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Medical Treatment Capability Analysis using Queuing Theory in a Biochemical Terrorist Attack

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Abstract Biochemical terrorist attack will cause a large number of victims flock to the nearest hospital in a short period of time. It is necessary that these victims get medical treatment in the "golden hour". In this paper, we proposed a method by formulating this into a transient non-steady-state system of Queuing Theory. The method can provide model and algorithms for the decision support systems of emergency management by calculating the arrival rate of victims and the medical treatment capability.

Keywords Biochemical Terrorist Attacks; Medical Treatment Capability; Queuing Theory

1 Introduction

The '9.11' Event has made anti-terrorism becomes one of the most important issues throughout the world. Terrorist attacks by biological and chemical (biochemical) means, such as the Japanese subway Sarin gas incident and the U.S. anthrax incident, had also resulted in very bad effects. If it is not handled properly, it will bring on tremendous loss of personnel and property. For example, on June 27, 1994, Matsumoto city in Japan was attacked by Sarin gas, during which seven people were killed. However, due to inadequate information about the prevention and preparation, Sarin gas terrorist incidents occurred again in the Tokyo subway on March 20, 1995, resulting in 12 deaths and more than 6,000 victims. When such terrorist attacks occurred, the self-help and medical treatment of victims are quite different from those of other disasters. Therefore, capability analysis of hospitals becomes a very important issue in biochemical terrorist attacks. In the above-mentioned attack, there were a large number of wounded flooded into St. Luke's Hospital near Tokyo subway Hibiya Line, Tsukiji Station in a short period of time, which had brought tremendous pressure to the hospital [1].

Lots of researches on biochemical terrorist attacks and related issues have been carried out. Most of these literatures focus on three aspects: (1) the pattern and content of emergency terrorist attacks [2]∼[9], which is a general discussion; (2) researches on modeling and simulation, layout of ambulances, and the hospital relief [10]∼[17], mainly for general terrorist attack, etc; (3) researches on handling chemical transport incident and consulting toxicology emphasizes on medical knowledge and treatment technology[18]∼[21].

Two problems will be studied in this article:

(1) Analysis and calculation of the number of victims flooding into the first treated hospitals when biochemical terrorist attacks occur;

(2) Calculation of the relief capacity, the waiting time of victims in the preferred hospital within 'golden hour'.

2 Scenario and Research Purpose

With the above-mentioned Sarin incident in Japan and the subway disaster exercises in Beijing as references[1], [23], we set the scene of this study as follows. In a certain time, there are hundreds of people in a subway station (including people who are waiting for and transferring subway train). Irritant smell is emitted from a suspicious object suddenly, and then spread quickly throughout the subway station. Some people already have symptoms such as nauseation and lachrymation. A few people who are closer to the suspicious object have more serious symptoms and may collapse. At this time, the staff makes efforts to evacuate the crowd. Many passengers rush out of the subway for the nearest hospital for help. Because of the peculiarity of such events, it is unable to determine whether a person is injured by there is a trauma or not. Therefore almost all the persons in the station (victims in this paper) rushed out to hospitals. Most victims will empirically rush to a hospital closer to the subway station,

In this paper, we calculate the number of victims arrived at the hospital each time based on this scene using the method of queuing theory, and we use this as a basis for the judgment to initial emergency response. Then, according to the probability calculation of the number of victims in "golden hour" the hospital medical treatment capacity, calculate the number of victims staying in the hospital. This will provide a basis for taking further measures (to send doctors or transfer to another hospital, etc.).

When biochemical terrorist attacks occur, there will be many victims flow into the hospital in a short time that some victims must wait for treatment. If the waiting time over the "golden hour", which is the best treatment time, it may affect the victim's prognosis effect, and even caused casualties increasing. Therefore, in accordance with the conditions of the hospital, the number of victims, etc., the average waiting time and other parameters will be calculated, which produce the medical treatment capability. Those victims that exceeded the maximum treatment capacity are required to be transfer to other hospitals as soon as possible.

3 The proposed method

3.1 Calculation of arrived victim number in transient acts system

Generally speaking, in the conventional condition, the queuing theory is the theoretical research in a relatively longer time interval of circumstances, such as the number of services of a hospital or a barber shop within one day. However, in the case studied in this paper, the arrival time of customers follows a Poisson distribution and the random interarrival and service time follow the exponential distribution. The system can be achieved in a steady-state system after a period of adjustment. The limit value π_j of the transfer-probability of n(the probability of the Markov chain in state j), which does not depend on initial state i, can be used to establish a queuing system.

Biochemical terrorist attacks are characterized by low probability and large uncertainty. Therefore, we can not use conventional queuing system when predicting the condition of the crowd flow to the hospital when an attack took place. As people's psychological panic and convergence, the number of people rushed to the hospital in a very short time follows the probability distribution of the transient distribution. Therefore, the parameters in the queuing system keep changing over time, making the system in a non-steady-state. As a result, we need to establish a transient acts system (TAS) to estimate the number of victims arrived at the hospital at each moment, which determines whether the number of victims arrived at the hospital is beyond the available treated number in "golden hour" or not.

We assume that victims arrive at the first hospital at moment t, and the inter-arrival time is exponential and that the arrival rate is $\lambda(t)$; there are $s(t)$ parallel servers in the hospital, the service time is exponential and that the service rate is $\mu(t)$; and the hospital can accommodate the largest number of victims is N.

Setting the inter-arrival time as, and there is only one event (arrival or completing treatment) at most occur in $\Delta(t)$. If there are K victims at moment t in the system, and:

i) The arrival probability $\lambda(t)\Delta t$ is when only one victim arrives at the hospital during Δt;

ii) The arrival probability $o(\Delta t)$ is when more than one victims arrive at the hospital during Δt;

iii) Assuming that the arrival time between victims is independent during Δt, the service probability completed a victim is $\min(s(t),K)\mu(t)\Delta t$ in Δt;

iv) The service probability completed more than one victim is $o(\Delta t)$ in Δt.

If the arrival victims follow the first three assumptions, they will be subject to the non-homogeneous Poisson process. The value of Δt can be small enough to reduce the error of calculating the transient probability.

We use $p_i(t)$ to denote the probability of i victims in the system at the moment t based on the initial system. Therefore, $P_0(0) = 1$ if $i > 0$, $P_i(0) = 0$. Then according to the following equation, we can calculate the probability of i victims in the system at the moment $t + \Delta t$:

$$P_0(t+\Delta t) = (1-\lambda(t)\Delta t)P_0(t) + \mu(t)\Delta t P_i(t) \tag{1}$$

$$\begin{aligned} P_i(t+\Delta t) = {} & \lambda(t)\Delta t P_{i-1}(t) + (1-\lambda(t)\Delta t - \min(s(t),i)\mu(t)\Delta t P_i(t) \\ & + \min(s(t),i+1)\ \ \mu(t)\Delta t P_{i+1}(t) \\ & 1 \le i \le N-1 \end{aligned} \tag{2}$$

$$P_N(t+\Delta t) = \lambda(t)\Delta t P_{N-1}(t) + (1-\min(s(t),N)\mu(t)\Delta t P_N(t) \tag{3}$$

The above equation can calculate the probability of arrived or serviced victims at moment $t + \Delta t$. Then the average number of $E(t)$ for waiting for and getting treatment at one moment can be calculated by the following equation.

$$E(t) = \sum_{\mathrm{i=1}}^{\mathrm{n}} \lambda P_i(t) \tag{4}$$

3.2 Calculation of Medical Treatment Capability of a Hospital

In this section, the medical treatment capability will be calculated. According to the relationship between the average rate (λ) of victims and overall average service rate (μ) of hospitals, we can divide it into the following two cases:

(1) The medical treatment capacity of the hospital when (λ) < (μ).

(2) The medical treatment capacity of the hospital when (λ) > (μ).

3.2.1 Denotation

λ the arriving rate of victims;

μ the service rate per server (doctor);

Sparallel servers (doctors);

$c$the system limit, which is to say, the hospital capacity;

$N(t)$ the number of victims in the system at moment t;

ρ intensity of passenger flow;

$P_{ij}(\Delta t)$ the probability of $j$victims in time of Δt when there are already i victims in the system at moment t;

π_j $\lim\limits_{\mathrm{t}\to+\infty} \mathrm{p_{ij}}(\mathrm{t}) = \pi_\mathrm{j}$, the probability with queue length of j at any moment when system is steady state;

q_j the steady-state probability of j victims when victim gains access into the system;

L the average queue length when system is in steady state;

L_q the average waiting queue length when system is in steady state at any time;

$W(t)$ the distribution function of victims staying time when system is in steady state;

W victims staying time when system is in steady state;

W_q the average waiting time of victims when system is in steady state;

W_s the average service time per victim when system is in steady state;

$\rho_s$intensity of passenger flow of S servers;

π_s, $\pi_s = \lim\limits_{t\to\infty} P\{N(t)\} = S$ the probability with queue length of S at any time;

$W_q(t)$ the distribution function of the waiting time in the system.

3.2.2 Calculation of medical treatment capacity of hospital when $\lambda<\mu$

When $\lambda<\mu$, the medical treatment capability of a hospital can be calculated by the queuing system (M / M / S / GD / ∞ / ∞) method. In the system, both the arrival and the service time follow the exponential distribution, the service time of each of the victims is independent, parallel desk (including doctors and nurses) is S, and the system is unlimited.

Because $\lambda<\mu$, the system is in a steady state. Then the average waiting time of the victims in hospitals can calculated as follows.

$$\mathrm{W_q(t)} = \frac{\rho_\mathrm{S}}{\lambda(1-\rho_\mathrm{S})^2}\pi_\mathrm{S} \tag{5}$$

The average stay time of victims in the hospital is

$$\mathrm{W} = \mathrm{W_q} + \frac{1}{\mu} \tag{6}$$

In this case, all victims can be rescued within the golden hour. We simply calculated the average waiting time and the average number of waiting victims.

3.2.3 Calculation of the medical treatment capacity of hospital when$\lambda>\mu$

When $\lambda>\mu$, the medical treatment capability of the hospital is insufficient, which can be calculated by the queuing system (M / M / S / GD / c / ∞) method that the system is limit. In the system, both the arrival and the service time follow the exponential distribution, the service time of each of the victims is independent, parallel desk (including doctors and nurses) is *S*, the system limit is *c*, and the victims are unlimited. Because the system is a limited one, when the system has c victims, no more victims can be posted into the system. As a result, even if the arriving rate is greater than the service rate, the system will not lead to "explosive" and be in a steady state.

The discussion about the parameters of the system is as follows.

The system follows the rule of “first in, first service”. q_i denotes the steady probability of j victims are in the system.

$$q_j = \frac{\pi_j}{1-\pi_c} \qquad j = 1,2,...,c-1 \tag{7}$$

The waiting time follows $j-c$+1-Erlang distribution, the distribution function of waiting time is

$$\begin{aligned} W_q(t) &= P\left\{W_q \le t\right\} \\ &= \begin{cases} \sum\limits_{j=0}^{S-1} q_j & t = 0 \\ \sum\limits_{j=0}^{S-1} q_j + \sum\limits_{j=S}^{c-1} q_j \int\limits_0^t \frac{S\mu(S\mu x)^{j-S}}{(j-S)!} e^{-S\mu x} dx & t > 0 \end{cases} \end{aligned} \tag{8}$$

Then the calculation of average waiting time of the victims in the hospital is

$$W_q(t) = \sum_{j=S}^{c-1} \frac{j-S+1}{S\mu} q_j \tag{9}$$

The calculation of the average stay time of the victims in the hospital is

$$W = W_q + \frac{1}{\mu} \tag{10}$$

However, because , not all of the victims in the system can be treated in the "golden hour". Therefore, we try to calculate the number of victims who are treated in the "golden hour" by the distribution functions of the waiting time, and then according to the probability in the conditions, the victim number can be treated in the hospital is determined.

For this reason, when, we can obtain (11) from (8), which denotes the probability that the waiting time that is smaller than 1.0 hour.

$$
\begin{aligned}
W_q(t) = P\left\{W_q \le t\right\} = \sum_{j=0}^{S-1} \tfrac{\pi_j}{1-\pi_c} + \tfrac{\pi_0}{(1-\pi_c)S!}(\rho^S(1-e^{-S\mu t}) \\
+\tfrac{\rho^{S+1}}{S}(1-e^{-S\mu t}-S\mu te^{-S\mu t}) \\
+\tfrac{\rho^{S+2}}{S^2}(1-e^{-S\mu t}-S\mu te^{-S\mu t}-\tfrac{1}{2!}(S\mu t)^2e^{-S\mu t}) \\
+\ldots+\tfrac{\rho^{c-1}}{S^{c-S-1}}(1-e^{-S\mu t}S\mu te^{-S\mu t}-\ldots-\tfrac{(S\mu t)^{c-S-1}}{(c-S-1)!}e^{-S\mu t}))
\end{aligned}
\tag{11}
$$

The development of (5)∼(10) can be found in [23], and the development of (11) can be found in Appendix 1.

4 Experiments

In this section, we validate the proposed method by simulation methodology. 4.1 is for arrival rate simulation and 4.2 for medical treatment capacity.

4.1 Simulation of arrival rate calculation

According to equation (4), the arrived victim number can be calculated at each moment in the first hospital when the incident occurs.

Suppose that a biochemical terrorist attack occurs in a subway station, and the nearest hospital from the subway station is the first treated hospital. In this case, according to the combinational factors, such as the distance between the first hospital and the subway station, the moving rate of crowd, and road conditions, the rate of λ the victims arrived at the hospital in different times can be calculated, as shown in table 1. These data also based on experience and the literature [22] is modified.

Table 1: The rate of λ the victims arrived at the hospital in different times

Internal time(min)	Arrived rate λ (person/Hr)
0——5	0
5——10	720
10——20	900
20——25	600
25——30	180
30——60	0

Then, in accordance with method in [23], the probability distribution of the arrived victims by the equation (1), (2), and (3) can be simulated in different time. The partial results are listed in Table 2.

Then, as the results of Table 2, arrivals over time within an hour can be calculated by (4).The result is described in Figure 1.

From Figure 1, it can be seen that there is a sharp increase of arrivals at the fifth minute after the first victim arrived. There are about 250 victims arrived at the hospital within 30 minutes, and almost all victims have arrived at the hospital in half an hour. At the same

Table 2: Probability distribution of the arrived victims in different time

Incremental time		The probability of the arrived victims in different time: Pro(person)					
s	hour	Prob(0)	Prob(1)	Prob(2)	Prob(3)	Prob(250)	Prob(251)
0	0	1	0	0	0	0	0
2	0.00	1	0	0	0	0	0
300	0.08	1	0	0	0	0	0
600	0.17	9.64E-18	5.5E-17	2.53E-16	1.04E-15	0	0
900	0.25	1.78E-39	1.17E-38	6.26E-38	3E-37	1.73E-44	3.44E-45
1200	0.33	2.95E-60	1.78E-59	9.24E-59	4.42E-58	3.47E-07	5.67E-07
1500	0.42	9.77E-71	5.3E-70	2.42E-69	1.03E-68	0.035609	0.18014
1800	0.50	9.48E-70	3.21E-69	1.12E-68	3.93E-68	0.129262	0.350354
2100	0.58	1.26E-64	2.83E-64	9.14E-64	2.93E-63	0.012013	0.002168
2400	0.67	3.74E-60	7.55E-60	2.27E-59	6.77E-59	0.000144	1.34E-05
2700	0.75	4.27E-56	7.86E-56	2.22E-55	6.22E-55	1.32E-06	8.3E-08
3000	0.83	2.23E-52	3.76E-52	1E-51	2.66E-51	1.08E-08	5.14E-10
3300	0.92	5.95E-49	9.25E-49	2.35E-48	5.94E-48	8.33E-11	3.18E-12
3600	1	8.88E-46	1.28E-45	3.11E-45	7.51E-45	6.17E-13	1.97E-14
Conditions: 1 Arrivals submit to the non-homogeneous Poisson process; 2 The hospital has 10 doctors and can serve 60 people per hour; 3 The hospital can accommodate up to 252 people; 4 The first victim arrives at the hospital needing 5 minutes from the site of the incident.							

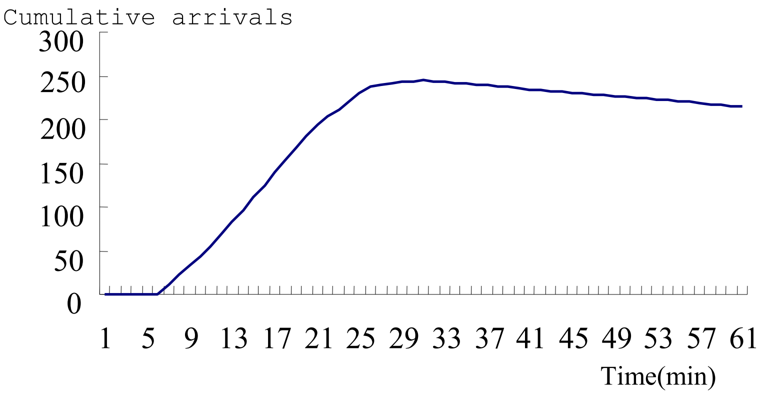

Figure 1: Average number of waiting victims and those being treated

time, as some victims have been rescued, the number of victims decreased in the system. The simulation results are consistent with our experience. This method can be applied to estimate the number of victims in hospitals or on the way at each moment to provide data for decision supporting system.

4.2 Simulation of treatment capacity calculation

Assume that there are 300 victims waiting for medical treatment in the hospital. doctors who can serve victims per hour, and each doctor can serve 6 victims per hour; the service time follows the exponential distribution; waiting area can accommodate ; passenger traffic density of is 50; the "golden hour" is 60 minutes. Table 3 describes the service number, the average waiting time, and the probability of treatment within 60 minutes.

Table 3: Average waiting time and probability of victims.

Number of doctors	Number of treatment	The average waiting time(Hr)	The probability of treatment in 60 minutes	Number of doctors	Number of treatment	The average waiting time(Hr)	The probability of treatment in 60 minutes
10	50	0.66	0.99	20	110	0.74	1.00
	53	0.71	0.99		115	0.79	0.99
	55	0.75	0.98		120	0.83	0.88
	58	0.80	0.95		125	0.87	0.93
	60	0.83	0.92		130	0.91	0.85
	65	0.91	0.77		135	0.95	0.71
	70	1.00	0.53		140	0.99	0.54

Table 3 indicates that if the probability of treatment within the "golden hour" is larger than 0.95, then when there are 10 doctors, the number of getting treatment will be 58, and when there are 20 doctors, the number of getting treatment is 125. However, if there are hundreds of victims exist in the system, victims behind the fifty-ninth victim in the waiting queue for the 10 doctors will have two choices: i) They must be transferred as soon as possible, the capacity of the transferred destination hospital for treatment can also be used this method; ii) More doctors are requested to serve victims. For example, in this case the first hospital only has 10 doctors, if there are another 10 doctors, 125 victims can be treated. Although in theory, by calculation of (2), sufficient doctors could come to the hospital from other hospitals to meet the needs of victims, we still need to take into account the restrictions of the hospital's limitations (for example, beds of emergency room, rescue equipment, etc.), so transferring victims to other hospitals for treatment will be the most practical way.

5 Conclusions

In this paper, we studied the arriving rate of victims of the preferred hospital and the medical treatment capability of the hospital in biochemical terrorist attacks. We established a transient system which satisfies the emergency circumstances by the method of queuing theory. According to the relationship between the arriving rate and service rate, the number of victims arrived at the hospital at each moment and the waiting time have also been calculated respectively.

Particularly, we discussed the calculation of medical treatment capacity of hospital when $\lambda > \mu$, which more accords with biochemical emergency and actual conditions. The method for calculating medical treatment capacity of hospitals is provided using the probability distribution function of waiting time when the waiting time of the victims obeys j-c+1-Erlang distribution.

These studies have set the scene as an example to justify the feasibility. Future studies include the functions of arriving rate, multiple queues and multiple servers, and even multiple hospitals, etc.. What's more, according to the location, time, the surrounding environment, and other conditions of incidents, the arrival rate of victims will also be simulated to better satisfying the actual situation.

In this paper, the model and algorithm research as a part of model and algorithm database will be applied into the development of applications on emergency management decision supporting system for simulation.

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