The 7th International Symposium on Operations Research and Its Applications (ISORA'08)
Lijiang, China, October 31–Novemver 3, 2008

Optimal Configurations of Cell Automata to Generate Test Stimuli for VLSI

Masanori Fushimi[1,*] Tohru Hadano[2,†]

[1]Department of Information Systems and Mathematical Sciences,
Nanzan University, 27 Seirei, Seto, Aichi 489-0863, Japan
[2]Graduate School of Mathematical Sciences and Information Engineering,
Nanzan University, 27 Seirei, Seto, Aichi 489-0863, Japan

Abstract It is impossible to test a VLSI chip for malfunction by exhaustively checking all the states that the chip can assume. So a chip is usually checked by some mechanism of random sampling. In this paper, we are concerned with the use of linear CA, i.e. hybrids of rules 90 and 150 CA, for generating random test patterns to test VLSI chips. If the number of cells of 90/150 CA is n, the maximum possible period of the output sequence of any cell is 2^n-1. It is also known that the output sequence of any cell of a maximum-period CA is the same as the output sequence of another cell of the same CA except the phase shift. Since the output sequences of all the cells are used in parallel as random test patterns, it is important that the phase shift between any pair of output sequences is sufficiently large. This paper reports the computational results of the search for such CA's as well as the cross correlations of any pair of output sequences.

1 Introduction

Since millions of gates are integrated into a VLSI chip, it is impossible to test a chip for malfunction by exhaustively checking all the states that the chip can assume. So a chip is usually checked by some mechanism of random sampling. It is natural to include a testing mechanism into a chip in order to shorten the time required for test procedures. This method of testing is called "built-in self-test" (BIST). When we include a random pattern generator into a VLSI chip, it is desirable to make the area that the generator takes as compact as possible. In this respect, considerable interest has recently developed in the use of cell automata (CA) for BIST. In this paper, we are concerned with the use of linear CA, i.e. hybrids of rules 90 and 150 CA. If the number of cells of 90/150 CA is n, the maximum possible period of the output sequence of any cell is 2^n-1, and a method of designing maximum-length 90/150 CA was proposed by Tezuka and Fushimi [7] in 1994. It is also known that the output sequence of any cell of such CA is the same as the output sequence of another cell of the same CA except the phase shift. The output sequences of all the cells are used in parallel as test stimuli in BIST. So it is important that the phase shift between any pair of output sequences is sufficiently large. This paper reports the

*fushimi@nanzan-u.ac.jp
†m08mm008@nanzan-u.ac.jp

computational results of the search for such CA's as well as the cross correlations of any pair of output sequences.

2 90/150 Cellular Automata Sequences

We consider an n-cell 90/150 cellular automaton (CA) with null boundary conditions. Let the state of the k-th cell at discrete time t be denoted by $x_k(t)$. Then the state transition of the CA is described by the following recurrence:

$$x_k(t+1) = x_{k-1}(t) + c_k x_k(t) + x_{k+1}(t) (\bmod 2), 1 \leq k \leq n,$$

where c_k=0 or 1, and $x_0(t) = x_{n+1}(t) = 0$ for any t. In matrix notation, this can be written as a state transition equation over the Galois field GF(2)

$$X(t+1) = AX(t)$$

where $X(t) = (x_1(t), x_2(t), ..., x_n(t))$ is a column vector, and the state transition matrix A is tridiagonal with $c_1, c_2, ..., c_n$ as main diagonal and 1's as subdiagonal elements. It is well known that the sequence attains the maximum possible period $2^n - 1$ if and only if the characteristic polynomial of A

$$p_n(x) = \det(xI + A)$$

is primitive over GF(2), where I is the identity matrix of order n. Hereafter we consider only such maximum-period 90/150 CA's.

It is also known that the output sequences of all the cells of such a CA are the same as the linear feedback shift register (LFSR) sequence generated by $p_n(x)$ except the phase shifts. Methods of computing these phase shifts with respect to the output sequence of the 1-st cell have been proposed by several authors, e.g. [2,6]. Using these methods, we can compute phase shifts between any pair of cells, and then find the minimum among all these phase shifts, which we call the minimum spacing (MS). In order to use the vector sequence $X(t)$ as the test stimuli for BIST of VLSI, it is desirable that its MS is big enough. Fushimi et al. [3] tries to find the configuration of CA with maximum MS for various numbers of cells. Specifically, they list the optimal configuration for each n in the range $17 \leq n \leq 24$, and the configuration with the maximum MS among 1000 randomly chosen configurations for each n in the range $25 \leq n \leq 32$. The optimal configurations found by them are shown in Table 1.

3 Correlations among Output Sequences of Different Cells

The autocorrelation function of any LFSR sequence *over the whole period* is known to be similar to the autocorrelation function of white noise, i.e. the correlation value is almost equal to 0 unless the phase shift is a multiple of the period. On the other hand, the autocorrelation function of an LFSR sequence *over a partial period* is not known theoretically and must be computed numerically if it is needed. When we use the vector sequence $X(t)$ as the test stimuli, we usually do not use the sequence over the whole period, and it is very important to compute the correlations among output sequences of

Table 1: Data for Optimal Configurations and Computing Correlations

degree n	diagonal elements of A	MS	length of a block	no. of blocks
17	1000 1101 1001 0000 1	2,787	2,787	1
18	1010 1100 0101 0000 01	4,657	4,657	1
19	1010 0111 1001 1111 101	9,205	9,205	1
20	1000 1010 1100 0001 0101	20,523	20,523	1
21	1000 1011 1101 0110 1010 1	33,843	33,843	1
22	1011 1111 1011 1111 0100 01	73,913	73,913	1
23	1011 1110 1100 1110 1111 101	152,389	50,796	3
24	1010 1100 1101 1000 1111 0101	224,094	56,023	4
25	1010 0000 1111 1000 1101 0010 1	288,967	57,793	5
26	1000 1111 0111 0111 1001 1111 01	382,874	54,696	7
27	1010 1000 0000 0101 0000 0100 001	494,737	49,473	10
28	1010 1101 1100 1101 1111 0001 1101	1,548,576	103,238	15
29	1010 1101 1100 1101 0111 1111 0010 1	1,748,427	116,561	15
30	1010 0011 0111 0101 0101 1111 1000 01	3,453,738	115,124	30
31	1010 1000 1001 0000 1100 0101 1010 001	10,697,022	200,000	50
32	1000 1000 1010 0100 1100 1100 1011 0001	7,822,043	156,440	50

different cells over a partial period. Thus we have performed extensive computations for all CA sequences listed in Table 1. For a sequence with very large MS, we divided MS into several blocks with equal length and computed correlations for each block. Table 1 shows the number of such blocks and their length for each sequence. The correlations over a partial period depend, of course, on the initial vector $X(0)$, and we have chosen several initial patterns for $X(0)$ as follows:

1. random bit patterns
2. all 1's
3. the central component (when n is odd) or one of the central components (when is n even) is 1 and all the other components are 0's

Table 2 shows, as an example, the correlations for case 3 with $n = 17$, where italicized figures indicate absolute values of negative correlations.

It is clear that all the cross correlations are negligibly small. No significant differences were observed among the computational results for different initial patterns, so that the tables of the correlations for the other initial patterns are omitted. It has also turned out that there are no significant differences among the computational results for the blocks with the same n shown in Table 1. Table 3 shows the maximum (in absolute value) cross correlations for case 3) for the first block for every n shown in Table 1.

4 Conclusion

We have shown the optimal configurations of 90/150 cell automata for generating random patterns to test VLSI chips. They are optimal in the sense that the minimum phase difference among the output sequences of all the cells is maximum. We have performed

Table 2: An example of correlations among output sequences of cells.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1.00	0.01	0.01	0.01	0.02	0.01	0.01	0.02	0.03	0.01	0.04	0.01	0.01	0.01	0.02	0.01	0.00
2		1.00	0.01	0.02	0.00	0.03	0.01	0.02	0.02	0.04	0.02	0.03	0.03	0.02	0.00	0.02	0.01
3			1.00	0.01	0.00	0.03	0.01	0.03	0.01	0.03	0.02	0.00	0.02	0.01	0.01	0.02	0.04
4				1.00	0.01	0.02	0.00	0.03	0.00	0.00	0.01	0.01	0.01	0.02	0.01	0.01	0.02
5					1.00	0.00	0.02	0.02	0.02	0.03	0.00	0.01	0.01	0.01	0.01	0.02	0.01
6						1.00	0.01	0.00	0.03	0.01	0.02	0.04	0.01	0.00	0.02	0.00	0.04
7							1.00	0.02	0.03	0.00	0.01	0.00	0.00	0.02	0.02	0.01	0.02
8								1.00	0.00	0.01	0.00	0.03	0.01	0.01	0.01	0.00	0.01
9									1.00	0.00	0.03	0.03	0.01	0.03	0.01	0.01	0.00
10										1.00	0.01	0.02	0.02	0.01	0.00	0.02	0.02
11											1.00	0.02	0.01	0.01	0.02	0.00	0.03
12												1.00	0.02	0.02	0.00	0.01	0.01
13													1.00	0.01	0.01	0.01	0.02
14														1.00	0.03	0.02	0.01
15															1.00	0.01	0.01
16																1.00	0.01
17																	1.00

Table 3: The maximum (in absolute value) cross correlations for the first block. Italicized figures indicate negative correlations.

n	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
	0.043	*0.05*	0.029	*0.02*	0.015	*0.01*	*0.01*	0.011	0.012	0.012	0.017	0.095	0.097	0.081	0.007	0.07

extensive computations to check the cross correlations of output sequences of all the cells, and verified they are negligibly small in all the cases. This research was supported by Nanzan University Pache Research Subsidy I-A-2 for the 2008 academic year.

References

[1] P. H. Bardell, Analysis of cellular automata used as pseudorandom pattern generators, *Proceedings of the International Test Conference*, pp. 762-768, 1990.

[2] S.-J. Choi et al., Computing phase shifts of maximum-length 90/150 cellular automata sequences, *Lecture Notes in Computer Science*, **3305**, Springer, 2004, pp.31-39.

[3] M. Fushimi, T. Furuta, and A. Ito, Search for 90/150 cellular automata sequences with maximum minimum-phase-spacing, *Proceedings of International Workshop on Recent Advances in Stochastic Operations Research II*, March 5-6, 2007, Nanzan University, Nagoya, pp.61-65.

[4] M. Fushimi, Y. Mano, and T. Kuroda, Parallel computations and random numbers with special reference to random number generation for built-in self-test for VLSI (in Japanese), in *The Institute of Statistical Mathematics Cooperative Research Report: Applied Characterization of Random Number Generators and Related Topics*, March 2008, pp.58-65.

[5] S. W. Golomb, *Shift Register Sequences*, Holden-Day, 1967.

[6] P. Sarkar, Computing shifts in 90/150 cellular automata sequences, *Finite Fields and Their Applications*, **9**(2), pp. 175-186, 2003.

[7] S. Tezuka and M. Fushimi, A method of designing cellular automata as pseudorandom number generators for built-in self-test for VLSI, *Contemporary Mathematics*, **168**, pp. 363-367, 1994.

[8] S. Wolfram, Statistical mechanics of cellular automata, *Reviews of Modern Physics*, **55**(3), pp. 601-644, 1983.