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A Method of Designing Energy Tax Rate Based On Game Theory*

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Abstract In this paper, we propose a method of designing energy tax rate based on game theory. Through this method, the effect of energy tax on encouraging manufacturers' improvement in energy efficiency is enhanced. Consequently policy makers can reduce the average burden of manufacturers brought by energy tax. Through utilizing game theory in a numerical example, we explain the effect of energy tax on encouraging manufacturers to improve their energy efficiency. The result shows that, given a fixed average tax rate, energy tax with differential tax rate according to manufacturers' energy efficiency is more effective than that with uniform tax rate. Then, based on differential tax rate, a method to determine a proper average tax rate is presented by utilizing theory of Nash equilibrium in game theory. In addition, practicality of the method is simply discussed in the last section.

Keywords Energy tax, Game theory, Nash equilibrium, Differential rate, Energy efficiency

1 Introduction

Energy problem is undoubtedly one of the most important problems people all over the world are facing today. Energy resources, especially fossil fuels, are crucial to everyone on the earth. Unfortunately, however, these resources are nonrenewable. Besides, great consumption of fossil fuels is also causing severe environmental problems. Therefore, improving energy efficiency is an important goal of every country.

Although all people know that energy conservation is necessary, few of them will voluntarily alter their activities for it, especially when their present profit may be decreased. Energy tax such as carbon tax[1] have been implemented to change the situation. However, the result is not as good as expected[2]. Furthermore, in some countries, it is hard to enact energy tax because of strong objection from industry[3]. In China, the proposal of energy tax is under discussion now. During cooperation with State Administration of Taxation of China in the project of central tax revenues forecasting, one of the most important objectives of the project is to research on relationship between tax revenue and important economic factors[4][5]. the problem of how to design an effective tax system with acceptable rate for both government and industry is also greatly concerned.

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Actually, the implementation of energy tax is not only to collect money for environment conservation, but also to encourage manufacturers to improve their energy efficiency. In current energy tax systems, manufacturers in the same country are usually taxed with uniform rate. Although it is simple for implementation, its effect on stimulating energy saving is weak because its penalty for manufacturers with lower energy efficiency is not intense enough. However, if the tax rate is increased, the burden of all manufacturers will be increased and it is easy to cause rejection from industry. Therefore, policy makers need to balance the need for both energy conservation and industry development.

Because of this, a method of designing tax rate based on game theory is proposed in this paper. In the method, energy tax rate of each manufacturer in an industry is determined dynamically according to its energy efficiency and the penal policy. The penal policy here means relation between tax rate and energy efficiency. Its intensity is adjustable. The higher the intensity is, the more tax that manufacturers with lower energy efficiency is going to pay. Besides, the average tax rate for total energy consumption in the industry is fixed in advance. Therefore, the final profit of each manufacturer is effected not only by its own energy efficiency, but also by that of others. In other words, all manufacturers are in a game. Based on Nash equilibrium in game theory[6], the lower bound of the average tax rate is determined. Policy makers can then choose the most proper average tax rate according to their selection of the intensity of the penal policy. Through this way, manufacturers with higher energy efficiency can benefit more from energy tax.

The rest of this paper is organized as follows: in section 2, advantages of energy tax with differential rate over that with uniform rate is shown by comparison with a numerical example; in section 3, the method to determine the proper average tax rate is shown ; section 4 concludes the paper.

2 The advantage of energy tax with differential rate over that with uniform rate

In addition to the characters of energy tax with differential rate introduced in last section, energy efficiency is represented by energy consumption per unit of product here. Since both the data of production and consumption can be monitored easily in practice, the evaluation of energy efficiency could be done with little difficulty. Besides, the penal policy should be set following rules below:

Rule 1 manufactories with higher energy efficiency should have lower tax rate;
Rule 2 manufactories without energy consumption should not be taxed.

The comparison of effect of the two energy tax systems on encouraging improvement in energy efficiency will be shown through a numerical example.

Example: Assume that there are two coal consuming manufactories in a city, which are in identical state in production. Each manufacturer has two options now. One is to improve production equipment which will add extra cost while decreasing coal consumption of unit product by 20%; the other is to do nothing and keep the current energy efficiency. Besides, because of competition with manufactories in other cities, the price of their product can not be increased no matter which option they choose. Finally, both manufacturers are rational, which means they will choose to improve production equipment only when

	Cost of coal	Other costs	**Total cost**	**price**	**Profit**
Keep	2	6	8	9.6	20%
Improve	1.6	6.5	8.1	9.6	18.5%

Table 1: Cost information

	Cost of coal	Other costs	tax	**Total cost**	**price**	**Profit**
Keep	2	6	0.4	8.4	9.6	14.29%
Improve	1.6	6.5	0.32	8.42	9.6	14.01%

Table 2: Cost information with uniform-rate energy tax

they could get relatively more profit after the improvement. The cost and profit per unit product are listed in Table 1.

From Table 1,it can be concluded immediately that these two manufactories are definitely reluctant to improve their production equipment, if local government do nothing about the situation. Now, suppose that local government is going to tax them for coal consumption, and a tax rate of 20% for total consumption is needed to solve local energy problems. Then, the way of collecting the money from the two manufactories will be crucial to the effect of the tax.

1. Energy tax with uniform rate
   In this system, both manufactories will be taxed by 20% for each unit of coal they have consumed. Consequently, the cost and profit of one unit product will be changed as listed in table 2.
   We can conclude from Table 2 that, although profit difference between the two choices will be diminished from 1.5% to 0.28%, the best choice of rational manufacturer is still to keep the current state. Therefore, coal tax implemented in the way of uniform rate, can only get the money, while doing nothing effective to stimulate coal saving. In fact, it can be figured out that, in this system, a tax rate of 25% at least is needed to accomplish both objectives.
2. energy tax with differential rate
   Now suppose the tax rate is linearly related to energy efficiency . That means, let r_1 and r_2 be coal tax rates of the two manufactories, let c_1 and c_2 be their coal consumption of unit product, then, they satisfy

$$\frac{r_1}{r_2} = \frac{c_1}{c_2} \tag{1}$$

   Besides, in order to get the money needed, they should also satisfy

$$c_1 * r_1 + c_2 * r_2 = (c_1 + c_2) * 20\% \tag{2}$$

   Therefore, the tax rate of each manufactory will be determined not only by its own energy efficiency but also by that of the other manufactory, which will be called its opponent below. Corresponding to different strategy each manufactory takes, there are four cases of values of c_1 and c_2. They are as follows:

		OPPONENT'S STRATEGY	
		Improve	**Keep**
STRATEGY	**Improve**	14.01%	14.54%
	Keep	13.76%	14.29%

Table 3: Game matrix of each manufacturer

$$\begin{cases} c_1 = 1.6 \\ c_2 = 1.6 \end{cases} \quad \begin{cases} c_1 = 1.6 \\ c_2 = 2 \end{cases} \quad \begin{cases} c_1 = 2 \\ c_2 = 1.6 \end{cases} \quad \begin{cases} c_1 = 2 \\ c_2 = 2 \end{cases}$$

Set the values of c_1 and c_2 in equation (1) and (2) separately with data above, we can get the game matrix of the two manufactories as summarized in Table 3.
It can be concluded from Table 3 that, for each manufacturer, no matter which strategy its opponent takes, its incentive will be to improve production equipment and decrease coal consumption. That means decreasing coal consumption is in a Nash equilibrium in this game. Therefore, the objective of simulating coal saving is also accomplished by energy tax with differential rate.

The example above shows clearly that, energy tax with differential rate is more effective than that with uniform rate in stimulating energy saving. Actually, it accomplish the work by structuring a game between the two manufacturers and make them competitors of each other in energy conservation.

In fact, energy tax with differential rate can still accomplish the work when lower average tax rate is needed. If relationship between tax rate and energy efficiency is linear, as currently used in the example, the average tax rate could be decreased to 16.7%. It could even be decreased to 5%, if intensity of penalty is being increased to the extreme. The method to get the result will be explained in detail in section 3.

3 Determine the proper average tax rate utilizing game theory

In the energy tax system with differential rate, the average tax rate and penal policy need to be determined. Actually, once penal policy is given, the lowest average tax rate is determined. On the contrary, once given an average tax rate, the finest penal policy is determined. Here, we only concentrate on the average tax rate needed for changing manufacturers' incentives. In this section, we will introduce the method to determine the most appropriate average tax rate which could accomplish the work.

3.1 Constraints on the average tax rate based on Nash equilibrium

It is usually believed in game theory that people will choose strategy which he could benefit more from. Hence, in order to change manufacturers' incentives, government should structure a game for them utilizing energy tax, and in the game, strategies that manufacturers are needed to take should be in a Nash equilibrium.

We still use the example in section 2 for illustration. The game between the two manufacturers could be summarized as in Table 4. Because of the symmetry of the two

		OPPONENT'S STRATEGY	
		Improve	**Keep**
STRATEGY	**Improve**	$\sigma(1.6,1.6)$	$\sigma(1.6,2)$
	Keep	$\sigma(2,1.6)$	$\sigma(2,2)$

Table 4: Game matrix of each manufacturer

manufactories, only payoff of one manufacturer is included in the game matrix. Payoff of the other one is the same. Besides, the payoff function σ in the game is defined as total cost of a unit of product. Now, let c_1,c_2 be coal consumption of unit product of the two manufacturers separately, r_1,r_2 be tax rate, R be the average tax rate. Besides, let φ represents penal policy, A represent relation between coal consumption and manufacturer's cost except that of coal, therefore $A(x)$ is defined as below

$$A(x) = \begin{cases} 6 & x = 2 \\ 6.5 & x = 1.6 \end{cases}$$

then, there is

$$\begin{cases} \sum_{i=1}^{2} c_i * r_i & = & R * \sum_{i=1}^{2} c_i \\ r_i & = & k * \varphi(c_i), \quad i = 1,2 \\ \sigma(c_1,c_2) & = & c_1 * (1+r_1) + A(c_1) \end{cases} \tag{3}$$

With equations (3), all payoffs in the game could be figured out. Then, the necessities for strategy of improvement being in a Nash equilibrium are

$$\begin{cases} \sigma(1.6,1.6) & < & \sigma(2,1.6) \\ \sigma(1.6,2) & < & \sigma(2,2) \end{cases} \tag{4}$$

which means improvement of equipment could bring relatively low cost to manufactory.

Since variable r_1 and r_2 could be eliminated and represented by R and φ through equation (3), the combination of equation (3) and (4) is actually the constraints on average tax rate and penal policy. It can be concluded consequently that the range of effective average tax rate is solely decided by penal policy now, and we could expand the range by increase the intensity of penal policy.

3.2 determine lower bound of average tax rate

Basing on the constraints above, we formulate the problem of seeking for the lower bound of average tax rate as a linear programming model below.

$$\begin{array}{llll} & \textbf{min} & R & \\ \textbf{s.t.} & \sum_{i=1}^{2} c_i * r_i & = & R * \sum_{i=1}^{2} c_i \\ & r_i & = & k * \varphi(c_i), \quad i = 1,2 \\ & \sigma(c_1,c_2) & = & c_1 * (1+r_1) + A(c_1) \\ & \sigma(1.6,1.6) & \leq & \sigma(2,1.6) \\ & \sigma(1.6,2) & \leq & \sigma(2,2) \\ & r_1 \geq 0, & r_2 \geq 0, & R \geq 0 \end{array}$$

As analyzed in last subsection, the range of proper average tax rate is decided by penal policy, which is φ here. Hence, once φ is fixed, the optimal solution to the **LP** is fixed.

For example, we define φ as $\varphi(x) = x$. Then for each R, there are

$$\sigma(1.6, 1.6) = 8.1 + 1.6 * R \tag{5}$$
$$\sigma(2, 1.6) = 8 + 2.2 * R \tag{6}$$
$$\sigma(1.6, 2) = 8.1 + 1.4 * R \tag{7}$$
$$\sigma(2, 2) = 8 + 2 * R \tag{8}$$

Hence, the **LP** could be simplified to:

$$\begin{array}{rrcl} & \textbf{min} & & R \\ \textbf{s.t.} & 8.1 + 1.6 * R & \leq & 8 + 2.2 * R \\ & 8.1 + 1.4 * R & \leq & 8 + 2 * R \end{array}$$

Then it can be figured out that the lower bound of average tax rate is 16.7%.

If this is still not low enough, we need adjust the penal policy. Before adjusting, it is necessary to define the range of φ. Here, according to the rules set for it, we could take $\Omega = \{\varphi | \varphi(x) = x^t, t \in (0, \infty)\}$ as its possible set. Replace $\varphi(x)$ with x^t in the constraints of **LP**. In addition, in order to analyze the relation between R and t more clearly, we replace numbers in the example with symbols as follows:

C_I : consumption after improvement
C_K : consumption without improvement

$$A(x) = \begin{cases} A_I & x = C_I \\ A_K & x = C_K \end{cases}$$

Then, its constraints can be simplified into one inequality:

$$R \geq (C_I + A_I - C_K - A_K) \frac{C_K^{t+1} + C_I^{t+1}}{C_K^{t+2} - C_I^{t+2}}. \tag{9}$$

let

$$f(t) = (C_I + A_I - C_K - A_K) \frac{C_K^{t+1} + C_I^{t+1}}{C_K^{t+2} - C_I^{t+2}}.$$

Because it can be proved that $f'(t) < 0$, $f(t)$ is a monotonously decreasing function. Hence, the lower bound of average tax rate can be decreased by increasing t. However, it can not be decreased unlimitedly. Since

$$\lim_{t \to \infty} f(t) = \frac{C_I + A_I - C_K - A_K}{C_K}, \tag{10}$$

there should be

$$R \geq \frac{C_I + A_I - C_K - A_K}{C_K}, \text{ for all } \varphi \in \Omega.$$

In addition, the two manufacturers' tax rates are

$$r_I = 0, \quad r_K = R\frac{C_I + C_K}{C_K}.$$

This means manufacturer with lower coal efficiency will pay alone for coal consumption of both, because of the severest penal policy. It can be seen that the penal policy is really the severest one.

Finally, replace symbols with numbers, and we can get that the lower bound of average tax rate for the example is 5%.

Given the lower bound of average tax rate, what policy makers need now is to choose the most proper rate according to specific economic situation. In the meantime, policy makers also know the corresponding penal policy they should take. Even when the lower bound of average tax rate is still not accepted by manufacturers, it helps policy makers to realize the necessity of finding other ways to improve the situation.

4 Conclusion

In this paper, we have introduced a method of designing energy tax rate. The energy tax designed in this way has three main characters. Firstly, the tax rate of each manufactory is dynamically determined according to its energy efficiency. Through this way, the actions beneficial for energy conservation will be encouraged by lower tax rate. Secondly, manufacturers are designed to be taxed according to its relative energy efficiency. As a result, the tax rate of each manufactory is not only decided by its own energy efficiency but also by that of others. The competition between them enhances the stimulation to decrease energy consumption. Finally, range of average tax rate are figured out based on Nash equilibrium in game theory. This makes sure that the energy tax is effective on changing manufacturers' incentive. In addition, lower average tax rate can be achieved by more severe penal policy, which makes the differential-rate energy tax system more flexible.

The example we used in this paper is an extremely simple case. We used it just for illustration of our idea. The complexity of computation will surely be much more higher when the method is put into practice. However, it is usually not necessary for a tax system to be effective for every specific manufacturer. Therefore, the complexity of computation can be decreased greatly according to specific target of policy makers. For example, manufactories are firstly classified into several groups according to their energy efficiency. Then, energy tax system is only needed to be effective for the group with the lowest energy efficiency. Consequently, only incentives of manufacturers in that group need to be considered, and the game matrix is greatly simplified.

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