + g'_{vv}) \\ &\quad - f'_{uu}g'_{uu} + f'_{vv}g'_{vv}]|_{(0,0,s_0)} \end{aligned}$$

where

$$\begin{aligned} f'_{uuu} &= 6a_6, f'_{uvv} = g'_{uuv} = g'_{vvv} = 0, f'_{vv} = 2a_5N^2, \\ f'_{uu} &= 2(a_3 - 1 + a_4M + a_5M^2) + 6a_6u, \\ f'_{uv} &= a_4N + 2MNa_5, f'_{vv} = 2b_5N^2 \\ g'_{uv} &= -\frac{M}{N}f'_{uv} + \frac{1}{N}g^1_{uv}, g'_{uu} = -\frac{M}{N}f'_{uu} + \frac{1}{N}g^1_{uu}, \\ g^1_{uu} &= -\frac{M}{N}f'_{vv} + \frac{1}{N}g^1_{vv}, g^1_{uv} = b_4N + 2MNb_5, \end{aligned}$$

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Thus, we obtain $\mu = -\frac{\tilde{a}(s_0)}{\tilde{p}(s_0)}$, According to Poincare-Andronov theorem, we have the following conclusions:

Theorem 3.1: Suppose that the condition $\frac{\alpha}{\beta} + c < 1$ *and* $(1 - c - \frac{\alpha}{\beta})^2 - 4c(\frac{\alpha}{\beta} + \frac{h}{c} - 1) > 0$ *is satisfied.*

1) *$\tilde{a}(s_0)$ determines the stability of bifurcated periodic solutions. If $\tilde{a}(s_0) < 0 (> 0)$, the bifurcated periodic solutions are stable (unstable);*
2) *μ determines the directions of Hopf bifurcation. If $\mu > 0$ (< 0), then the Hopf bifurcation is supercritical (subcritical).*

We can also come to a conclusion that the ecological system is difficult to destroy with small disturbance when the parameter $\tilde{a}(s_0)$ meets $\tilde{a}(s_0) < 0$.

IV. Turing instability for system with diffusion effects

In this part, we will derive conditions for the diffusion-driven instability with respect to the equilibrium solution, the modified Leslie-Gower prey-predator model under the assumption that $s > s_0$. Consider the following system with the no-flux boundary condition in a one-dimensional bounded domain Ω.

$$\begin{cases} \frac{\partial x}{\partial t} = x(1 - x - \frac{\alpha y}{m + x} - \frac{h}{c + x}) + d_1 \Delta x \\ \frac{\partial y}{\partial t} = \rho y(1 - \frac{\beta y}{m + x}) + d_2 \Delta y \end{cases} \quad (12)$$

$$\frac{\partial x}{\partial \vec{n}} = \frac{\partial y}{\partial \vec{n}} = 0.$$

$\vec{n}$ is the unit outer normal to $\partial\Omega$. d_1 and d_2 are the diffusion coefficients. We can obtain the linearized system of (12) at E^*:

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = D \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} + J^* \begin{pmatrix} x \\ y \end{pmatrix}$$

To this end, let

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} e^{\lambda t} cos(kl)$$

where k is the growth rate of perturbation in time t, α_1, α_2 is the amplitude and k is the wave number of the solutions. Denote $J_k = J^* - k^2 \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}$ Then the characteristic equation of (12) at the interior equilibrium solution E^* is

$$\lambda^2 - \textbf{trace}J_k \lambda + \textbf{det}J_k = 0 \quad (13)$$

where

$$\begin{aligned} \textbf{trace}J_k &= a_1 + b_2 - (d_1 + d_2)k^2 \\ \textbf{det}J_k &= d_1 d_2 k^4 - (b_2 d_1 + a_1 d_2)k^2 + a_1 b_2 - a_2 b_1 \end{aligned}$$

It is well known that the interior equilibrium solution E^* of (12) is unstable when (13) has at least one root with positive real part. Noticing that $\textbf{trace}J_k < 0$ when $s > s_0$ and hence $\textbf{trace}J_k = \textbf{trace}J^* - (d_1 + d_2)k^2 < 0$, since $d_1, d_2 > 0$. Therefore, (13) has no imaginary root with positive real parts, thus E^* is unstable if (13) has at least a positive real root. For the sake of convenience, let $F(k^2) = \textbf{det}J_k$. when $F(k^2) < 0$, (13) has two real roots in which one is positive and another is negative.Note that

$$d_1 d_2 > 0, k^2 > 0$$

Therefore, $F(k^2)$ will take the minimum value at $k^2 = k^2_{min}$ when $b_2 d_1 + a_1 d_2 > 0$, where

$$k^2_{min} = -\frac{b_2 d_1 + a_1 d_2}{2 d_1 d_2}$$

In this case, we can obtain

$$b_2^2 \frac{d_1^2}{d_2^2} + (4a_2 b_1 - 2a_1 b_2)\frac{d_1}{d_2} + a_1^2 > 0 \quad (14)$$

Hence, k^2_{min} will be negative when (14) is satisfied and for the wave numbers close to k^2_{min} the growth rate of perturbation λ can be positive. Thus, (14) is the criterion for Turing instability of (12).From (14), we have

$$0 < \frac{d_1}{d_2} < \frac{1}{s}(\frac{2\alpha}{\beta(m + x^*)} - s_0) - \frac{1}{s}\sqrt{(\frac{2\alpha}{\beta(m + x^*)} - s_0)^2 - s_0^2}$$

or

$$\frac{d_1}{d_2} > \frac{1}{s}(\frac{2\alpha}{\beta(m + x^*)} - s_0) - \frac{1}{s}\sqrt{(\frac{2\alpha}{\beta(m + x^*)} - s_0)^2 - s_0^2}$$

It is easy to get from $a_4 d_1 + a_1 d_2 > 0$ and that

$$\frac{1}{s}(\frac{2\alpha}{\beta(m + x^*)} - s_0) - \frac{1}{s}\sqrt{(\frac{2\alpha}{\beta(m + x^*)} - s_0)^2 - s_0^2} < \frac{s_0}{s}$$

and

$$\frac{1}{s}(\frac{2\alpha}{\beta(m + x^*)} - s_0) + \frac{1}{s}\sqrt{(\frac{2\alpha}{\beta(m + x^*)} - s_0)^2 - s_0^2} > \frac{s_0}{s}$$

Therefore, we have the following result:

$$\frac{1}{s}(\frac{2\alpha}{\beta(m + x^*)} - s_0) - \frac{1}{s}\sqrt{(\frac{2\alpha}{\beta(m + x^*)} - s_0)^2 - s_0^2} < \frac{s_0}{s}$$

Theorem 4.1: Suppose that the condition $0 < \frac{s_0}{s} < 1$ (the interior equilibrium of (3) is stable). Then E^* is an unstable interior equilibrium solution of (12), that is, Turing instability occurs, if

$$0 < \frac{d_1}{d_2} < \frac{1}{s}(\frac{2\alpha}{\beta(m + x^*)} - s_0) - \frac{1}{s}\sqrt{(\frac{2\alpha}{\beta(m + x^*)} - s_0)^2 - s_0^2}$$

We can suppose that the ecological system cause unstable with diffusion when the the diffusion coefficient meets the above conditions.

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V. Examples and Numerical Simulations

In this section, we present some examples and numerical simulations to verify our theoretical results proved in the previous sections by using matlab programm. We first give the numerical simulations for the following particular case of system (3) for fixed parameter values $\alpha = 0.4, \beta = 1, c = 0.05, h = 0.1, m = 0.1$. It is easy to see that $1 - c - \dfrac{\alpha}{\beta} = 0.55 > 0, (1-c-\dfrac{\alpha}{\beta})^2 - 4c(\dfrac{\alpha}{\beta} + h/c - 1) = 0.0225 > 0$, hence (3) has a interior equilibrium $E_1^* = (0.35, 0.45)$.it follows from Theorem 2.1 that E^* is asymptotically stable when $\rho = 0.185$, $\dfrac{s_0}{s} = 0.9722 < 1$, the results are shown in Fig.1-2. and E^* is unstable when $\rho = 0.174$, $\dfrac{s_0}{s} = 1.7278 > 1$, the results are shown in Fig.3-4. the Hopf bifurcation at $s = s_0(\rho = 0.179861)$ is subcritical and the bifurcating periodic solutions are local asymptotically stable. The numerical simulation are shown in fig.5.

Furthermore, we choose $\rho = 0.185$ and $\dfrac{s_0}{s} = 0.9722 < 1$,the interior equilibrium $E^* = (0.35, 0.45)$ is stable. Considering the reaction-diffusion model with no-flux boundary conditions. We only choose the diffusion coefficients d_1 and d_2. Then we let

$$R_0 = \frac{1}{s}(\frac{2\alpha}{\beta(m+x^*)} - s_0) - \frac{1}{s}\sqrt{(\frac{2\alpha}{\beta(m+x^*)} - s_0)^2 - s_0^2}$$

The result are showed in fig.6 and fig.7.

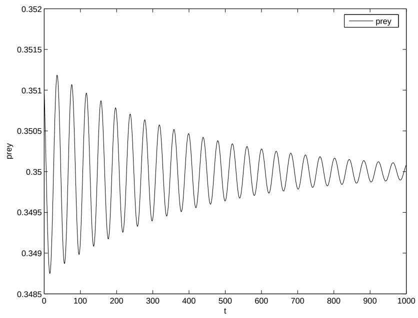

Fig. 1. The trajectory portrait of (2) in the t-prey when $\rho = 0.185, \dfrac{s_0}{s} = 0.9722 < 1$.

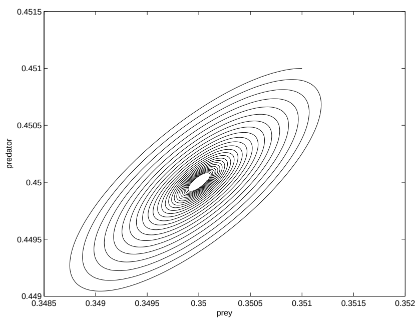

Fig. 2. The trajectory portrait of (2) in the prey-predator when $\rho = 0.185, \dfrac{s_0}{s} = 0.9722 < 1$.

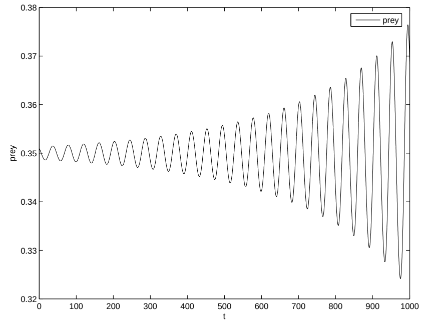

Fig. 3. The trajectory portrait of (2) in the t-prey. $\rho = 0.174, \dfrac{s_0}{s} = 1.7278 > 1$.

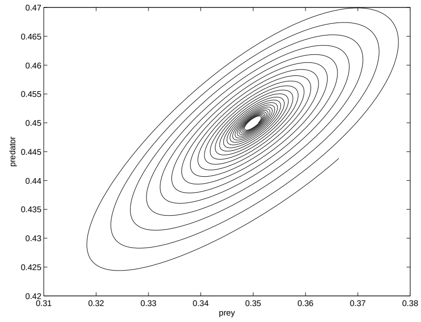

Fig. 4. The trajectory portrait of (2) in the prey-predator. $\rho = 0.174, \dfrac{s_0}{s} = 1.7278 > 1$.

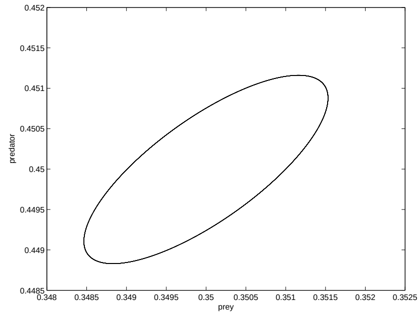

Fig. 5. The trajectory portrait of (2) in the prey-predator. $\rho = 0.1798611, \dfrac{s_0}{s} = 1$.

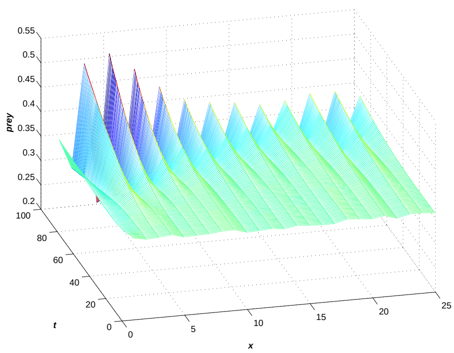

Fig. 6. Numerical simulations of an unstable equilibrium solution of system of (12) in the prey-predator. $\rho = 0.85, d_1 = 0.2, d_2 = 1.2, \dfrac{d_1}{d_2} = 0.167 < R_0 = 0.20657$.

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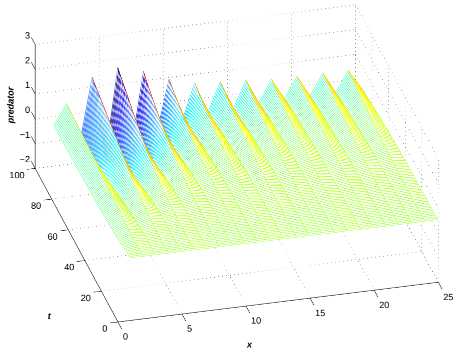

Fig. 7. Numerical simulations of an unstable equilibrium solution of system (12) in the prey-predator under $\rho = 0.85, d_1 = 0.2, d_2 = 1.2, \frac{d_1}{d_2} = 0.167 < R_0 = 0.20657$.

VI. Conclusion

This paper introduces modified Leslie-Gower prey-predator model. we study the Hopf bifurcation and the stability of the system. Our results reveal the conditions on the parameters so that the periodic solution exist surrounding the interior equilibrium. It show that s_0 is a critical value for the parameter s. Furthermore, the direction of Hopf bifurcation and the stability of bifurcated periodic solutions are investigated. Turing instability of the interior equilibrium solution is studied for the diffusion model with the Neumann boundary condition. Then diffusion-driven instability of the equilibrium solution and bifurcating periodic solution occur when $\frac{d_1}{d_2} < R_0$. Numerical simulations are carried out to demonstrate the results obtained. The global branch of periodic solutions bifurcating from the Hopf bifurcation point needs further investigation.

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