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Australian Dollars Exchange Rate and Gold Prices: An Interval Method Analysis

Ai Han[1] Shanying Xu[1] Shouyang Wang[1]

[1]Institute of Systems Science, Academy of Mathematics and Systems Sciences,
Chinese Academy of Sciences, Beijing, 100190, China

Abstract This paper proposes an interval method to explore the relationship between the exchange rate of Australian dollar against US dollar and the gold price, using weekly, monthly and quarterly data. With the interval method, interval sample data are formed to present the volatility of variables. The ILS approach is extended to multi-model estimation and the computational schemes are provided. The empirical evidence suggests that the ILS estimates well characterize how the exchange rate relates to the gold price, both in the long-run and short-run. The comparison between the interval and point method indicates that the difference between the OLS and the ILS estimates is increasing from weekly data to quarterly data, since the lowest frequency point data lost the most information of volatility.

1 Introduction

The perception that freely floating exchange rates, after the breakdown of the Bretton Woods System, are rarely, if ever, at their equilibrium levels, has generated an increasing interest in modeling the relationships between exchange rates movements and economic parameters $[1-6]$. This paper concerns about the exchange rate movements between Australian dollar and US dollar (AUS/USD thereof), which are two important currencies in the foreign exchange rate market. The gold price is selected as the main factor that affects the AUS/USD nominal spot rate because there is no other industrial endeavor that has had such a profound economic effect on the Australian economy $[7,8]$. Therefore, the fluctuation of international gold price is crucial for the value of Australian dollar. In specifically determining the long-term and short-term relationships between the AUS/USD exchange rate and the gold price, the cointergration and error correction models are employed.

Previous literature concerning about the exchange rate uses primary measurements of time series data, that is, to quantify points. Yet in a financial market, many variables are bounded by intervals for a given period. For instance, the exchange rate has its daily (or weekly, or monthly) bounds, the lowest and the highest, and varies in between during a given trading period. Representing the variations with snap-shot points, say the closing price, only reflects a particular number at a particular time; it does not properly reflect its variability during the trading period. To address this issue, this paper proposes to use an interval method, which is technically based on interval analysis $[9-12]$, to conduct the investigation. Specifically, we first implement the interval least squares approach (ILS thereof) $[13]$ into the multi-model estimation to produce the estimates.

The aim of this paper is to study the long-term and short-term relationships between the exchange rate of AUS/USD and the gold price by means of the interval method. Section 2 provides the interval methodology. Section 3 presents the data. The empirical results are reported in Section 4. Section 5 concludes this paper.

2 Methodology

2.1 Theoretical Model

The standard models in classical point-based econometrics, consisting of cointegration model (1) and error correction model (ECM thereof) (2), are employed to motivate our investigation of the link between exchange rates and gold prices, which are assumed to hold the population of interest:

$$y_t = \beta_0 + \beta_1 x_t + e_t \tag{1}$$

where x_t and y_t denote the interval inputs of the gold price and the exchange rate of AUS/USD in the t^{th} period, respectively. β_0 and β_1 are point-valued parameters. The cointegration model allows for testing whether the long-run equilibrium relationship between the exchange rate of AUS/USD and the gold price exists.

$$dy_t = \alpha_0 + \alpha_1 e_{t-1} + \alpha_2 dx_t + u_t \tag{2}$$

where dx_t and dy_t denote the interval difference of the interval valued data calculated with interval arithmetic (see Section 2.2); e_{t-1} denotes the interval valued error correction term calculated with interval arithmetic from model (1) (see Section 2.2). α_0, α_1 and α_2 are point-valued parameters, and α_1 should be negative. The ECM is employed to detect dynamic short-term relationship between the exchange rate of AUS/USD and the gold price.

2.2 Differences between Interval Method and Point Method

First, with the interval method, all inputs are in terms of intervals rather than points. For instance, in foreign exchange rate market, the lowest and the highest spot rate, denoted *low* and *high* respectively, are selected to form the interval sample data of x_t and y_t in a given period as $p_t = [low_t, high_t]$. The interval inputs incorporate the extra information besides the pointed closing price-*last*, which is the most widely used point data.

As for dy_t and dx_t, they are not simply the computing results of interval subtraction, but what we call "interval difference (ID thereof)" that bears more economic significance. An ID indicates the interval changes of a variable between two consecutive periods. Suppose in period t and period $t+1$, the interval sample data are $[1,2]$ and $[0,3]$ respectively, then the ID $= [0,3] - [1,2] = [0-2, 3-1] = [-2,2]$. The lower bound -2 denotes the largest range of decrease from 2 to 0, while the upper bound 2 denotes the largest increasing range from 1 to 3. Therefore, an ID includes the whole range of variation in the two periods.

Remark 2.1. If $high_t = low_t = last_t$, this is the case when the variable does not vary at all in a given period, which is pretty rare in any financial market. With interval arithmetic, the point like number is substituted by a "degenerate" interval, i.e., $p_t = [last_t, last_t]$.

Secondly, population parameters are estimated with the ILS approach which is technically based on interval arithmetic, instead of ordinary least squares (OLS thereof). With interval arithmetic, the interval inputs, which contain more information than the point ones, are added, subtracted, multiplied, etc. The computed intervals of the interval arithmetic operations are exactly the ranges of the corresponding real operations. Thus, with the $[low_t, high_t]$ interval constructing approach, the whole value range of variables is incorporated into the estimation process.

Based on the ILS approach proposed in [13], we originally extend its application into multi-model estimation. The computation schemes are illustrated as follows:

Step 1. Input interval data pairs of $x_t = [\underline{x}_t, \bar{x}_t]$ and $y_t = [\underline{y}_t, \bar{y}_t]$ to model (1)

Step 2. Evaluate the interval matrix A_1 with interval arithmetic

$$\mathbf{A_1} = \begin{pmatrix} [n,n] & [\sum_{t=1}^{T} \underline{x}_t, \sum_{t=1}^{T} \bar{x}_t] \\ [\sum_{t=1}^{T} \underline{x}_t, \sum_{t=1}^{T} \bar{x}_t] & \sum_{t=1}^{T} [\underline{x}_t, \bar{x}_t]^2 \end{pmatrix}$$

Step 3. Calculate the interval vector b_1 with interval arithmetic

$b_1 = ([\sum_{t=1}^{T} \underline{y}_t, \sum_{t=1}^{T} \bar{y}_t] \ \ \sum_{t=1}^{T} [\underline{y}_t, \bar{y}_t][\underline{x}_t, \bar{x}_t])$

Step 4. Solve the interval linear system of equations $A_1(\hat{\beta}_0\ \hat{\beta}_1)^T = b_1$ for $(\hat{\beta}_0\ \hat{\beta}_1)^T$. While intuitively matching the center of two interval vectors and in the interval linear system of equations, we initially assume that $(\hat{\beta}_0\ \hat{\beta}_1)^T$ is a scalar vector. Let $A_{1,mid}$ be the midpoint matrix of A_1, and $b_{1,mid}$ be the midpoint vector of b_1. We then solve the non-interval linear system of equations $A_{1,mid}(\hat{\beta}_0\ \hat{\beta}_1)^T = b_{1,mid}$. The numerical estimations of the coefficients are obtained by using Gaussian elimination with scaled partial pivoting.

Step 5. Calculate the interval inputs of model (2) following formulas (3)-(5):

$$e_t = y_t - \hat{y}_t = [\underline{y}_t, \bar{y}_t] - [\underline{\hat{y}}_t, \bar{\hat{y}}_t] = [\underline{y}_t - \bar{\hat{y}}_t, \bar{y}_t - \underline{\hat{y}}_t] \tag{3}$$

where $\hat{y}_t$ denotes the fitted interval of y_t in the corresponding period.

$$dx_t = x_t - x_{t-1} = [\underline{x}_t, \bar{x}_t] - [\underline{x}_{t-1}, \bar{x}_{t-1}] = [\underline{x}_t - \bar{x}_{t-1}, \bar{x}_t - \underline{x}_{t-1}] \tag{4}$$

$$dy_t = y_t - y_{t-1} = [\underline{y}_t, \bar{y}_t] - [\underline{y}_{t-1}, \bar{y}_{t-1}] = [\underline{y}_t - \bar{y}_{t-1}, \bar{y}_t - \underline{y}_{t-1}] \tag{5}$$

Remark 2.2. Instead of constructing the confidence interval of y_t with the fitted point-based value and the standard error, with the interval method, $\hat{y}_t$ is directly obtained with formula (6), through which all information of independent variables is incorporated in the computing process.

$$\hat{y}_t = [\underline{\hat{y}}_t, \bar{\hat{y}}_t] = [\hat{\beta}_0, \hat{\beta}_0] + \hat{\beta}_1 [\underline{x}_t, \bar{x}_t] \tag{6}$$

Step 6. Evaluate the interval matrix A_2 with interval arithmetic

$$\mathbf{A_2} = \begin{pmatrix} [n,n] & [\sum_{t=2}^{T} \underline{e}_{t-1}, \sum_{t=2}^{T} \bar{e}_{t-1}] & [\sum_{t=2}^{T} \underline{dx}_t, \sum_{t=2}^{T} \bar{dx}_t] \\ [\sum_{t=2}^{T} \underline{e}_{t-1}, \sum_{t=2}^{T} \bar{e}_{t-1}] & \sum_{t=2}^{T} [\underline{e}_{t-1}, \bar{e}_{t-1}]^2 & \sum_{t=2}^{T} [\underline{e}_{t-1}, \bar{e}_{t-1}][\underline{dx}_t, \bar{dx}_t] \\ [\sum_{t=2}^{T} \underline{dx}_t, \sum_{t=2}^{T} \bar{dx}_t] & \sum_{t=2}^{T} [\underline{e}_{t-1}, \bar{e}_{t-1}][\underline{dx}_t, \bar{dx}_t] & \sum_{t=2}^{T} [\underline{dx}_t, \bar{dx}_t]^2 \end{pmatrix}$$

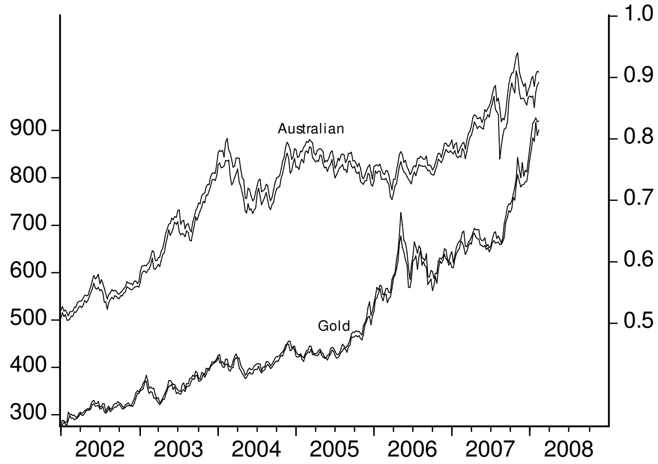
Step 7. Calculate the interval vector b_2 with interval arithmetic

$$b_2 = ([\sum_{t=2}^{T} \underline{dy}_t, \sum_{t=2}^{T} \bar{d}y_t] \quad \sum_{t=2}^{T} [\underline{dy}_t, \bar{d}y_t][\underline{e}_{t-1}, \bar{e}_{t-1}] \quad \sum_{t=2}^{T} [\underline{dy}_t, \bar{d}y_t][\underline{dx}_t, \bar{d}x_t])$$

Step 8. Solve the interval linear system of equations $A_2(\hat{\alpha}_0\ \hat{\alpha}_1\ \hat{\alpha}_2)^T = b_2$ for $(\hat{\alpha}_0\ \hat{\alpha}_1\ \hat{\alpha}_2)^T$.

3 Data Preprocessing

Three sets of data, weekly, monthly and quarterly, have been constructed for the exchange rate of AUS/USD and the New York COMEX Cash Gold Prices. The sample period, after adoption of the Euro, is from January 6, 2002 to February 10, 2008. All data are taken from the Reuters 3000 Xtra database. Each variable includes *low, high* and *last* in a given period. *low* and *high* are used to form the lower and upper bounds of interval sample data, while the *last* are used to form the point sample data to conduct the primary point method for comparison.



Note. Gold price in US dollars (Gold, left vertical axis), the exchange rate of AUS/USD (Australian, right vertical axis); January 6, 2002-February 10, 2008.

Figure 1: Weekly interval sample data

Figure 1 shows the weekly interval gold price series and intervals of the exchange rate of AUS/USD. The behavior of the gold price has been that of general appreciation, which is seen to range from about [US\$278, US\$280] at the beginning of 2002 to the maximum of [US\$917, US\$927] on February 3, 2008. The trend in Australian dollar has been that of general appreciation from [0.5, 0.52] at the beginning of 2002 to [0.76, 0.80] in February 2004; since then, it has been relatively stable, within the interval of [0.7, 0.8] in the following three years. Given the large range of fluctuations, all intervals have been converted into logarithms by interval arithmetic.

Table 1 Kwiatkowsky-Phillips-Schmidt-Shin test statistic of point sample data

Table 1A KPSS test statistic of levels of variables

Null Hypothesis: Tested variable is stationary

Levels of variables	Weekly	Monthly	Quarterly
x	2.03894**	1.130843**	0.705179*
y	1.729955**	0.970938**	0.598407*

Table 1B KPSS test statistic of the first differences of variables

Null Hypothesis: Tested variable is stationary

First Differences of Variables	Weekly	Monthly	Quarterly
dx	0.141746	0.175302	0.241667
dy	0.098122	0.175893	0.155948

Table 1C KPSS test statistic of the residuals of model. (1)

Null Hypothesis: Tested variable is stationary

First Differences of Variables	Weekly	Monthly	Quarterly
Residuals	0.391381	0.222923	0.139329

Note. The critical values of the KPSS test statistics with a constant are 0.739, 0.463 and 0.347 at 1%, 5% and 10% levels, respectively. '**'and '*' indicate significance at 1% and 5% level, respectively.

4 Empirical Results and Analysis

We implement the above computation schemes in C++ to conduct empirical experiments. And a comprehensive comparison between the ILS estimates and the OLS estimates is made.

In testing for stationarity, the Kwiatkowsky Phillips Schmidt Shin (KPSS) test is implemented for the point sample data. Table 1A and Table 1B indicate that both series are 1^{st} order stationary, while Table 1C shows that residuals calculated from model (1) are stationary, which indicates that the two point time series are cointegrated and the ECM is constructed. The ILS is conducted to the couple of models with interval sample data following Section 2.2. The OLS estimated parameters are reported in Table 2, while the ILS results are listed in Table 3.

Table 2 and Table 3 indicate some interesting findings. First, both the ILS and the OLS estimates represent a positive long-run relationship between the gold price and the exchange rate of AUS/USD. Assuming a sustained rise in gold price, which implies the increasing equilibrium of the exchange rate of AUS/USD, will increase Australia's export revenue from gold in the long run. This lends strong support to a stronger-than-expected Australian economy, which is more likely to lead to a tighter monetary policy that results in the appreciation of Australian dollar.

Secondly, the short-term variability of the exchange rate of AUD/USD is attributed to two parts: the error correction term e_{t-1}, and the contemporaneous variability of gold price dx_t. The speed-of-adjustment coefficient, i.e., α_1, suggests that approximately 1.6

percent (7.5 or 18.4) of the change in the exchange rate per week, month or quarter, can be attributed to the disequilibrium between actual and equilibrium levels. The ILS coefficient of e_{t-1} is negative, which is consistent with the point-based econometric theory. The ILS estimate of α_2 show that contemporaneous variability of the gold price induces a positive change in the exchange rate. Counteraction of the two results in the performance of dy_t.

Table 2: The OLS estimates of weekly, monthly and quarterly point sample data

	Weekly	Monthly	Quarterly
β_0	-2.930382**	-2.935939**	-2.92697**
	(0.087807)	(0.189648)**	(0.354652)
β_1	0.422235**	0.423247**	0.421776**
	(0.014300)	(0.030869)	(0.057664)
α_0	0.000741	0.003375	0.016309
	(0.000711)	(0.003083)	(0.012589)
α_1	-0.017988**	-0.076158**	-0.229122*
	(0.009191)	(0.037278)	(0.122134)
α_2	0.268771**	0.278966**	0.128683
	(0.029921)	(0.067909)	(0.185506)

Note. The OLS estimates from fitting the following models are reported in the above table.

$y_t=\beta_0+\beta_1*x_t+e_t$

$dy_t=\alpha_0+\alpha_1*e_{t-1}+\alpha_2*dx_t+u_t$

Numbers in parentheses are the standard error of estimates.'**'and '*' indicate significance at 5% and 10% levels, respectively.

Thirdly, there is an increasing difference between the OLS and the corresponding ILS estimate while moving from weekly to quarterly data. Since the OLS estimates are calculated from point sample data and thus is incapable of characterizing the volatility of either variable. Conversely, each interval input contains more variability information of the corresponding variable. Hence, the ILS estimates calculated from T pairs of sets, instead of T pairs of points, are more realistic. Since the variables fluctuate in a larger range when the length of the time interval is longer, quarterly point sample data involve much more loss of information. Consequently, quarterly data generally results in the biggest difference between the OLS and the ILS estimates.

Table 3: The ILS estimates of weekly, monthly and quarterly interval sample data

	Weekly	**Monthly**	**Quarterly**
β_0	-2.93445	-2.87709	-2.69903
β_1	0.422752	0.413338	0.383921
α_0	0.000821	0.004391	0.016938
α_1	-0.01602	-0.0755	-0.18404
α_2	0.252568	0.19172	0.151077

Note. The ILS estimates from fitting the following models are reported in the above table.

$y_t=\beta_0+\beta_1*x_t+e_t$

$dy_t=\alpha_0+\alpha_1*e_{t-1}+\alpha_2*dx_t+u_t$

5 Concluding Remarks

Traditional econometric models are employed to capture the relationships between the AUS/USD exchange rate and the gold price. In the presence of volatility in time series data, we propose to use interval sample data rather than point ones. The ILS approach is originally extended into multi-model estimation and the computational schemes are provided. The empirical results indicate that the ILS estimates well characterize how the AUS/USD exchange rate relates to the gold price, both in the long-run and the short-run.

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