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Network Dimensioning Problems of Applying Achievement Functions

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Abstract In this paper, we focus on approaches in network dimensioning where it allocates bandwidths and attempts to provide a proportionally fair treatment of all the competing classes. We will show that an achievement function can map different criteria onto a normalized scale subject to various utilities. The achievement function can be coped with the Ordered Weighted Averaging method. Moreover, it may be interpreted as a measure of QoS on All-IP networks. Using the achievement function, one can find a Pareto optimal allocation of bandwidths on the network under the available budget, which provides the so-called proportional fairness to each class. Consequently, this results in the similar satisfaction level to every connection in all classes.

Keywords multiple criteria optimization; achievement function; proportional fairness

1 Introduction

Taking a single global Internet Protocol (IP) based networks to carry all types of services, telecommunication is moving toward a converged network to replace the traditional separated packet switched and circuit switched networks [17]. This revolutionary converged All-IP networks not only reduces network deployment and management costs, but also offers a great deal of opportunity to introduce various new services that are not possible on the traditional separated networks.

The idea of a single shared physical network that will support multiple heterogeneous applications with different traffic characteristics and different Quality of Service (QoS) requirements, is widely regarded as a way to meet the telecommunication challenges of the future [3], [19]. Packet-switched networks have been proposed to offer the QoS guarantees in integrated-services networks because individual packets may exhibit a significant variation in network service quality.

Packet switched networks suffer three major quality problems in offering time-sensitive services: long delay time, jitter, packet loss. The Universal Mobile Telecommunications System (UMTS) offers teleservices (like speech or SMS) and bearer services which provide the capability for information transfer between access points. It

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is possible to negotiate and renegotiate the characteristics of a bearer service at connection establishment or during ongoing connection. Bearer services have different QoS parameters such as maximum transfer delay, delay variation and bit error rate. Thus, UMTS network services have different QoS classes for four types of traffic:

1. Conversational class (voice, video telephony, video gaming)
2. Streaming class (multimedia, video on demand, webcast)
3. Interactive class (web browsing, network gaming, database access)
4. Background class (email, SMS, downloading)

UMTS [17] has specified four different traffic classes according to their QoS requirements for different applications as shown in Table 1. An UMTS network consists of three interacting domains; Core Network (CN), UMTS Terrestrial Radio Access Network (UTRAN) and User Equipment (UE). The main function of the core network is to provide switching, routing and transit for user traffic. A core network also contains the databases and network management functions. The basic Core Network architecture for UMTS is based on GSM network with GPRS. UMTS uses channels with a fixed bandwidth of 5 Mbps, which is transmitted from a base station to all mobile stations [17]. Each station transmits its information using Code Division Multiple Access (CDMA). The available bandwidth can be divided between the users according to their needs. Therefore, a problem of network dimensioning with elastic traffic [11] requires to allocate bandwidth to maximize flows fairly.

Table 1: The Characteristics of UMTS Service Classes

Traffic Classes	Sensitivity to Jitter	Sensitivity to Delay	Sensitivity to Packet Loss
Conversational	high	high	low
Streaming	high	high	low
Interactive class	low	low	high
Background class	very low	low	high

Fair resource allocation problems are concerned with the allocation of limited bandwidth among competing activities so as to achieve the best overall performances of the system but providing fair treatment of all the competitors [8]. We introduce the methodology that allow the decision maker to explore a set of solutions that could satisfy users' preferences with respect to throughput and fairness (see [4], [5], [11], [12]). The formulation and analysis is carried out in a general utility-maximizing framework. Assume there are m classes in different QoS requirements. In this work, we will adopt the approach called proportional fairness (see [4], [5], [12]) to maximize the sum of logarithms of the bandwidth θ_i for each class i, $i = 1, \ldots, m$. The optimization model of the Proportional Fairness method takes the following form:

$$\max \sum_{i=1}^{m} \log(\theta_i) \tag{1}$$

Different users have different objectives for the network QoS. There are a number of characteristics that qualify QoS [20], including minimum queue delay, minimum delay variations, maximum capacity of consistent data throughput, etc. Multiple criteria decision methods are applied in some decision models to aggregate different criteria measurements and subjective preference information from the decision-makers (DMs). A more straightforward technique to represent the DMs' preferences in a decision model is through goals or reference points where the DMs can specify aspiration levels, i.e., desirable or preferable values for each criterion.

In a multiple criteria decision-making situation we often search for Pareto optimal solutions. In a typical multiple criteria optimization of realistic size, especially one with more than two objectives, the range of efficient solutions can be enormous. One scheme for dealing with multiple criteria models that permits more balanced handling of the objectives is simply to combine them in a weighted sum. Multiple objective functions can be combined into a single composite one by summing objectives with positive weights on maximizing and negative weights on minimizing [15]. Signs orient all objectives in the same direction, and weights reflect their relative importance. If a single weighted-sum objective model derived from a multiple-objective optimization produces an optimal solution, the solution is undominated by any other one; simply we call it a Pareto optimal solution of the multiple-objective model. In this work, we use the method of weighted sums to solve (1). When compared to weights, reference points provide a more direct way for the DMs to express their desires and, thus, to affect the solution.

In short, we deal with the problem of dimensioning bandwidth for elastic data applications in packet-switched communication networks, which can be considered as a multiple-objective optimization model. Users' satisfaction is summarized by means of their achievement functions where each user is allowed to request more than one type of service. The objective of the optimization problem is to determine the amount of required bandwidth for each class to maximize the users' satisfaction with a proper aggregate utility function. In this regard, we will focus on the following subjects:

i) how to construct the achievement functions involving utility functions;
ii) how to transform the different criterion measurement onto a normalized scale;
iii) how to allocate budgets with proportional fairness on UMTS networks.

The remaining parts of this paper are organized as follows. In the next section we introduce the network dimensioning problems where we construct the utility functions function to transform the different measurements onto a normalized scale. A numerical example is given in Section 3. Finally, in Section 4, we remark on summaries of this study.

2 Fair bandwidth allocation on network dimensioning problems

Consider a core network topology $G = (V, E)$, where V and E denote the set of nodes and the set of links in the network respectively. Given the total budget B and the maximal possible capacity of each link U_e, the objective is to allocate optimal bandwidth for all connections subject to QoS requirements b_i. Suppose there are m different classes which have their own QoS requirement. In each class, every connection is allocated the same bandwidth and it has the same QoS requirement. Suppose each connection is delivered between the same source and destination in this (core) network. Under a limited available budget, we want to allocate the bandwidth x_e in order to provide each class with maximal possible QoS.

2.1 Network constraints

Denote by S^i a set of connections in class i. Suppose there is the number of connections in S^i is K_i, i.e., $|S^i| = K_i$. Given the marginal cost κ_e of bandwidths for each link $e \in E$, it shall allocate the bandwidths in order to provide each class with maximal possible QoS. First, let x_e and θ_i be the bandwidth allocated to the link e and the connection j of class i respectively. Then these decision variables must be nonnegative:

$$x_e \geq 0, \; \forall\, e \in E \tag{2}$$

$$\theta_i \geq 0, \; \forall\, j \in S^i, \text{ for } i = 1, \ldots, m. \tag{3}$$

Furthermore, it requires the following constraints on the network:

$$\sum_{e \in E} \kappa_e x_e = B \tag{4}$$

and

$$x_e \leq U_e, \;\; \forall e \in E. \tag{5}$$

In each class i, every connection takes the same bandwidth and has the same bandwidth requirement and it produces $\theta_{i,1} = \theta_{i,2} = \cdots = \theta_{i,K_i}$. Let $\theta_i (= \theta_{i,1} = \theta_{i,2} = \cdots = \theta_{i,K_i})$ be the bandwidth allocated to each connection of class i. Thus, the constraint follows

$$\theta_i \geq b_i. \tag{6}$$

Next, for each connection j of class i, denote by p_j the routing path from the source o to destination d. To determine whether link e is chosen we define the binary decision variable

$$\chi_j^i(e) = \begin{cases} 1 & \text{if link } e \in p_j \\ 0 & \text{if link } e \notin p_j. \end{cases} \tag{7}$$

Thus, it yields the following constraint:

$$\sum_i \sum_j \chi_j^i(e) \theta_i = x_e, \; \forall\, e \in E. \tag{8}$$

Moreover, for each class i, it has

$$\theta_i \cdot \sum_e \kappa_e \chi_j^i(e) = c^i, \tag{9}$$

where c^i is a budget allocated to each connection of class i. Then, it gives

$$\sum_i (K_i \cdot c^i + \pi^i) = B, \tag{10}$$

where π^i is the reserved budget for each class i. Let $E_o \subseteq E$ be the set of links connected with the source node o, then we have

$$\sum_{e \in E_o} \chi_j^i(e) = 1, \; \forall\, i, j. \tag{11}$$

Let $E_d \subseteq E$ be the set of links connected with the destination node d, then we have

$$\sum_{e \in E_d} \chi_j^i(e) = 1, \; \forall\, i, j. \tag{12}$$

Let $E_v \subseteq E$ be the set of links that flow into the node v and $E'_v \subseteq E$ be the set of links that flow out of the node v, then we have

$$\sum_{e \in E_v} \chi_j^i(e) = \sum_{e \in E'_v} \chi_j^i(e), \; \forall\, i, j. \tag{13}$$

Let $\mathbf{x} = \{(x_e, \theta_i, \chi_j^i(e)) |\; \forall\, j \in S^i, \text{ for } i = 1, \ldots, m, \forall e \in E\} \in \mathbb{R}^n$ denote the vector of decision variables and $Q = \{\mathbf{x}|\mathbf{x} \text{ satisfies constraints } (2) - (13)\}$ denote the feasible set. We consider a resource allocation problem defined as an optimization problem with m objective functions, i.e.,

$$\max\{\mathbf{f}(\mathbf{x}) : \mathbf{x} \in Q\}, \tag{14}$$

where $\mathbf{f}(\mathbf{x})$=$\{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})\}$ is a vector-function that maps the decision space $\mathbb{R}^n$ into the criterion space $\mathbb{R}^m$.

2.2 Achievement functions with proportional fairness

In order to transform the different measurements onto a normalized scale, we construct the achievement function μ_i for each criteria i which can be viewed as an extension of the fuzzy membership function in terms of a strictly monotonic and concave utility function as shown in Figure 1 (see [11], [16], etc.)

We assume that the decision maker specifies requirements in aspiration and reservation levels by introducing desired and required values for several outcomes. Depending on the specified aspiration and reservation levels, a_i and r_i, respectively, we construct our achievement function of θ_i as follows:

$$\mu_i(\theta_i) = \log_{\alpha_i} \frac{\theta_i}{r_i}, \text{ where } \alpha_i = \frac{a_i}{r_i}. \tag{15}$$

Formally, we define $\mu_i(\cdot)$ over the range $[0, \infty)$, with $\mu_i(0) = -\infty$ and $\mu'_i(0) = \infty$. Depending on the specified reference levels, this achievement function can be interpreted as a measure of the decision maker's satisfaction with the value of the i-th

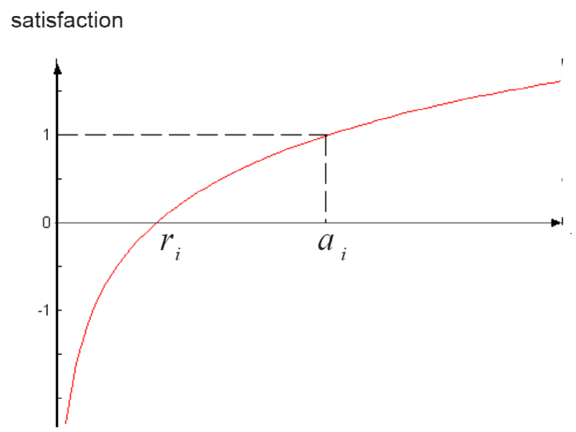

Figure 1: The Graph of an Achievement Function $\mu_i(\theta_i)$

criteria. It is a strictly increasing function of θ_i, having value 1 if $\theta_i = a_i$, and value 0 if $\theta_i = r_i$. The achievement function can map the different criterion values onto a normalized scale of the decision maker's satisfaction. Moreover, the logarithmic achievement function will be intimately associated with the concept of proportional fairness (see [5], [11], and [12].)

Proposition 1. *The achievement function $\mu_i(\theta_i)$ is continuous, increasing, and concave.*

These results are entirely consistent with those assumptions on the utility functions for end-to-end flow control in [5], where the objective is to maximize the aggregate source utility over their transmission rates. In addition, it takes an analytical approach devised to address decision-making problems where targets have been assigned to all the attributes. In other words, the DM may seek a satisfactory and sufficient solution through it. A key element is the achievement function that represents a mathematical expression of the unwanted deviation variables. Moreover, when some conditions hold the corresponding solution represents a balanced allocation among the achievement of the different goals. We will formulate the mathematical model of the fair bandwidth allocation by using the achievement function in the following sections.

2.3 Multiple objectives of applying achievement functions

Achievement functions are derived on the basis of reference points to project an arbitrary point to the set of nondominated attainable solutions. The achievement function is constructed in such a way that if the reference point is dominated, the optimization will advance past the reference point to a nondominated solution. By using the concept of the utility functions as mentioned, we can construct the achievement functions μ_i^T and μ_i^D for the throughput T and delay D respectively for each class i. Given the weight β_i^T and β_i^D of the throughput and delay for each class i and $\beta_i^T + \beta_i^D = 1$ for each $\beta_i^T, \beta_i^D \in (0,1)$. Then, for each class i, we consider the individual objective function f_i:

$$\begin{aligned}
f_i(\mathbf{x}) \triangleq & \beta_i^T \mu_i^T(\theta_i) + \beta_i^D \mu_i^D(\frac{\theta_i}{\lambda_i}) \\
= & \beta_i^T \log_{\alpha_i^T} \frac{\theta_i}{r_i^T} + \beta_i^D \log_{\alpha_i^D} \frac{\theta_i/\lambda_i}{r_i^D} \\
= & \beta_i^T \frac{\log \frac{\theta_i}{r_i^T}}{\log \alpha_i^T} + \beta_i^D \frac{\log \frac{\theta_i}{\lambda_i r_i^D}}{\log \alpha_i^D} \\
= & \log(\frac{\theta_i}{r_i^T})^{\frac{\beta_i^T}{\log \alpha_i^T}} + \log(\frac{\theta_i}{\lambda_i r_i^D})^{\frac{\beta_i^D}{\log \alpha_i^D}} \\
= & \log[(\frac{\theta_i}{r_i^T})^{\frac{\beta_i^T}{\log \alpha_i^T}} \cdot (\frac{\theta_i}{\lambda_i r_i^D})^{\frac{\beta_i^D}{\log \alpha_i^D}}] \\
= & \log[\frac{(\theta_i)^{(\frac{\beta_i^T}{\log \alpha_i^T} + \frac{\beta_i^D}{\log \alpha_i^D})}}{(r_i^T)^{(\frac{\beta_i^T}{\log \alpha_i^T})} \cdot (\lambda_i r_i^D)^{(\frac{\beta_i^D}{\log \alpha_i^D})}}] \\
= & \log[\frac{(\theta_i)^{G_i}}{H_i}] \\
= & G_i \log \theta_i - \log H_i
\end{aligned}$$

where λ_i represents the demand of bandwidth per unit time for class i. Let

$$\alpha_i^T \triangleq a_i^T / r_i^T$$

$$\alpha_i^D \triangleq a_i^D / r_i^D$$

and

$$G_i \triangleq \beta_i^T / \log \alpha_i^T + \beta_i^D / \log \alpha_i^D$$

be constant numbers as well as

$$H_i \triangleq (r_i^T)^{(\frac{\beta_i^T}{\log \alpha_i^T})} \cdot (\lambda_i r_i^D)^{(\frac{\beta_i^D}{\log \alpha_i^D})}.$$

Note θ_i, a function of $\mathbf{x}$, is the bandwidths allocated to class i. The individual objective function f_i is the function of the allocation pattern $\mathbf{x}$.

Next, it shows how to transform the multiple-objective problems to a single objective optimization subject to the fairness for each class. We will apply an approach by analyzing aggregation of outcomes $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_m(\mathbf{x}))$. This approach is introduced by Yager [21] as the so-called Ordered Weighted Averaging (OWA) Method. First, it defines the ordering map $\blacksquare : \mathbb{R}^m \to \mathbb{R}^m$ such that

$$\blacksquare(\mathbf{f}(\mathbf{x})) = (\, \Phi_1(\mathbf{f}(\mathbf{x})), \Phi_2(\mathbf{f}(\mathbf{x})), \ldots, \Phi_m(\mathbf{f}(\mathbf{x}))\,),$$

where

$$\Phi_1(\mathbf{f}(\mathbf{x})) \leq \Phi_2(\mathbf{f}(\mathbf{x})) \leq \ldots \leq \Phi_m(\mathbf{f}(\mathbf{x})).$$

There exists a permutation τ of set $S = \{1, 2, \ldots, m\}$ such that

$$\Phi_i(\mathbf{f}(\mathbf{x})) = f_{\tau(i)}(\mathbf{x})$$

for $i = 1, \ldots, m$. Then we define the cumulative ordering map $\tilde{\blacksquare}(\mathbf{f}(\mathbf{x})) = (\tilde{\Phi}_1(\mathbf{f}(\mathbf{x})), \ldots, \tilde{\Phi}_m(\mathbf{f}(\mathbf{x})))$ defined as $\tilde{\Phi}_i(\mathbf{f}(\mathbf{x})) = \sum_{k=1}^{i} \Phi_k(\mathbf{f}(\mathbf{x}))$, for $i = 1, 2, \ldots, m$. The coefficients of vector $\tilde{\blacksquare}(\mathbf{f}(\mathbf{x}))$ express, respectively: the smallest outcome, the total of the two smallest outcomes, the total of the three smallest outcomes, etc. Vector $\tilde{\blacksquare}(\mathbf{f}(\mathbf{x}))$ can be viewed graphically with a piecewise linear curve connecting point $(0,0)$ and points $(\frac{i}{m}, \frac{\tilde{\Phi}_i(\mathbf{f}(\mathbf{x}))}{m})$ for $i = 1, 2, \ldots, m$. Such a curve represents the absolute Lorenz curve as shown in Figure 2. Fair solutions to problem (14) can be expressed as

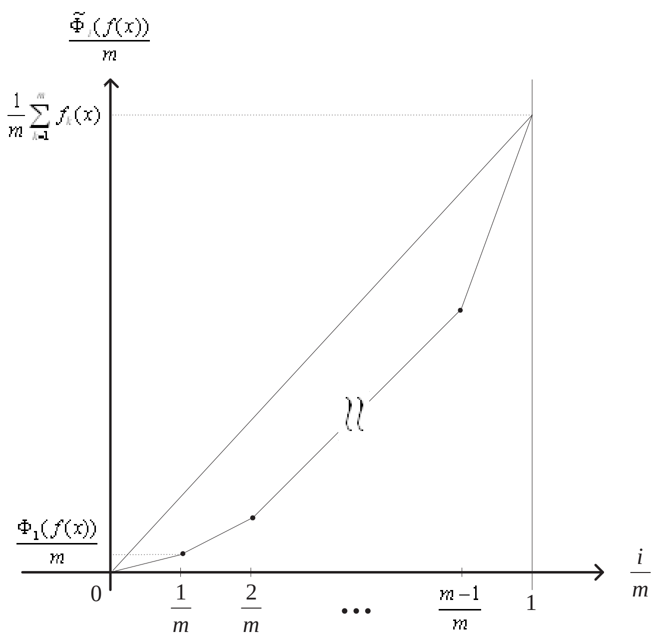

Figure 2: The Graph of a Absolute Lorenz Curve

Pareto-optimal solutions for the multiple criteria problem with objectives $\tilde{\blacksquare}(\mathbf{f}(\mathbf{x}))$

$$\max \{(\tilde{\Phi}_1(\mathbf{f}(\mathbf{x})), \tilde{\Phi}_2(\mathbf{f}(\mathbf{x})), \ldots, \tilde{\Phi}_m(\mathbf{f}(\mathbf{x}))) : \mathbf{x} \in Q\}. \qquad (16)$$

Theorem 2. *A feasible solution $\mathbf{x} \in Q$ is a fair solution of the resource allocation problem (14), if and only if it is a Pareto-optimal solution of the multiple criteria problem (16).*

The proof of this theorem is given in [11].

2.4 Majorization

We introduce the concept of majorization (see [9], [10]) to provide the fairness. For the n-dimensional decision vector $\mathbf{x}=(x_1,\ldots,x_n)$ of reals, let $x_{(1)} \leq \ldots \leq x_{(n)}$ denote the components of $\mathbf{x}$ in increasing order.

Definition 1. For $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^n$, $\mathbf{x} \leq_M \mathbf{y}$ if $\sum_{i=1}^{n} x_{(i)} = \sum_{i=1}^{n} y_{(i)}$ and $\sum_{i=1}^{k} x_{(i)} \geq \sum_{i=1}^{k} y_{(i)}$, for $k = 1,\ldots,n-1$. When $\mathbf{x} \leq_M \mathbf{y}$ then $\mathbf{x}$ is said to be **majorized** by $\mathbf{y}$.

If $\mathbf{x} \leq_M \mathbf{y}$, then the allocation $\mathbf{x}$ is more fair than $\mathbf{y}$. Next, we have the following definition.

Definition 2. A function $g : \mathbb{R}^n \to \mathbb{R}$ is called **Schur-concave**, if $\mathbf{x} \leq_M \mathbf{y}$ implies $g(\mathbf{x}) \geq g(\mathbf{y})$.

Thus, we present the following theorem which is adopted from [10].

Theorem 3. *Let h be an arbitrary real function and define $g(\mathbf{x})=\sum_{i=1}^{n} h(x_i)$ for $\mathbf{x} \in \mathbb{R}^n$, then g is Schur-concave if and only if h is concave.*

Recall that the achievement functions, μ_i and $\hat{\mu}_i$, are concave functions. According to this definition, it is easy to prove that the function f_i is Schur-concave.

In the following, we will introduce the concept of fairness by using the fair aggregation function (see [5], [10], [11], [12], [21]). Typical solution concepts for multiple criteria problems are defined by aggregation functions $g : \mathbb{R}^m \to \mathbb{R}$ to be maximized. Thus, (14) becomes

$$\max\{g(\mathbf{f}(\mathbf{x})) : \mathbf{x} \in Q\} \tag{17}$$

The simplest aggregation functions commonly used for the multiple criteria problem (14) are defined as the weighted sum of outcomes

$$g(\mathbf{f}(\mathbf{x})) = \sum_{i=1}^{m} w_i f_i(\mathbf{x}), \tag{18}$$

or the worst outcome

$$g(\mathbf{f}(\mathbf{x})) = \min_{i=1,\ldots,m} f_i(\mathbf{x}). \tag{19}$$

An aggregation (17) is fair if it is defined by a strictly increasing and strictly Schur-concave function g.

Definition 3. An aggregation function g satisfying all the following requirements (20), (21) and (22), we call the corresponding problem (17) a **fair aggregation** of problem (14). For all $i \in S = \{1,2,\ldots,m\}$,

$$g(f_1(\mathbf{x}),\ldots,f_{i-1}(\mathbf{x}),f_i'(\mathbf{x}),f_{i+1}(\mathbf{x}),\ldots,f_m(\mathbf{x})) < g(f_1(\mathbf{x}),\ldots,f_m(\mathbf{x})), \tag{20}$$

whenever $f_i'(\mathbf{x}) < f_i(\mathbf{x})$. For any permutation π of S,

$$g(f_{\pi(1)}(\mathbf{x}),f_{\pi(2)}(\mathbf{x}),\ldots,f_{\pi(m)}(\mathbf{x})) = g(f_1(\mathbf{x}),f_2(\mathbf{x}),\ldots,f_m(\mathbf{x})) \tag{21}$$

For any $0 < \varepsilon < z_{i'} - z_{i''}$, we have

$$g(f_1(\mathbf{x}),\ldots,f_{i'}(\mathbf{x})-\varepsilon,\ldots,f_{i''}(\mathbf{x})+\varepsilon,\ldots,f_m(\mathbf{x})) > g(f_1(\mathbf{x}),f_2(\mathbf{x}),\ldots,f_m(\mathbf{x})). \tag{22}$$

It is straightforward to see that $g(\mathbf{f}(\mathbf{x})) = \sum_{i=1}^{m} w_i f_i$ is a fair aggregation function according to this definition. Every optimal solution to the fair aggregation (17) of a resource allocation problem (14) defines some fair allocation scheme. In order to guarantee the consistency of the aggregated problem (17) with the maximization all individual objective functions in the original multiple criteria problem, the aggregation function must be strictly increasing with respect to every coordinate, i.e., (20). In order to guarantee the fairness of the solution concept, the aggregation function must be additionally symmetric (impartial), i.e., (21). Symmetric functions satisfying the requirement (22), are called strictly Schur-concave functions. Next, we give the following two theorems without proofs.

Theorem 4. *For a strictly concave, increasing function* $\mu_i : \mathbb{R} \to \mathbb{R}$*, the function* $g(\boldsymbol{f}(\mathbf{x})) = \sum_{i=1}^{m} w_i f_i(\mathbf{x})$ *is a strictly monotonic and strictly Schur-concave function.*

Theorem 5. *For a strictly concave, increasing function* $\mu_i : \mathbb{R} \to \mathbb{R}$*, the optimal solution of the problem* $\max\{\sum_{i=1}^{m} w_i f_i(\mathbf{x}) : \mathbf{x} \in Q\}$ *is a fair solution for resource allocation problem (14).*

These proofs are omitted here due to the page limitation.

2.5 Mathematical models

In the following, we adopt an effective modeling technique for quantities $\tilde{\Phi}_i(\mathbf{f}(\mathbf{x}))$ with arbitrary i. In [11], for a given outcome vector $\mathbf{f}(\mathbf{x})$ the quantity $\tilde{\Phi}_i(\mathbf{f}(\mathbf{x}))$ may be found by solving the following linear program:

$$\begin{aligned} \tilde{\Phi}_i(\mathbf{f}(\mathbf{x})) = \max \quad & i t_i - \sum_{k=1}^{m} d_k \\ \text{subject to} \quad & t_i - f_k(\mathbf{x}) \le d_k,\ k = 1,\ldots,m \\ & d_k \ge 0,\ k = 1,\ldots,m, \end{aligned} \tag{23}$$

where t_i is an unrestricted variable and nonnegative variables d_k represent their downside deviations from the value of t_i for several values $f_k(\mathbf{x})$. For example, the worst outcome of $i = 1$ may be defined by the following optimization:

$$\tilde{\Phi}_1(\mathbf{f}(\mathbf{x})) = \max\ \{t_1 : t_1 \le f_i(\mathbf{x}) \text{ for } i = 1,\ldots,m\}. \tag{24}$$

Formula (23) provides us with a computational formulation for the worst conditional mean $M_{\frac{k}{m}}(\mathbf{f}(\mathbf{x}))$ defined as the mean outcome for the k worst-off services, i.e.,

$$M_{\frac{k}{m}}(\mathbf{f}(\mathbf{x})) = \frac{1}{k}\tilde{\Phi}_k(\mathbf{f}(\mathbf{x})), \text{ for } k = 1,\ldots,m. \tag{25}$$

For $k = 1$, $M_{\frac{1}{m}}(\mathbf{f}(\mathbf{x})) = \tilde{\Phi}_1(\mathbf{f}(\mathbf{x})) = \Phi_1(\mathbf{f}(\mathbf{x}))$ which represents the minimum outcome. For $k = m$, $M_{\frac{m}{m}}(\mathbf{f}(\mathbf{x})) = \frac{1}{m}\tilde{\Phi}_m(\mathbf{f}(\mathbf{x})) = \frac{1}{m}\sum_{i=1}^{m}\Phi_i(\mathbf{f}(\mathbf{x})) = \frac{1}{m}\sum_{i=1}^{m} f_i(\mathbf{x})$ which represents the mean outcome.

For modeling various fair preferences one may use some combinations of the cumulative ordered outcomes $\tilde{\Phi}_i(\mathbf{f}(\mathbf{x}))$. In specific, for the weighted sum we obtain

$$\sum_{i=1}^{m} w_i \tilde{\Phi}_i(\mathbf{f}(\mathbf{x})). \tag{26}$$

Note that, due to the definition of map $\tilde{\Phi}_i$, the above function can be expressed in the form with weights $v_i = \sum_{j=i}^{m} w_i$ $(i = 1, \dots, m)$ allocated to coordinates of the ordered outcome vector. When substituting w_i with v_i, (26) becomes $\sum_{i=1}^{m} v_i \Phi_i(\mathbf{f}(\mathbf{x}))$, where $\sum_{i=1}^{m} v_i = 1$ and $v_i \geq 0$, $\forall i = 1, \dots, m$.

Applying the OWA method to problem (14), we get

$$\max \{\sum_{i=1}^{m} v_i \Phi_i(\mathbf{f}(\mathbf{x})) : \mathbf{x} \in Q\}. \tag{27}$$

If weights v_i are strictly decreasing and positive, that is $v_1 > v_2 > \dots > v_{m-1} > v_m > 0$, then each optimal solution of the OWA problem (27) is a fair solution of (14). Actually, formulas (23) and (26) allow us to formulate the following mathematical programming of the original multiple criteria problem:

$$\begin{aligned}
\text{maximize} \quad & \sum_{i=1}^{m} w_i \psi_i \\
\text{subject to} \quad & \psi_i = it_i - \sum_{k=1}^{m} d_{ki}, \ \forall i = 1, \dots, m \\
& t_i - d_{ki} \leq f_k(\mathbf{x}), \ \forall i, \ k = 1, \dots, m \\
& d_{ki} \geq 0, \ \forall i, \ k = 1, \dots, m \\
& t_i \text{ unrestricted}, \ \forall i = 1, \dots, m, \\
& \mathbf{x} \in Q,
\end{aligned} \tag{28}$$

where $w_m = v_m$, $w_i = v_i - v_{i+1}$ for $i = 1, \dots, m-1$, $v_i \in (0,1)$ for each i, and $\sum_{i=1}^{m} v_i = 1$. The individual function ψ_i is the first i sum of the ordered multiple objective functions $\blacksquare(\mathbf{f}(\mathbf{x}))$ in the allocation pattern $\mathbf{x} \in Q$.

2.6 Piecewise linear form of the achievement functions

In implementation, it is convenient to handle a linear function rather than a concave function. Depending on the specified aspiration and reservation levels for each class i, a_i and r_i, respectively, we construct the achievement function $\hat{\mu}_i(\theta_i)$ of bandwidth θ_i as a piecewise linear function (29). Between r_i and a_i, We have break points $r_i = k_{i,0} \leq k_{i,1} \leq \dots \leq k_{i,n-1} \leq k_{i,n} = a_i$. We assume $k_{i,l} - k_{i,l-1}$ are the same for all $l = 1, \dots, n$. Moreover, we denote the point b_i to represent the minimal bandwidth requirement for each class i. We will give the following propositions for the achievement function.

Lemma 6. *Let κ be the cheapest cost per unit bandwidth given in an end-to-end path. Suppose the total budget is B. There exists a finite number $M_i \leq B/\kappa K_i$ such that $\theta_i \leq M_i$, $\forall$ i, where K_i is the number of connections in class i.*

Since pages are limited, proofs of the following results are skipped and will be provided under request. We define $\hat{\mu}_i(\cdot)$ over the range $[0, M_i]$. Depending on the specified reference levels, this achievement function can be interpreted as a measure of the decision maker's satisfaction with the value of the i-th criteria [7]. It is a strictly increasing function of θ_i, having value 1 if $\theta_i = a_i$, and value 0 if $\theta_i = r_i$.

Practically, we define

$$\hat{\mu}_i(\theta_i) = \begin{cases} -M & \text{for } 0 \le \theta_i < b_i \\ \rho_0 \cdot (\theta_i - k_{i,0}) & \text{for } b_i \le \theta_i < r_i \\ \rho_1 \cdot (\theta_i - k_{i,1}) + \mu_i(k_{i,1}) & \text{for } r_i \le \theta_i < k_{i,1} \\ \rho_2 \cdot (\theta_i - k_{i,2}) + \mu_i(k_{i,2}) & \text{for } k_{i,1} \le \theta_i < k_{i,2} \\ \vdots & \\ \rho_{n-1} \cdot (\theta_i - k_{i,n-1}) + \mu_i(k_{i,n-1}) & \text{for } k_{i,n-2} \le \theta_i < k_{i,n-1} \\ \rho_n \cdot (\theta_i - k_{i,n}) + 1 & \text{for } k_{i,n-1} \le \theta_i < a_i \\ \rho_M \cdot (\theta_i - M_i) + \mu_i(M_i) & \text{for } a_i \le \theta_i \le M_i. \end{cases} \tag{29}$$

Denote $\alpha_i = \frac{a_i}{r_i}$, we have parameters $\mu_i(M_i) = \log_{\alpha_i} M_i/r_i$ and $\mu_i(k_{i,l}) = \log_{\alpha_i} k_{i,l}/r_i$ for $l = 1, \ldots, n-1$. Moreover, the parameters $\rho_0 = M/(r_i - b_i)$, $\rho_M = (\mu_i(M_i) - 1)/(M_i - a_i)$ and

$$\rho_l = \frac{n \log_{\alpha_i}(k_{i,l}/k_{i,l-1})}{a_i - r_i}, \text{ for } l = 1, \ldots, n,$$

represent the slope on l-th line segment for $l = 0, 1, \ldots, n$ and M. The slope of each segment represents a different marginal rate of satisfaction. The increasing (decreasing) rate case means that the DM wishes to attach a larger (smaller) marginal satisfaction depending on how far the achievement of the goal is from its target.

Next, we present an appealing property of the achievement function (15), which holds in the bandwidth allocation problem we are studying.

Proposition 7. *The achievement function $\hat{\mu}_i(\theta_i)$ is continuous, increasing, and concave.*

Given the budget and a network, one may choose proper values of r_i and a_i such that $\theta_i \in [r_i, a_i]$ for each connection, namely, the bandwidth θ_i in terms of transmission rates is always manageable between r_i and a_i. Thus, $\hat{\mu}_i(\theta_i)$ is well defined. Next, we focus on investigating the properties when $\theta_i \in [r_i, a_i]$.

Lemma 8. *Let $\hat{\mu}_i^{(n)}(\theta_i) : [r_i, a_i] \to [0, 1]$, where n means the number of break points, be defined as the achievement function (15) restricted on $[r_i, a_i]$. Then the sequence of functions $\{\hat{\mu}_i^{(n)}(\theta_i)\}_{n=1}^{\infty}$ converges uniformly to $\mu_i(\theta_i) = \log_{\alpha_i}(\theta_i/r_i)$ on $[r_i, a_i]$.*

Proof. Let $\varepsilon > 0$ be given. Choose $N = \frac{(r_i - a_i)}{\varepsilon \rho_M}$ such that $n \ge N$, implies

$$\begin{aligned} |\hat{\mu}_i^{(n)}(\theta) - \mu_i(\theta_i)| \le & |\hat{\mu}_i^{(n)}(k_{i,l}) - \hat{\mu}_i^{(n)}(k_{i,l-1})| \\ = & |\mu_i(k_{i,l}) - \mu_i(k_{i,l-1})| \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{\rho_l} |k_{i,l} - k_{i,l-1}| \\
&\leq \frac{1}{\rho_M} \cdot \frac{r_i - a_i}{n} \\
&< \varepsilon,
\end{aligned}
$$

for each $\theta \in [r_i, a_i]$. The first inequality holds because both $\hat{\mu}_i^{(n)}(\theta)$ and $\mu_i(\theta_i)$ are increasing functions. Moreover, there must be an interval $[k_{i,l-1}, k_{i,l}) \subseteq [r_i, a_i]$ contains θ for each $\theta \in [r_i, a_i]$. Thus, by the definition of uniformly convergence, $\{\hat{\mu}_i^{(n)}\}$ converges uniformly to $\log_{\alpha_i}(\theta_i/r_i)$ on $[r_i, a_i]$. □

Theorem 9. *If $r_i \leq \theta_i \leq a_i$, then the ε-proportionally fair bandwidth allocation obtained by using (15) as objective function approaches to proportional fairness as $n \to \infty$.*

Proof. When using the logarithm function $\log_{\alpha_i}(\theta_i/r_i)$ as the associated objective function for each class i, Luh and Wang [6] showed that this bandwidth allocation provided proportional fairness. By Lemma 8, the achievement function (15) converges uniformly to $\log_{\alpha_i}(\theta_i/r_i)$. So, the fair bandwidth allocation, obtained by using (15) as objective function, approaches to proportional fairness. □

The achievement function is intimately associated with the concept of proportional fairness. The achievement function (15) can also map the different criterion values onto a normalized scale of the decision maker's satisfaction. When taking the achievement function which is considered as a measure of QoS on All-IP networks, one can formulate the mathematical model of the fair bandwidth allocation in a network.

3 A Numerical Example

Consider a network topology $G = (V, E)$ (as shown in Figure 3), where $V = \{$node 1, node 2, ..., node 7$\}$ and $E = \{e_k,\ k = 1, 2, \ldots, 14\}$ denote the set of nodes and the set of links in the network respectively. Let node 1 and node 7 be the source and destination respectively. Each connection is delivered from o to d. Given the cost taking account of delay and the purchasing cost of bandwidth for each link: $\kappa_1 = \$5$, $\kappa_2 = \$6$, $\kappa_3 = \$10$, $\kappa_4 = \$5$, $\kappa_5 = \$4$, $\kappa_6 = \$11$, $\kappa_7 = \$6$, $\kappa_8 = \$8$, $\kappa_9 = \$6$, $\kappa_{10} = \$7$, $\kappa_{11} = \$12$, $\kappa_{12} = \$6$, $\kappa_{13} = \$5$, and $\kappa_{14} = \$6$. There are also given the maximal capacity of each link: $U_1 = 2,300$ kbps (i.e. kilobits/sec), $U_2 = 3,500$ kbps, $U_3 = 1,000$ kbps, $U_4 = 2,500$ kbps, $U_5 = 2,100$ kbps, $U_6 = 2,200$ kbps, $U_7 = 2,000$ kbps, $U_8 = 3,000$ kbps, $U_9 = 2,100$ kbps, $U_{10} = 2,700$ kbps, $U_{11} = 1,500$ kbps, $U_{12} = 1,800$ kbps, $U_{13} = 3,000$ kbps, and $U_{14} = 3,500$ kbps.

There are given three classes (as Table 2 shows), where class 1 has the highest priority and class 3 has the lowest priority. The maximal possible number of connections in each class is 10. Under the total available budget $B = \$130,000$, we want to allocate the bandwidths in order to provide each class with maximal possible quality of service (QoS) defined via (15).

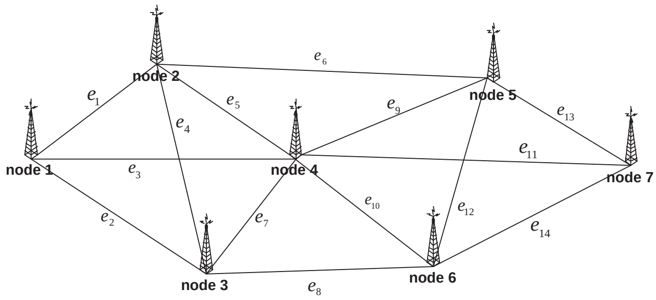

Figure 3: Sample Network Topology

Table 2: The Characteristics of Each Class

class	bandwidth requirement	aspiration level of bandwidth	reservation level of bandwidth	reserved budget	maximal number of connections
1	160 kbps	334 kbps	167 kbps	$18,000	10
2	80 kbps	166 kbps	83 kbps	$9,000	10
3	25 kbps	56 kbps	28 kbps	$3,000	10

3.1 A Mathematical Programming

Let x_k be the bandwidth allocated to the link $e_k \forall k = 1,2,\ldots,14$. We also let θ_i be the bandwidth allocated to the connection j of class i $\forall i = 1,2,3$. For each class i, we consider the objective function f_i as below:

$$f_1(\theta_1) = \log_2 \frac{\theta_1}{1670}, \tag{30}$$

$$f_2(\theta_2) = \log_2 \frac{\theta_2}{830}, \tag{31}$$

$$f_3(\theta_3) = \log_2 \frac{\theta_3}{280}, \tag{32}$$

where θ_i is the bandwidth allocated to class i. Suppose each objective is regarded as important as each other. Thus, each objective function has equal weight $w_1 = w_2 = w_3 = \frac{1}{3}$. Then, we can formulate the mathematical model as follows.

As the formulation of (MP1), we have the following mathematical model (MP2):

$$\begin{aligned}
\text{maximize} \quad & \frac{1}{3}\log_2 \frac{\theta_1}{1670} + \frac{1}{3}\log_2 \frac{\theta_2}{830} + \frac{1}{3}\log_2 \frac{\theta_3}{280} \\
\text{subject to} \quad & 5x_1 + 6x_2 + 10x_3 + 5x_4 + 4x_5 + 11x_6 + 6x_7 + 8x_8 + 6x_9 \\
& \quad + 7x_{10} + 12x_{11} + 6x_{12} + 5x_{13} + 6x_{14} = 130,000 \\
& (10c^1 + 18,000) + (10c^2 + 9,000) + (10c^3 + 3,000) = 130,000 \\
& \theta_i \cdot (5\chi_j^i(e_1) + 6\chi_j^i(e_2) + 10\chi_j^i(e_3) + 5\chi_j^i(e_4) + 4\chi_j^i(e_5)
\end{aligned}$$

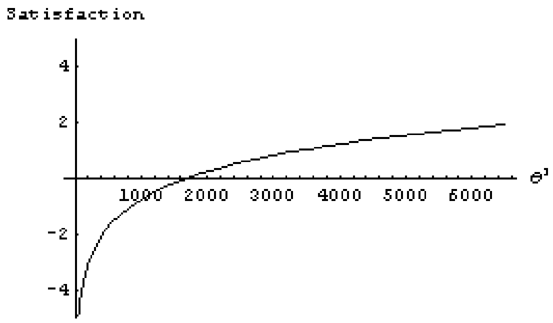

Figure 4: The Graph of $f_1(\theta_1)$

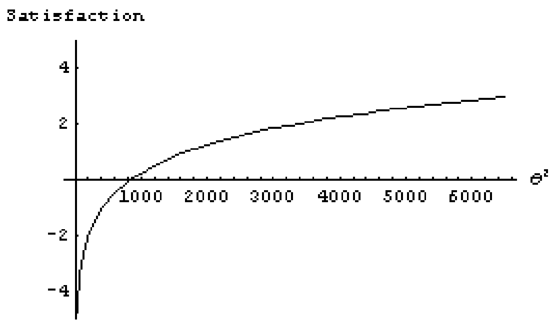

Figure 5: The Graph of $f_2(\theta_2)$

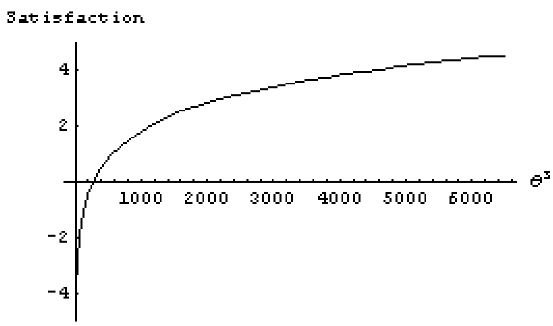

Figure 6: The Graph of $f_3(\theta_3)$

$$
\begin{aligned}
&+11\chi_j^i(e_6)+6\chi_j^i(e_7)+8\chi_j^i(e_8)+6\chi_j^i(e_9)+7\chi_j^i(e_{10})\\
&+12\chi_j^i(e_{11})+6\chi_j^i(e_{12})+5\chi_j^i(e_{13})+6\chi_j^i(e_{14}))=c^i,\\
&\quad \forall j=1,\ldots,10,\ \forall i=1,2,3
\end{aligned}
$$

$$
\begin{aligned}
&\theta_{1,1}=\theta_{1,2}=\cdots=\theta_{1,10}\geq 160\\
&\theta_{2,1}=\theta_{2,2}=\cdots=\theta_{2,10}\geq 80\\
&\theta_{3,1}=\theta_{3,2}=\cdots=\theta_{3,10}\geq 25\\
&\sum_{j=1}^{10}\theta_i=\theta_i,\ \forall i=1,2,3\\
&\sum_{i=1}^{3}\sum_{j=1}^{10}\chi_j^i(e_k)\cdot\theta_i=x_k,\ \forall k=1,\ldots,14\\
&\chi_j^i(e_k)=0 \text{ or } 1,\ \ \forall i=1,2,3,\ \forall j=1,\ldots,10,\text{and } k=1,\ldots,14\\
&0\leq x_k\leq U_k,\ \forall k=1,\ldots,14,
\end{aligned}
$$

where $U_1=2,300$, $U_2=3,500$, $U_3=1,000$, $U_4=2,500$, $U_5=2,100$, $U_6=2,200$, $U_7=2,000$, $U_8=3,000$, $U_9=2,100$, $U_{10}=2,700$, $U_{11}=1,500$, $U_{12}=1,800$, $U_{13}=3,000$, and $U_{14}=3,500$.

3.2 Numerical experiments with ILOG

Since the objective functions f_i in (30)-(32) are logarithmic functions which can not be solved by ILOG software. To overcome this problem, we replace f_i by piece-wise linear functions $\hat{f}_i$ for each $i=1,2,3$.

$$
\hat{f}_1(\theta_i)=\begin{cases}
2(\theta_1-835)-1 & \text{for } 0\leq\theta_1<835\\
\frac{1}{835}(\theta_1-835)-1 & \text{for } 835\leq\theta_1<1670\\
\frac{0.42}{557}(\theta_1-1670) & \text{for } 1670\leq\theta_1<2227\\
\frac{0.32}{557}(\theta_1-2227)+0.42 & \text{for } 2227\leq\theta_1<2784\\
\frac{0.26}{556}(\theta_1-2784)+0.74 & \text{for } 2784\leq\theta_i<3340\\
\frac{0.56}{1580}(\theta_1-3340)+1 & \text{for } 3340\leq\theta_1<4920\\
\frac{0.4}{1580}(\theta_1-4920)+1.56 & \text{for } 4920\leq\theta_1\leq 6500
\end{cases}
\tag{33}
$$

$$
\hat{f}_2(\theta_2)=\begin{cases}
2(\theta_2-415)-1 & \text{for } 0\leq\theta_2<415\\
\frac{1}{415}(\theta_2-415)-1 & \text{for } 415\leq\theta_2<830\\
\frac{0.42}{277}(\theta_2-830) & \text{for } 830\leq\theta_2<1107\\
\frac{0.32}{277}(\theta_2-1107)+0.42 & \text{for } 1107\leq\theta_2<1384\\
\frac{0.26}{276}(\theta_2-1384)+0.74 & \text{for } 1384\leq\theta_2<1660\\
\frac{1.30}{2420}(\theta_2-1660)+1 & \text{for } 1660\leq\theta_2<4080\\
\frac{0.67}{2420}(\theta_2-4080)+2.30 & \text{for } 4080\leq\theta_2\leq 6500
\end{cases}
\tag{34}
$$

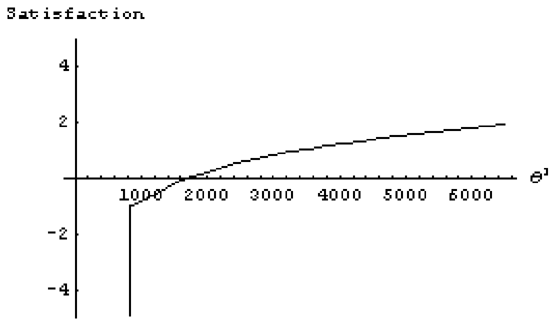

Figure 7: The Graph of $\hat{f}_1(\theta_1)$

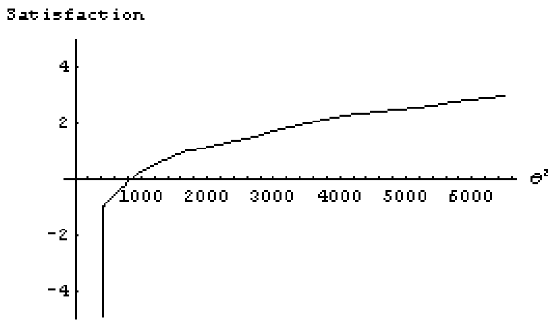

Figure 8: The Graph of $\hat{f}_2(\theta_2)$

$$\hat{f}_3(\theta_3) = \begin{cases} 2(\theta_3 - 140) - 1 & \text{for } 0 \leq \theta_3 < 140 \\ \frac{1}{140}(\theta_3 - 140) - 1 & \text{for } 140 \leq \theta_3 < 280 \\ \frac{0.41}{93}(\theta_3 - 280) & \text{for } 280 \leq \theta_3 < 373 \\ \frac{0.32}{93}(\theta_3 - 373) + 0.41 & \text{for } 373 \leq \theta_3 < 466 \\ \frac{0.27}{94}(\theta_3 - 466) + 0.73 & \text{for } 466 \leq \theta_3 < 560 \\ \frac{2.66}{2970}(\theta_3 - 560) + 1 & \text{for } 560 \leq \theta_3 < 3530 \\ \frac{0.88}{2970}(\theta_3 - 3530) + 3.66 & \text{for } 3530 \leq \theta_3 \leq 6500 \end{cases} \tag{35}$$

The break points for $\hat{f}_1(\theta_1)$ are 0, 835, 1670, 2227, 2784, 3340, 4920, and 6500, we proceed as follows:

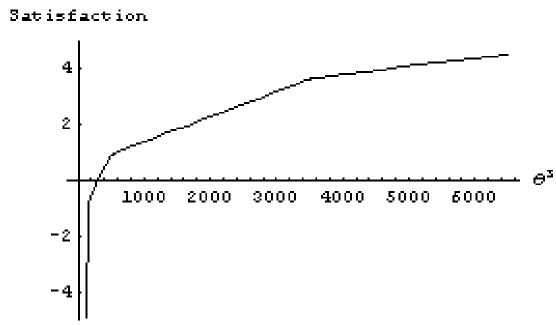

Figure 9: The Graph of $\hat{f}_3(\theta_3)$

Step 1 Replace $\hat{f}_1(\theta_1)$ by

$$\begin{aligned}\hat{f}_1(\theta_1) =& z_1^1\hat{f}_1(0)+z_2^1\hat{f}_1(835)+z_3^1\hat{f}_1(1670)+z_4^1\hat{f}_1(2227)\\ &+z_5^1\hat{f}_1(2784)+z_6^1\hat{f}_1(3340)+z_7^1\hat{f}_1(4920)+z_8^1\hat{f}_1(6500).\\ =&-1671z_1^1-1z_2^1+0.42z_4^1+0.74z_5^1+z_6^1+1.56z_7^1+1.96z_8^1.\end{aligned} \tag{36}$$

Step 2 Add the following constraints:

$$\theta_1 = 0z_1^1+835z_2^1+1670z_3^1+2227z_4^1+2784z_5^1+3340z_6^1+4920z_7^1+6500z_8^1 \tag{37}$$

$$z_1^1 \le y_1^1 \tag{38}$$

$$z_k^1 \le y_{k-1}^1 + y_k^1,\ \forall k = 2,\dots,7 \tag{39}$$

$$z_8^1 \le y_7^1 \tag{40}$$

$$\sum_{k=1}^{8} z_k^1 = 1 \tag{41}$$

$$\sum_{k=1}^{7} y_k^1 = 1 \tag{42}$$

$$y_k^1 = 0 \text{ or } 1, \forall k = 1,2,\dots,7 \tag{43}$$

$$z_k^1 \ge 0, \forall k = 1,2,\dots,8. \tag{44}$$

The break points for $\hat{f}_2(\theta_2)$ are 0, 415, 830, 1107, 1384, 1660, 4080, and 6500, we proceed as follows:

Step 3 Replace $\hat{f}_2(\theta_2)$ by

$$\begin{aligned}\hat{f}_2(\theta_2) =& z_1^2\hat{f}_2(0)+z_2^2\hat{f}_2(415)+z_3^2\hat{f}_2(830)+z_4^2\hat{f}_2(1107)\\ &+z_5^2\hat{f}_2(1384)+z_6^2\hat{f}_2(1660)+z_7^2\hat{f}_2(4080)+z_8^2\hat{f}_2(6500).\\ =&-831z_1^2-1z_2^2+0.42z_4^2+0.74z_5^2+z_6^2+2.30z_7^2+2.97z_8^2.\end{aligned} \tag{45}$$

Step 4 Add the following constraints:

$$\theta_2 = 0z_1^2 + 415z_2^2 + 830z_3^2 + 1107z_4^2 + 1384z_5^2 + 1660z_6^2 + 4080z_7^2 + 6500z_8^2 \tag{46}$$

$$z_1^2 \le y_1^2 \tag{47}$$

$$z_k^2 \le y_{k-1}^2 + y_k^2,\ \forall k = 2,\ldots,7 \tag{48}$$

$$z_8^2 \le y_7^2 \tag{49}$$

$$\sum_{k=1}^{8} z_k^2 = 1 \tag{50}$$

$$\sum_{k=1}^{7} y_k^2 = 1 \tag{51}$$

$$y_k^2 = 0 \text{ or } 1, \forall k = 1,2,\ldots,7 \tag{52}$$

$$z_k^2 \ge 0, \forall k = 1,2,\ldots,8. \tag{53}$$

The break points for $\hat{f}_3(\theta_3)$ are 0, 140, 280, 373, 466, 560, 3530, and 6500, we proceed as follows:

Step 5 Replace $\hat{f}_3(\theta_3)$ by

$$\begin{aligned}\hat{f}_3(\theta_3) =& z_1^3\hat{f}_3(0) + z_2^3\hat{f}_3(140) + z_3^3\hat{f}_3(280) + z_4^3\hat{f}_3(373)\\ &+ z_5^3\hat{f}_3(466) + z_6^3\hat{f}_3(560) + z_7^3\hat{f}_3(3530) + z_8^3\hat{f}_3(6500).\\ =& -281z_1^3 - 1z_2^3 + 0.41z_4^3 + 0.73z_5^3 + z_6^3 + 3.66z_7^3 + 4.54z_8^3.\end{aligned} \tag{54}$$

Step 6 Add the following constraints:

$$\theta_3 = 0z_1^3 + 140z_2^3 + 280z_3^3 + 373z_4^3 + 466z_5^3 + 560z_6^3 + 3530z_7^3 + 6500z_8^3 \tag{55}$$

$$z_1^3 \le y_1^3 \tag{56}$$

$$z_k^3 \le y_{k-1}^3 + y_k^3,\ \forall k = 2,\ldots,7 \tag{57}$$

$$z_8^3 \le y_7^3 \tag{58}$$

$$\sum_{k=1}^{8} z_k^3 = 1 \tag{59}$$

$$\sum_{k=1}^{7} y_k^3 = 1 \tag{60}$$

$$y_k^3 = 0 \text{ or } 1, \forall k = 1,2,\ldots,7 \tag{61}$$

$$z_k^3 \ge 0, \forall k = 1,2,\ldots,8. \tag{62}$$

Next, combining (36), (45), and (54), we can replace the objective function

$$\frac{1}{3}\log_2\frac{\theta_1}{1670} + \frac{1}{3}\log_2\frac{\theta_2}{830} + \frac{1}{3}\log_2\frac{\theta_3}{280}$$

by

$$\frac{1}{3}\hat{f}_1(\theta_1)+\frac{1}{3}\hat{f}_2(\theta_2)+\frac{1}{3}\hat{f}_3(\theta_3)$$
$$=\frac{1}{3}(-1671z_1^1-1z_2^1+0.42z_4^1+0.74z_5^1+z_6^1+1.56z_7^1+1.96z_8^1)$$
$$+\frac{1}{3}(-831z_1^2-1z_2^2+0.42z_4^2+0.74z_5^2+z_6^2+2.30z_7^2+2.97z_8^2)$$
$$+\frac{1}{3}(-281z_1^3-1z_2^3+0.41z_4^3+0.73z_5^3+z_6^3+3.66z_7^3+4.54z_8^3)$$

We proceed to consider the following constraints:

$$\theta_i\cdot(5\chi_j^i(e_1)+6\chi_j^i(e_2)+10\chi_j^i(e_3)+5\chi_j^i(e_4)+4\chi_j^i(e_5)$$
$$+11\chi_j^i(e_6)+6\chi_j^i(e_7)+8\chi_j^i(e_8)+6\chi_j^i(e_9)+7\chi_j^i(e_{10})$$
$$+12\chi_j^i(e_{11})+6\chi_j^i(e_{12})+5\chi_j^i(e_{13})+6\chi_j^i(e_{14}))=c^i,$$
$$\forall j=1,\dots,10,\ \forall i=1,2,3 \quad (63)$$

and

$$\sum_{i=1}^{3}\sum_{j=1}^{10}\chi_j^i(e_k)\cdot\theta_i=x_k,\ \forall k=1,\dots,14. \quad (64)$$

Since 0-1 variables $\chi_j^i(e_k)$ multiplied by decision variables θ_i are nonlinear, we replace $\chi_j^i(e_k)\theta_i$ by nonnegative variables $A_j^i(e_k)$. Then (63) and (64) become

$$5A_j^i(e_1)+6A_j^i(e_2)+10A_j^i(e_3)+5A_j^i(e_4)+4A_j^i(e_5)$$
$$+11A_j^i(e_6)+6A_j^i(e_7)+8A_j^i(e_8)+6A_j^i(e_9)+7A_j^i(e_{10})$$
$$+12A_j^i(e_{11})+6A_j^i(e_{12})+5A_j^i(e_{13})+6A_j^i(e_{14})=c^i,$$
$$\forall j=1,\dots,10,\ \forall i=1,2,3 \quad (65)$$

and

$$\sum_{i=1}^{3}\sum_{j=1}^{10}A_j^i(e_k)=x_k,\ \forall k=1,\dots,14. \quad (66)$$

Simultaneously,

$$\theta_{1,1}=\theta_{1,2}=\cdots=\theta_{1,10}\geq 160, \quad (67)$$

$$\theta_{2,1}=\theta_{2,2}=\cdots=\theta_{2,10}\geq 80, \quad (68)$$

and

$$\theta_{3,1}=\theta_{3,2}=\cdots=\theta_{3,10}\geq 25 \quad (69)$$

can be rewritten respectively as

$$A_j^1(e_k)\geq 160\chi_j^1(e_k),\ \forall k=1,\dots,14,\ \forall j=1,\dots,10, \quad (70)$$

$$A_j^2(e_k) \geq 80\chi_j^2(e_k),\ \forall k = 1,\ldots,14,\ \forall j = 1,\ldots,10, \tag{71}$$

and

$$A_j^3(e_k) \geq 25\chi_j^3(e_k),\ \forall k = 1,\ldots,14,\ \forall j = 1,\ldots,10. \tag{72}$$

Then we have the constraints of the form

$$-A_j^1(e_k) + 160 \leq 0 \tag{73}$$

$$-A_j^1(e_k) \leq 0, \tag{74}$$

$$-A_j^2(e_k) + 80 \leq 0 \tag{75}$$

$$-A_j^2(e_k) \leq 0, \tag{76}$$

and

$$-A_j^3(e_k) + 25 \leq 0 \tag{77}$$

$$-A_j^3(e_k) \leq 0. \tag{78}$$

Adding the two constraints (79) and (80) to the model will ensure that at least one of (73) and (74) is satisfied:

$$-A_j^1(e_k) + 160 \leq M \cdot \chi_j^1(e_k) \tag{79}$$

$$-A_j^1(e_k) \leq M \cdot (1 - \chi_j^1(e_k)). \tag{80}$$

Similarly, adding the two constraints (81) and (82) to the model will ensure that at least one of (75) and (76) is satisfied:

$$-A_j^2(e_k) + 80 \leq M \cdot \chi_j^2(e_k) \tag{81}$$

$$-A_j^2(e_k) \leq M \cdot (1 - \chi_j^2(e_k)). \tag{82}$$

Next, adding the two constraints (83) and (84) to the model will ensure that at least one of (77) and (78) is satisfied:

$$-A_j^3(e_k) + 25 \leq M \cdot \chi_j^3(e_k) \tag{83}$$

$$-A_j^3(e_k) \leq M \cdot (1 - \chi_j^3(e_k)). \tag{84}$$

In (79)-(84), $\chi_j^i(e_k)$ is a 0-1 variable for each i, j and M is a number chosen large enough to ensure that

$$-A_j^1(e_k) + 160 \leq M,$$

$$-A_j^1(e_k) \leq M,$$

$$-A_j^2(e_k) + 80 \leq M,$$

$$-A_j^2(e_k) \leq M,$$

$$-A_j^3(e_k) + 25 \leq M,$$

and

$$-A_j^3(e_k) \le M$$

are satisfied.

From the above discussion, we present a Mixed-Integer programming model (MP3):

$$\begin{aligned}
\text{maximize} \quad & \frac{1}{3}(-1671z_1^1 - 1z_2^1 + 0.42z_4^1 + 0.74z_5^1 + z_6^1 + 1.56z_7^1 + 1.96z_8^1) \\
& + \frac{1}{3}(-831z_1^2 - 1z_2^2 + 0.42z_4^2 + 0.74z_5^2 + z_6^2 + 2.30z_7^2 + 2.97z_8^2) \\
& + \frac{1}{3}(-281z_1^3 - 1z_2^3 + 0.41z_4^3 + 0.73z_5^3 + z_6^3 + 3.66z_7^3 + 4.54z_8^3) \\
\text{subject to} \quad & 5x_1 + 6x_2 + 10x_3 + 5x_4 + 4x_5 + 11x_6 + 6x_7 + 8x_8 + 6x_9 \\
& + 7x_{10} + 12x_{11} + 6x_{12} + 5x_{13} + 6x_{14} = 130,000 \\
& (10c^1 + 18,000) + (10c^2 + 9,000) + (10c^3 + 3,000) = 130,000 \\
& 5A_j^i(e_1) + 6A_j^i(e_2) + 10A_j^i(e_3) + 5A_j^i(e_4) + 4A_j^i(e_5) \\
& + 11A_j^i(e_6) + 6A_j^i(e_7) + 8A_j^i(e_8) + 6A_j^i(e_9) + 7A_j^i(e_{10}) \\
& + 12A_j^i(e_{11}) + 6A_j^i(e_{12}) + 5A_j^i(e_{13}) + 6A_j^i(e_{14}) = c^i, \\
& \quad \forall j = 1,\dots,10,\ \forall i = 1,2,3 \\
& -A_j^1(e_k) + 160 - M\chi_j^1(e_k) \le 0,\ \forall j = 1,\dots,10,\ \forall k = 1,\dots,14 \\
& -A_j^1(e_k) - M(1 - \chi_j^1(e_k)) \le 0,\ \forall j = 1,\dots,10,\ \forall k = 1,\dots,14 \\
& -A_j^2(e_k) + 80 - M\chi_j^2(e_k) \le 0,\ \forall j = 1,\dots,10,\ \forall k = 1,\dots,14 \\
& -A_j^2(e_k) - M(1 - \chi_j^2(e_k)) \le 0,\ \forall j = 1,\dots,10,\ \forall k = 1,\dots,14 \\
& -A_j^3(e_k) + 25 - M\chi_j^3(e_k) \le 0,\ \forall j = 1,\dots,10,\ \forall k = 1,\dots,14 \\
& -A_j^3(e_k) - M(1 - \chi_j^3(e_k)) \le 0,\ \forall j = 1,\dots,10,\ \forall k = 1,\dots,14 \\
& \sum_{j=1}^{10} \theta_i = \theta_i,\ \forall i = 1,2,3 \\
& \sum_{i=1}^{3}\sum_{j=1}^{10} A_j^i(e_k) = x_k,\ \forall k = 1,\dots,14 \\
& \chi_j^i(e_k) = 0 \text{ or } 1,\ \forall i = 1,2,3,\ j = 1,\dots,10, \text{ and } k = 1,\dots,14 \\
& A_j^i(e_k) \ge 0,\ \forall i = 1,2,3,\ j = 1,\dots,10, \text{ and } k = 1,\dots,14 \\
& 0 \le x_k \le U_k,\ \forall k = 1,\dots,14 \\
& \theta_1 - 835z_2^1 - 1670z_3^1 - 2227z_4^1 - 2784z_5^1 - 3340z_6^1 - 4920z_7^1 - 6500z_8^1 = 0 \\
& \theta_2 - 415z_2^2 - 830z_3^2 - 1107z_4^2 - 1384z_5^2 - 1660z_6^2 - 4080z_7^2 - 6500z_8^2 = 0 \\
& \theta_3 - 140z_2^3 - 280z_3^3 - 373z_4^3 - 466z_5^3 - 560z_6^3 - 3530z_7^3 - 6500z_8^3 = 0 \\
& z_1^i - y_1^i \le 0,\ \forall i = 1,2,3
\end{aligned}$$

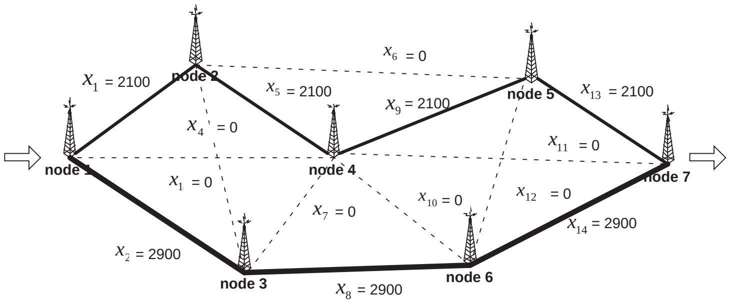

Figure 10: The Allocation of Bandwidths in Sample Network Topology

$$z_k^i - y_{k-1}^i - y_k^i \leq 0,\ \forall k = 2,\ldots,7,\ \forall i = 1,2,3$$
$$z_8^i - y_7^i \leq 0,\ \forall i = 1,2,3$$
$$\sum_{k=1}^{8} z_k^i = 1,\ \forall i = 1,2,3$$
$$\sum_{k=1}^{7} y_k^i = 1,\ \forall i = 1,2,3$$
$$y_k^i = 0 \text{ or } 1,\ \forall k = 1,2,\ldots,7,\ \forall i = 1,2,3$$
$$z_k^i \geq 0,\ \forall k = 1,2,\ldots,8,\ \forall i = 1,2,3,$$

where $U_1 = 2,300$, $U_2 = 3,500$, $U_3 = 1,000$, $U_4 = 2,500$, $U_5 = 2,100$, $U_6 = 2,200$, $U_7 = 2,000$, $U_8 = 3,000$, $U_9 = 2,100$, $U_{10} = 2,700$, $U_{11} = 1,500$, $U_{12} = 1,800$, $U_{13} = 3,000$, and $U_{14} = 3,500$.

Using this model, we can find a pareto optimal allocation of bandwidth on the network (as Figure 10) under a budget $B = \$130,000$. The Pareto optimal solution is: $x_3 = x_4 = x_6 = x_7 = x_{10} = x_{11} = x_{12} = 0$ kbps, $x_1 = x_5 = x_9 = x_{13} = 2,100$ kbps, $x_2 = x_8 = x_{14} = 2,900$ kbps, $\theta_{1,j} = 300$ kbps, $\theta_{2,j} = 150$ kbps, $\theta_{3,j} = 50$ kbps for all j. We find the bandwidth allocated to class 1 is $\theta_1 = 3,000$ kbps, the bandwidth allocated to class 2 is $\theta_2 = 1,500$ kbps, and the bandwidth allocated to class 3 is $\theta_3 = 500$ kbps. This allocation can provide proportional fairness to every class, and the satisfaction of each class equals 0.848. The optimal paths (in Figure 10) are 1-2-4-5-7 and 1-3-6-7, and the cost per unit bandwidth of the optimal path is \$20. We also find the bottleneck links are e_5 and e_9.

4 Conclusions

In this work, we present an approach for the fair resource allocation problem in All-IP networks that offer multiple services to users. Users' utility functions are summarized by means of achievement functions. We find that the achievement function can map different criteria onto a normalized scale. The achievement function also can work in the Ordered Weighted Averaging method. Moreover, it may be interpreted as a measure of QoS on All-IP networks. Using the bandwidth allocation model, we can find a Pareto optimal allocation of bandwidth on the network under a limited available budget, and this allocation can provide the so-called proportional fairness to every class. That is, this allocation can provide the similar satisfaction to each user in all classes. We also find the bandwidth allocated to each class.

Most of multiple criteria optimization reported in the literature use a weighted or a lexicographic achievement function. This selection is usually made in a mechanistic way without theoretical justification. For each type of achievement function underlies a different philosophy on the DMs' preferences. If the selection of the achievement function is improper, then it is very likely that the DM will not accept the solution. Therefore, the right choice of the achievement function is a key element for the success of the optimization model.

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