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A randomized distributed algorithm for total scheduling problem*

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Abstract In multihop radio network, total scheduling occurs when stations communicate one-to-one and broadcast simultaneously. In this paper, we prove a global upper bound for total scheduling by a simple construction method; A randomized distributed algorithm is also presented.

Keywords

1 Introduction

A radio network consists of processors (stations) that communicate among themselves using radio transmission. Packet radio networks, cellular phone networks, and satellite networks are such typical examples.

In a radio network, the stations share a common radio channel over which communication takes place. Owing to the multihop nature of most radio networks, spatial reuse is possible in the sharing or assignment of channels. The channel assignment considered here assigns transmission rights using time division multiplexing (TDM). In this method, transmissions that will not collide may overlap in time, thereby obtaining channel reuse in time.

So, it is necessary to construct a scheduling, a sequence of fixed-length time slots, where each possible transmission is assigned a time slot in such a way that transmissions assigned to the same time slot do not collide. We are interested in the problem of minimizing the number of time slots in such a scheduling.

First, we consider what is meant by a collision. In particular, transmissions may collide in two ways: primary and secondary interference. Primary interference occurs when a station must perform more than one operation in a single time slot, such as receiving from two different transmitters at the same time or transmitting and receiving at the same time (see Fig.1). Secondary interference occurs when a transmission from a neighboring transmitter unwittingly interferes at the receiving end of a communication between a transmitter and a receiver (see Fig.2).

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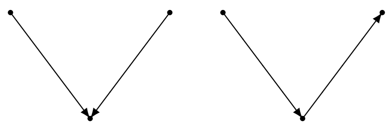

Fig.1 Primary interference

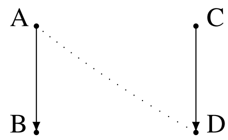

Fig.2 Secondary interference

Under these two different types of interferences, many earlier works [1][8][6][7] dealt with the construction of two different types of scheduling, link scheduling and broadcast scheduling. In link scheduling, the transmission of a station is intended for only one specific transceiver within its range and the scheduling needs to ensure that there are no collisions. In broadcast scheduling, each transceiver is scheduled to ensure collision free transmission of messages to all transceivers within its range.

In this paper, we considered a total scheduling proposed in [9], which occurs when stations communicate one to one and broadcast simultaneously. The rest of this paper is organized as follows. In section 2, a radio network is modeled as a directed graph and the scheduling problems in radio network can be interpreted as coloring problems of the corresponding directed graph. In section 3, a global upper bound for total scheduling is given by a simple construction method; A randomized (Las-Vegas) distributed algorithm is also given in section 4.

2 Preliminary

It is a natural way to represent a radio network by a directed graph $G = (V,A)$. Here, V is the vertices set denoting the stations in the radio network, A is the arcs set, where for any two distinct vertices $u, v \in V$, $(u, v) \in A$ if and only if v is in the transmission range of u. Note that, because of the different transmission power, $(u, v) \in A$ does not necessarily imply $(v, u) \in A$.

In the context, the scheduling problems in radio network can be interpreted as coloring problems of the corresponding directed graph $G = (V,A)$, where the colors assigned correspond to time slots in the scheduling problem.

Link scheduling problem

Link scheduling corresponds to one of arc-coloring $f : A \to Z^+$ such that any pair of arcs (a,b), (c,d) may be colored the same, i.e. $f(a,b) = f(c,d)$, if and only if the following two conditions hold at the same time:

L1: a,b,c,d are mutually distinct, and

L2: $(a,d) \notin A$ and $(c,b) \notin A$.

Broadcast scheduling problem

Broadcast scheduling corresponds to one of vertex coloring $f : V \to Z^+$ such that any pair of vertices a, b may be colored the same, i.e. $f(a) = f(b)$, if and only if the following two conditions hold at the same time:

B1: $(a,b) \notin A$ and $(b,a) \notin A$, and

B2: there is no vertex c such that $(b,c) \in A$ and $(a,c) \in A$.

Total scheduling problem

Total scheduling occurs when stations communicate one to one and broadcast simultaneously. An interpretation of such scheduling is one of total coloring $f : V \cup A \to Z^+$ such that the following four conditions hold:

T1: If $f(a,b) = f(c,d)$ for any pair of arcs (a,b) and (c,d), then conditions L1, L2 must hold simultaneously,

T2: If $f(a) = f(b)$ for any pair of vertices a,b, then conditions B1, B2 must hold simultaneously,

T3: If $(a,b) \in A$, then $f(a) \neq f(a,b)$, and $f(b) \neq f(a,b)$.

T4: If $f(a,b) = f(c)$ for any arc (a,b) and any vertex $c(\neq a,b)$, then $(c,a) \notin A$, $(c,b) \notin A$, and there does not exist any vertex d such that $(a,d) \in A$ and $(c,d) \in A$.

3 Global bound

For every directed graph $G = (V,A)$, define the simple undirected *interference graph* of G, $I(G) = (V_{I(G)}, E_{I(G)})$, as follows: $V_{I(G)} = V \cup A$, there is an edge between any pair of elements in $V \cup A$ according to the following rules:

Rule I: For any pair of arcs (a,b) and (c,d), there is an edge between them in $I(G)$, if at least one of the following two conditions hold:

(1) $|\{a,b\} \bigcap \{c,d\}| \geq 1$

(2) $(a,d) \in A$ or $(c,b) \in A$;

Rule II: For any pair of vertices a,b, there is an edge between them in $I(G)$, if at least one of the following two conditions hold:

(1) $(a,b) \in A$, or $(b,a) \in A$

(2) there exists an vertex c such that $(a,c) \in A$ and $(b,c) \in A$;

Rule III: For any arcs (a,b) and any vertex c, there is an edge between them in $I(G)$, if at least one of the following conditions hold:

(1) $c = a$ or $c = b$;

(2) $(c,a) \in A$ or $(c,b) \in A$;

(3) there exists a vertex d such that $(a,d) \in A$ and $(c,d) \in A$.

We assume that in the radio network each stations knows the identity of its neighbors in the network, $\triangle_{in}$, $\triangle_{out}$, as well as n, the number of stations in the network.

Lemma 3.1.
The maximum degree of $I(G)$ is at most $\triangle_{out}^2 \triangle_{in} + \triangle_{out} \triangle_{in} + \triangle_{out} + 2\triangle_{in}$.

Proof. For any vertice (a,b) in $I(G)$, by rule I, III, we have

$$\begin{aligned} d_{I(G)}((a,b)) &\leq 2(\triangle_{in} + \triangle_{out} - 1) + 2(\triangle_{in} - 1)(\triangle_{out} - 1) \\ &+ 2 + (\triangle_{in} - 1) + \triangle_{in} + (\triangle_{in} - 1)(\triangle_{out} - 1) \\ &= 3\triangle_{in}\triangle_{out} + \triangle_{in} - \triangle_{out} + 2 \end{aligned}$$

For any vertice $a \in I(G)$, by rule II, III,

$$\begin{aligned} d_{I(G)}(a) &\leq \triangle_{in} + \triangle_{out} + \triangle_{out}(\triangle_{in} - 1) + \triangle_{out}^2 + \triangle_{out}(\triangle_{in} - 1)(\triangle_{out} - 1) \\ &+ \triangle_{out} + \triangle_{in} + \triangle_{out}(\triangle_{in} - 1) \\ &= \triangle_{in}\triangle_{out}^2 + \triangle_{out}(\triangle_{in} + 1) + 2\triangle_{in}. \end{aligned}$$

So, the theorem is true. □

Lemma 3.2.
$\chi(I(G))$ time solts are necessary and sufficient for the total scheduling.

Proof. Let $r = \chi(I(G))$ and $\varphi : V_{I(G)} \to \{1, \cdots, r\}$ be a proper r-coloring of $I(G)$.

Construct the following scheduling S.

If an element in $I(G)$, (x,y) or z is colored by φ with color i, $1 \leq i \leq r$, then processor x transmit the message designated to processor y at time slot i during the scheduling S; or processor z broadcast at time slot i during the scheduling S.

Elements x and (x,y) are adjacent in $I(G)$, so in each time slot, x do at most one job: either broadcast, or transmit message to designated neighbor.

Since the arcs originating from the vertex x are adjacent in $I(G)$, at each time slot, x transmit message to at most one designated neighbors.

Since φ is a proper coloring on the vertex set of $I(G)$, in the scheduling S, each message will not be assigned two different time slots.

So the scheduling we obtain is well-defined.

It remains to prove that all of the transmission and broadcast succeed.

Assume that at time slot i of the scheduling S, processor x transmit message to y and processor z broadcast. The rule I, II, III guarantee that at time slot i, transmission x to y succeed and processor z broadcast successfully.

For the other direction, assume that S is an total scheduling of r rounds. Color the vertex (x,y) or z of $I(G)$ with color i, where i is the first time slot in S in which transmission x to y succeeds or z broadcast successfully. The construction of $I(G)$ and arguments as before prove that this a proper coloring of $I(G)$. □

Theorem 3.3.
For every directed graph G, there is a (polynomial time constructible) total scheduling of $O(\triangle_{out}^2 \triangle_{in})$ time-slots.

Proof. By lemma 3.1, 3.2 and Brook's theorem (see [2]), there is a total scheduling of $O(\triangle_{out}^2 \triangle_{in})$ time slots, which can be constructed by greedy algorithm in polynomial time. □

4 Distributed algorithm

First, we define a procedure. The following procedure with the same r value and an appropriate out-neighbor y will be applied at every station x simultaneously.

Procedure Random Transmit-Broadcast((L_{xy}, B_x), r): In each round i, $1 \leq i \leq r$, station x transmit message to its out-neighbor y with probability p; broadcast with probability q; keep silent with probability $1 - p - q$, where the parameters r, p, and q will be defined in the following text.

Our randomized algorithm consists of $\triangle_{out}$ phases. Let $y_1, \cdots, y_k$ be out-neighbors of x in the network. In phase i, $(1 \leq i \leq k)$, each x applies the procedure Random transmit-broadcast ((L_{xy_i}, B_x), r).

For each directed edge (x,y), define A_{xy} to be the event that the station y fails to receive message from x during all r rounds in some phase.

For each vertex x, denote A_x by the event that the station x fails to broadcast during all $r\triangle_{out}$ rounds in all $\triangle_{out}$ phases.

Let $p = q = \frac{1}{2\triangle_{in}\triangle_{out}}$, we have

Lemma 4.1.
For any arc $(x,y) \in A$, $Pr(A_{xy}) \leq exp(-\frac{r}{4e\triangle_{in}\triangle_{out}})$.

Proof. The probability that x transmit to y with indegree d_{in} successfully in some single transmission step of the procedure is bounded below by

$$p(1-p-q)^{d_{in}} \geq \frac{1}{2\triangle_{in}\triangle_{out}}(1-\frac{1}{\triangle_{in}\triangle_{out}})^{\triangle_{in}} \geq \frac{1}{2\triangle_{in}\triangle_{out}}(\frac{1}{2e})^{\frac{1}{\triangle_{out}}} \geq \frac{1}{4e\triangle_{in}\triangle_{out}}.$$

Therefore,

$$Pr(A_{xy}) \leq (1-\frac{1}{4e\triangle_{in}\triangle_{out}})^r \leq exp(-\frac{r}{4e\,\triangle_{in}\,\triangle_{out}}).$$

□

Lemma 4.2.
For any vertex $x \in V$, $Pr(A_x) \leq exp(-\frac{r}{4e\triangle_{in}})$.

Proof. The probability that x broadcast successfully in some single transmission step of the procedure is bounded below by

$$q(1-p-q)^{\triangle_{in}\triangle_{out}} \geq \frac{1}{2\triangle_{in}\triangle_{out}}(1-\frac{1}{\triangle_{in}\triangle_{out}})^{\triangle_{in}\triangle_{out}} \geq \frac{1}{4e\triangle_{in}\triangle_{out}}.$$

Therefore,

$$Pr(A_x) \leq (1-\frac{1}{4e\triangle_{in}\triangle_{out}})^{r\triangle_{out}} \leq exp(-\frac{r}{4e\triangle_{in}}).$$

□

Let $r = 4e\triangle_{in}\triangle_{out}ln\frac{2n\triangle_{out}}{h}$, for some safety parameter $0 < h < 1$, by Lemma 4.1, 4.2, we get

Lemma 4.3.
The randomized distributed algorithm succeeds in total scheduling with probability $1-h$, where $0 < h < 1$.

Proof. The failure of the randomized distributed algorithm means that at least one of the bad events $\{A_{xy}, A_x, xy \in A, y \in V\}$ happened, so we have

$$\begin{aligned} Pr((\cup_{(x,y)\in A}A_{xy}) \cup (\cup_{x\in V}A_x)) &\leq \sum_{(x,y)\in A} Pr(A_{xy}) + \sum_{x\in v} Pr(A_x) \\ &\leq n\triangle_{out}exp(-\frac{r}{4e\triangle_{in}\triangle_{out}}) + n \times exp(-\frac{r}{4e\triangle_{in}}) \\ &\leq n\triangle_{out}\frac{h}{2n\triangle_{out}} + n(\frac{h}{2n\triangle_{out}})^{\triangle_{out}} \\ &\leq h \end{aligned}$$

□

Then we can derive the following theorem form above,

Theorem 4.4.
$\forall 0 < h < 1$, and $2 \leq max\{\triangle_{in}, \triangle_{out}\} \leq n$, the total scheduling has a randomized distributed algorithm requiring $O(\triangle_{in}\triangle_{out}^2 ln\frac{n}{h})$ time slots with success probability $1-h$ on any n-vertex directed graph G.

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