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A Dynamic Lot Size Model for Seasonal Products with Shipment Scheduling

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Abstract In this study we consider a dynamic lot size model for the case where single-item is produced and shipped by an overseas export company. We explore an optimal production scheduling with the constraint of production and shipment capacity so as to minimize the total cost over the finite planning horizon when the demands are deterministic. The optimal production schedule is obtained by a dynamic programming approach. We extend a dynamic lot size model to the case of incorporating shipping schedule into the model. And we deal with the model with backlogging and no backlogging, respectively. Some numerical examples are presented to illustrate optimal policies of the developed model under several demands and cost patterns.

Keywords Dynamic lot size model; Dynamic programming; Production Planning

1 Introduction

The production factory is operated to respond the pre-determined demands in advance and plans the production in order to satisfy the demands on time. The factory manager wishes to have a stable production level. Should the factory produce seasonal products, the demands fluctuate over a monthly or quarterly over the time horizon. Hence the production planning must be coordinated at the beginning of each period. Then he/she faces not only the production but also schedule the transportation problem. Usually they need a container or truck that should be filled up with the products. In other words, it is important for the company to make a schedule of transporting their products from the local factory to the consumer destinations. Therefore, the decision of a production and transportation must be made simultaneously because they depend on each other, reflecting the related costs during the seasons.

The Dynamic lot size problem is production scheduling over the periods so as to minimize total cost when the demands can change as the time goes but deterministic. The classical dynamic lot size model was first introduced by Wagner and Whitin[3]. Various extension models are based upon Wagner-Whitin model. There are some extented models

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which consider backlogging, production capacity limitation, quantity discount[2], replenishment[4], and multiple customers[8].In this study we consider a dynamic lot size model with shipping scheduling so as to minimize the total cost over the planning horizon for the case where single-item is produced and shipped by an overseas export company.

2 Model formulation

Before considering the problem with shipping scheduling, we explain a basic frameworks concerning a dynamic lot size model. At the begining of period a production is strated, and the demands are satisfied when it is shipped at the end of period. A shipping is one time in each period. Production and shipping have a limited capacity, and incur a setup cost. In addition, we consider the two cases of the relation between demands and shipping capacity. The first case is that the demands are less than the shipping capacity, that is, the demand cannot partially be satisfied. The second case is the one that the demands can be bigger than the shipping capacity. In this case, the demands can be spreaded, that is, the unsatisfied demands are delivered with delay. If the demands in period t exceeds shipping capacity then holdig cost and waiting cost incur in period t+1.

The planning horizon consists of n periods. Let i denotes each period, $i = 1, 2, \ldots, n$, we use the following notation:

K_i=setup cost for the production in period i
k_i=setup cost for the shipping in period i
D_i=demand in period i
I_i=units of inventory at the end of priod i
p=unit production cost
h=unit holding cost
s=unit shipping cost
w=unit waiting cost
R=production capacity
C=shipping capacity
Y_i=transportation cost after period i
V_i=the minimum of the total cost after period i

We assume all parameters are non-negative. p assumes the biggest in the unit costs, and the capacity of production and shipping capacity assumes $R \geq C$. We define

$$D(i, j) = D_i + \cdots + D_j$$

where $1 \leq i \leq j \leq n$. and define $D(i, j) = 0$ when $i > j$. The decision variable x_i at each period i is the amount of production. It is shown that the amount of production at period i can take either 0 or $D(i, j)$ for some $j \leq n$.

First of all, we discuss with the case where the demands in each period is $R \geq C \geq D_i$.we state the following lemmas for this case.

Lemma 1.
There is no shipping such that $I_j > 0$ when it has the shipping scheduling for $j - i + 1$ periods.

Proof. By assumption of $C \geq D_i$, there is no leftover of shipping for $j-i+1$ periods. That is, the schedule which produces and ships the demands of one period in each period exists at worst.

Lemma 2.
There exists an optimal production schedule $x = (x_1, \ldots, x_n)$ such that $x_i I_{i-1} = 0$ holds for each $i = 1, 2, \ldots, n$.

Proof. If $I_{i-1} = 0$, then there is a production in period i since no shortage of the demands in period i is allowed. That is, the production quantity in period i is the demands for $j-i+1$ periods which not exceed the production capacity. If $I_{i-1} > 0$, then the demands after period i which is produced in period i can be delayed to the next periods. The production cost does not increase while the holding cost exists. Therefore,there is no production for $j-i$ periods.

Assume without loss of generality that $Y_{n+1} = 0$ and $V_{n+1} = 0$ since there is no leftover at the end of planning horizon by Lemma 1.We drive the optimal solution by dynamic programming from this property. We have the transportation cost after period i as follows,

$$Y_i = \left\{ Y_{j+1} + k_i \delta\big(D(i,j)\big) + sD(i,j) - h \sum_{m=1}^{j-i} D(i+m, j) \right\}, \tag{1}$$

where $D(i,j) \leq C$,

$$\delta(d) = \begin{cases} 1 & \text{if } d > 0 \\ 0 & \text{otherwise} \end{cases} .$$

A production scheduling affects a shipping acheduling. A advance shipping can reduce the incured inventory holding cost by product the demands of several periods.

Hence, the minimum total cost after period i with transportation cost can be formulated as follows,

$$V_i = \min_{j \geq i} \left\{ V_{j+1} + K_i \delta\big(D(i,j)\big) + pD(i,j) + h \sum_{l=1}^{j-i} D(i+l, j) + Y_i - Y_{j+1} \right\}, \tag{2}$$

where $D(i,j) \leq R$.

We subtracts Y_{j+1} in Equation (2). Because Y_{j+1} is included both V_{j+1} and Y function. Then, we discuss with the case where the demands in period i $R \geq D_i \geq C$ and $R \leq C \leq D_i$. For $1 \leq i \leq j \leq n$, we define the new assumption as follows,

$$C(j+1) \geq D(1, j). \tag{3}$$

The following lemma is important to develop on algorithm in our model.

Lemma 3.
For $1 \leq i \leq j \leq n$, if we produce in priod i+1, then backordered demands in period i is met by a shipping in period i+1. And if we produce in period j, then there is a shipping in period j to meet backordered demands from previous periods.

Proof. If the demands in period i-1 is $D_{i-1} \geq C$,then there is a production in period i since the demnads in period i is not produced before period i. If a some demand is not met from shipping in period i, then it must be met from shipping in any future period by assumption (3). Therefore,the backordered demands cause holding and waitting costs. V_{n+1} is given by

$$V_{n+1} = \begin{cases} (h+w)I_n + Y_{n+1} & \text{if } I_n > 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where$Y_{n+1} = X_{n+1} + sI_n$.

By Lemma 3, we have the transportation cost after period i as follows.

$$Y_i = \left\{ Y_{i+1} + k_i\delta\big(D(i,i)\big) + sC \right\} \tag{5}$$

where $D_i \geq D(i,i) + I_{i-1} \geq C$

$$Y_i = \left\{ Y_{i+1} + k_i\delta\big(D(i,j)\big) + s\{ D(i,j) + I_{i-1}\} - h\sum_{m=1}^{j-i} D(i+m,j) \right\} \tag{6}$$

where $D_i \leq D(i,j) + I_{i-1} < C$.

If $I_{i-1} = 0$ is determinable, Equation (1) and (6) is the same equation. And we have the minimum total cost after period i with transportation cost as follows,

$$\begin{aligned} V_i = \min_{j \geq i} \Big\{ & V_{j+1} + K_i\delta\big(D(i,j)\big) + pD(i,j) \\ & + h\sum_{l=1}^{j-i} D(i+l,j) + (h+w)I_{i-1} + Y_i - Y_{j+1} \Big\} \end{aligned} \tag{7}$$

where $D(i,j) \leq R$.

3 Dynamic programming algorithm

In this section, we explain the algorithm for solving the vaule of V_i which can be described by dynamic programming with recursive property. We define $i,j = 1,2,\ldots,n$. Let V_{ij} denote the total cost when product from period i to period j. Let Y_{ij} denote the transportation cost when shipment from period i to period j. The initial value of each formulated model is defined by Lemma 1 and equation (4). If it computes the schedule of N periods, The number of solutions for V_i and Y_i is 2^{n-i}. We analysis V_i and Y_i in detail. In V_{ij} and Y_{ij}, period j has a transition from period i to period n. For any period i, we have

$$V_i = \min\left\{ V_{ii}, V_{ii+1}, \ldots, V_{in} \right\}.$$

V_{ij} consists of a various Y_{ij}. We also have

$$Y_i = \left\{ Y_{ii}, Y_{ii+1}, \ldots, Y_{in} \right\}$$

each V_{ij}. However Y_i is only the Y_{ij} which satisfies schedule of V_{ij}. The number of solutions for each j is 2^{n-j-1} where $(n-j-1)^+$. Therefore, we have $Y_{ij}^{(1)},\ldots,Y_{ij}^{(2^{n-j-1})}$ and $V_{ij}^{(1)},\ldots,V_{ij}^{(2^{n-j-1})}$ for each j of any period i.

The framework of computation has procedure as follows,

1. The computation starts from period n, and proceed to period 1.
2. V_i is computed after the computation of Y_i finished.
3. If the model consists a various demand pattern, it need to calculate a value of each demand pattern.

From these, the algorithm in from period 1 to period n can be shown as follows,

Step 1 Determine the planning horizon n. Simultaneously, input both the cost parameters and demands. Set n to i. Set Y_{i+1} and V_{i+1} as initial value from setting n=i.
Step 2 If the value of i is less than 0, terminate the program, otherwise set i to j.
Step 3 The first time it enters this step, set 1 to A. This step recurs $n-i+1$ times.
Step 4 The first time it enters this step, set 1 to B. This step recurs 2^{n-j-1} times where $(n-j-1)^+$.
Step 5 At first, calculate $Y_{ij}^{(B)}$ by fromulated eqations. Next, calculate $V_{ij}^{(B)}$ by fromulated eqations with Y_i which feasible from among the outputed solutions. Finally, select the minimum total cost from ever the computed V_i.
Step 6 Add 1 to B and go to Step 4. If the value of B exceeds 2^{n-j-1}, go to Step 7.
Step 7 Add 1 to j.
Step 8 Add 1 to A and go to Step 3. If the value of A exceeds $n-i+1$, go to Step 9.
Step 9 Subtract 1 from i and go to Step 2.

4 Example

The following example illustrates the algorithm discussed in section 3. we consider the five periods. we compute the two formulated models. The parameters are as follows:(K_i)=(500,750,1000,250,600,450);(X_i)=(80,100,200,150,120,130); (p,h,s,w,R,C) =(5,2,3,1,50,40);$I_0 = 0$.

The feasible solutions are restricted by production capacity and shipping capacity. Table 1 expresses by equations (1) and (2). In Table 1, the minimum total cost after period 1 is 2832. By tracing these equations, we have the solutions as follows:

$$\begin{aligned}
&V_6 = 0\\
&V_{45}^{(1)} = 0+250+240+50+414-0 = 954\\
&V_{23}^{(2)} = 954+750+170+32+584-414 = 2076\\
&V_{11}^{(6)} = 2076+500+110+730-584 = 2832
\end{aligned}$$

(×)						
×						
(×)	(×)					
×	×					
(×,×)	(×)	(×)				
×,×	×	×				
(846,×,650,×)	(584,×)	(466)	(×)			
3652,3202,3252,×	2526,2076	2332	954			
(962,×,766,×,730,×,×,×)	(816,×,620,×)	(662,×)	(414)	(195)	(0)	(Transport cost)
4482,4032,4082,×,3282,2832,×,×	3726,3276,3326,×	2732,2282	1404	820	0	Total cost
1 (22)	2 (18)	3 (16)	4 (23)	5 (25)	6 (0)	period (demand)

Table: 1: The optimal production schedule 1

$$
\begin{aligned}
&Y_6 = 0\\
&Y_{55}^{(1)} = 0+120+75 = 195\\
&Y_{44}^{(1)} = 195+150+69 = 414\\
&Y_{23}^{(1)} = 414+100+102-32 = 584\\
&Y_{11}^{(5)} = 584+80+66 = 730
\end{aligned}
$$

That is, the optimal production schedule is to produce 22 units in period 1, and 34 units in period 2, and 48 units in period 4. Then shipping schedule with the optimal production schedule is to ship 22 units in period 1, and 34 units in period 2, and 23 units in period 4, and 25 units in period 5. Table 2 expresses by equations (5), (6) and (7).In Table 1, the minimum total cost is 4005 after period 1. By tracing these equations,we have the solutions as follows:

$$
\begin{aligned}
&V_6 = 130+30 = 160\\
&V_{55}^{(1)} = 190+600+250+400-160 = 1280\\
&V_{44}^{(1)} = 1280+250+175+655-400 = 1960\\
&V_{23}^{(1)} = 1960+750+150+15+850-655 = 3080\\
&V_{11}^{(5)} = 3080+500+225+1050-850 = 4005
\end{aligned}
$$

(×) ×						
(×) ×	(×) ×					
(×,×) ×,×	(×) ×	(×) ×				
(×,×,×,×) ×,×,×,×	(850,×) 3080,×	(650) 2800	(×) ×			
(1260,×,1040,×, 1050,×,×,×) 5205,×,4805,×, 4005,×,×,×	(1060,×,840,×) 4280,×,3880,×	(870,×) 3200,×	(655) 1960	(400) 1280	(160) 190	(Transport cost) Total cost
1 (45) 0	2 (25) 5	3 (5) 0	4 (35) 0	5 (50) 0	6 (0) 10	period (demand) backlog

Table: 2: The optimal production schedule 2

$$
\begin{aligned}
Y_6 &= 130+30 = 160\\
Y_{55}^{(1)} &= 160+120+120 = 400\\
Y_{44}^{(1)} &= 400+150+105 = 655\\
Y_{23}^{(1)} &= 655+100+105-10 = 850\\
Y_{11}^{(5)} &= 850+80+120 = 1050
\end{aligned}
$$

That is,the optimal production schedule is to produce 45 units in period 1, and 30 units in period 2, and 35 units in period 4, and 50 units in period 5. Then shipping schedule with the optimal production schedule is to ship 40 units in period 1, and 40 units in period 2, and 35 units in period 4, and 40 units in period 5, and 10 units in period 6.

5 Conclusions

This paper presents a dynamic lot size model with scheduling of production and transportation in which the demands are satisfied by dispatch of vehicle within the framework of logistics. We show the two cases in which each period has the structure of a different cost-demand pattern. In particular, in the case of demand pattern that is more than shipping capacity, the algorithm structure is more complicated because of consideration two recursion patterns. For example it was shown that the production and transportation schedule can be provided simultaneously. However, we were not able to obtain many feasible solutions due to production and shipping capacity constraints. Furthermore it is a reason why a partial demand after period i+1 is not producted and shipped in period i. We need to extent this model under currently working.

Therefore, we have as the our future study for an extension model where a partial demand of deifferent periods is allowed to product and ship in any period. In other extension model, it would be interest to consider miltiple items and multiple vehicles.

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