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Applying Network Flow Optimization Techniques for Measuring the Robustness of Water Supply Network System in Tokyo

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1 Introduction

In this paper, we propose an evaluation method for measuring the robustness of water supply network system by applying network flow optimization techniques. Then we show numerical results obtained from applying optimization techniques to the water supply network in Tokyo.

Fig. 1 shows the main structure of water supply network in Tokyo. The network consists of the set of nodes corresponding to water intake points, intermediate points and demand points, and the set of edges corresponding to pipelines connecting two adjacent nodes.

Tokyo waterworks manages a large scale waterworks system. This system consists of water intake facilities at 3 main rivers, 11 main purification plants, 38 principal water supply stations and transmission/distribution pipelines. Since purification plants and water supply stations are supported by others, Tokyo waterworks supply water through a lot of transmission/distribution routes in a wide area in Tokyo. River water is transformed into potable water by purification plants, then it is pumped out of purification plants into the transmission pipelines. Finally, it flows into the distribution pipelines for being delivered to consumers, houses and buildings. These transmission pipelines consist of very large pipes connecting purification plants, water supplying stations and junction points. The transition pipelines are connected to the distribution pipelines at junction points. The total length of transition pipelines in Tokyo amount to almost 370 kilometer. The diameter of them is almost more than 1,000 millimeter, see Table 1. The total length of the distribution pipeline is 25,000 kilometer and 90% of them are less than 350 millimeter in diameter (Table 2). These pipelines are laid in almost all public roads in Tokyo. Tokyo Waterworks Bureau makes a water supply plan every month and prepares water flow data including the quantity and direction of the water flow in major transmission pipelines.

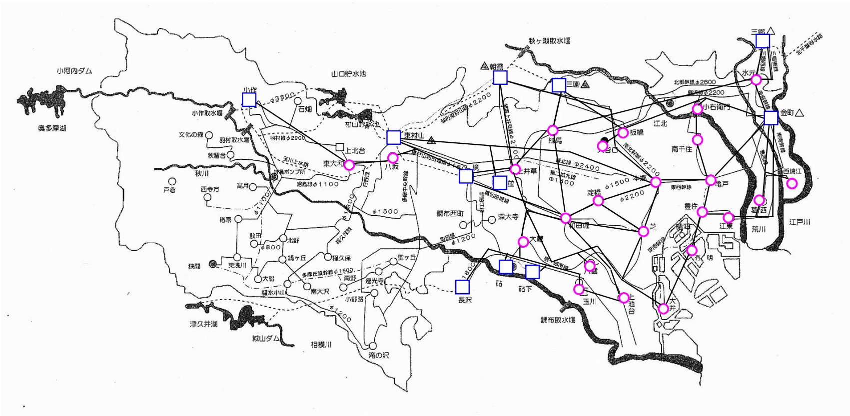


Figure 1: Water supply network in Tokyo (□: water intake points, ◯: water supply points)

Table 1: The diameters of water transition pipeline shonw in Fig. 1

diameter (mm)	≤1000	1000–1500	1500–2000	2000≤	total
length(km)	80	150	70	70	370

Water supply network is a necessary and indispensably important lifeline network for our daily lives. When a serious and large earthquake occurs, there are a lot of broken points in water pipelines. At any broken points, water flows out of pipelines becoming impossible to deliver water further. If other substituting pipelines survive in the damaged network, it would possibly send water to the final demand areas. In that situation, waterworks bureaus usually close the valves at broken points and let water flow through other pipelines to the final demand areas. They try to maximize the water supplied area. On January 17, 1995, water supply network in Kobe city had an extremely severe damage due to the Hanshin earthquake. In most areas in the Kobe city water supply service had been terminated instantaneously for about a month, taking to recover completely. Other lifeline networks such as electricity and gas supply systems were terminated for about a few days and a few weeks, respectively and very differently, then taking a day or so and three weeks to recover completely. In Kobe, many distribution pipelines were damaged due to the Hanshin

Table 2: The distribution of diameters of water supply network pipelines in whole Tokyo area

diameter(mm)	≤350	400–1000	1000–1500	1500-2000	2000≤	total
length(km)	22800	1800	400	200	200	25400

earthquake, thus the demand areas in the down streams of the damaged pipeline are out of water supply services. However, we can avoid the serious damage in case that we have a substituting route to meet the demand even if some pipelines were destroyed due to the broken pipeline. Thus, the problem we are interested in follows that how robust the water supply network is when some edge segments are broken in the sense that we can meet the demand of the water. We consider that measuring the robustness of the lifeline network structured system is necessary and important in order that we prepare the emergent situations due to various kinds of natural disasters such as earthquakes.

This work applies a similar idea as path counting method in [3, 4, 5] to a mutlti-terminal network flow problem (cf. [1]), which is presented in Sec. 2. In the same section we give Monte Carlo estimation methods [2] for the problem. In Sec. 3 we give the detail of our analysis on the Tokyo water supply network. Last section shows summary and possible extension in the future.

2 Network Flow Optimization Model

2.1 Network Flow Optimization Problem

A network flow optimization problem utilized in our model is mathematically called multi-source multi-sink maximum flow problem provided under the stochastically determined edge capacity conditions. A simple network example is illustrated in Fig. 2. Let $G = (V,E)$ be an undirected network composed of the node set V and

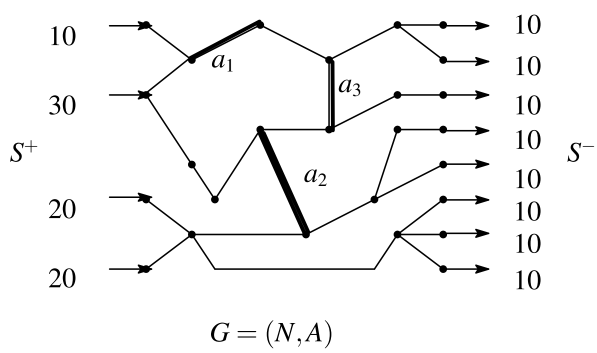


Figure 2: Unavailability by Edge Broken

edge set E with $|V| = n$ and $|E| = m$, respectively. Each edge $(i, j) \in E$, $i, j \in V$, has its capacity u_{ij} given by nonnegative integer. The flow goes through the edge (i, j) cannot exceed u_{ij}. Two subsets of nodes are designated source node set and sink node set, respectively. The set of source nodes and the set of sink nodes are denoted by S and T, respectively. Water flow goes into the system at source nodes, then drains out at sink nodes leaving the system. At each source node $i \in S$ the flow whose value is equal to or less than s_i is pushed into the water supply network system while it also runs into each sink node $j \in T$ with at most t_j units. The volume t_j indicates the "demand of water" at each sink node j. Hopefully, we assume the total demand is at most equal to the total supply: $\sum_{j \in T} t_j \leq \sum_{i \in S} s_i$.

In our water supply network problem, source nodes correspond to purification plants, while sink nodes correspond to final demand areas. The remaining nodes indicate facilities such as pump stations, pipes conjunctions and others. Table 3 gives a list showing the available water supply at each purification plant in the Tokyo Metropolitan area.

Table 3: Purification Plants in Tokyo. (2004.9.1)

River	Purification Plants	Supply(10^3m^3/day)	Region
Tone River · Arakawa River	Kanamachi	939	Joto
	Misato	900	
	Asaka	950	Yamate
	Misono	280	
	Higashimurayama	843	
Tama River			
	Ozaku	150	Tama
	Sakai	120	
	Kinuta	0	
	Kinuta-simo	34	
Other	Nagasawa	230	
	Suginami	4	Yamate

Mathematical formulation of the model described above is given as a multi-source multi-sink maximum network flow problem as follows.

$$\text{Maximize} \quad z = \sum_{i \in T} \left(\sum_j x_{ji} - \sum_j x_{ij} \right)$$

$$\text{subject to} \quad \sum_j x_{ji} - \sum_j x_{ij} \quad \begin{cases} \geq -s_i, & i \in S \\ = 0, & i \in V \setminus (S \cup T) \\ \leq t_i, & i \in T \end{cases} \qquad (1)$$

$$0 \leq x_{ij}, x_{ji} \leq u_{ij}, \qquad (i,j) \in E$$

where each x_{ij} is the decision variable designating the flow passing through each edge (i,j) from node i to j. The objective function is the amount of total flow running into sink nodes. We note that the above model is slightly different from usual multi-terminal maximum flow problem in that the total demand $\sum_{j \in T} t_j$ is not necessarily required to be satisfied with the total flow into sink node set, i.e. in the constraints (1) for each sink $i \in T$ the flow into the sink is less than or equal to the demand t_i, while the flow into sink is required to be greater than or equal to the prescribed demands in usual multi-terminal flow problems. This turns a key feature to develop the model into a stochastic environment.

We assume that each edge in the network is possibly broken with "same (constant) probability". Thus an idea for measuring the reliability of the network system is to compute the maximum flow on the network in such conditions that k edges out of m edges are randomly broken. To incorporate the random broken edges into the model (1), we introduce random variables $\{A_{ij} : (i,j) \in E\}$ each of which takes the value 0, or 1. We consider the edge (i,j) is alive when $A_{ij} = 1$, and (i,j) is broken when $A_{ij} = 0$. We modify the model (1) into the form containing these random variables.

$$\begin{array}{lll} \text{maximize} & Z = \sum_{i \in T} \left(\sum_j x_{ji} - \sum_j x_{ij} \right) & \\ \text{subject to} & \sum_j x_{ji} - \sum_j x_{ij} & \begin{cases} \geq -s_i, & i \in S \\ = 0, & i \in V \setminus (S \cup T) \\ \leq t_i, & i \in T \end{cases} \\ & 0 \leq x_{ij}, x_{ji} \leq A_{ij} u_{ij}, & \quad (i,j) \in E \end{array} \tag{2}$$

Since the realized maximum value of (2) depends on the random capacity $A_{ij}u_{ij}$, it naturally becomes a random variable, hence we use capital letter Z to denote the stochastic maximum flow value. We investigate how the maximum flow depends on the number of broken edges k by estimating the distribution of Z including the mean value of Z under the condition that the random variables A_{ij} satisfying $\sum A_{ij} = m - k$. This condition implies that the number of broken edges is fixed to k. Since we assume each edge is "broken" independently, conditional distribution function defined by

$$F_k(z) = \mathsf{P}\left\{ Z \leq z | \sum A_{ij} = m - k \right\} \tag{3}$$

provides the ratios of the network whose maximum flow value is less than or equal to z to all the $\binom{m}{k}$ networks obtained by deleting k edges randomly from the original network. The conditional expected maximum flow on the network G is defined similarly as

$$z_k = \mathsf{E}\left[Z | \sum A_{ij} = m - k \right]. \tag{4}$$

Finally we try to measure the reliability of the network by computing the degree of satisfying the total demand as follows:

$$r_k = \frac{z_k}{\sum_{i \in T} t_i} \tag{5}$$

We try to solve the reliability problem by calculating the coverage rate r_k iteratively with respect to k ranging from 0 to m. Even for one fixed k obtaining the exact value of r_k for actual water supply network system requires a huge amount of computation, thus becoming extremely difficult to carry out in reasonable amount of computation time.

We apply Monte Carlo method described in the following section in order to estimate approximately the coverage rate r_k.

2.2 Monte Carlo Estimation Method

As mentioned above, exact computation for obtaining the exact expected value for the mean of r_k requires to solve network flow optimization problems for 2^m different networks. Thus we apply a straightforward implementation of Monte Carlo method in order to solve the maximum flow problems repeatedly for these different network systems. Outline of the algorithm is given as follows. First we number all the edges of the network sequentially, so that we assume $E = \{1, 2, \ldots, m\}$. We begin with choosing k elements from E randomly, suppose chosen elements are $E_k = \{e_1, \ldots, e_k\}$. Set the value of A_{ij} to 1 if $(i, j) \in E_k$, or to 0 otherwise, then solve the problem (2) and obtain an estimate for Z. Next iteration starts from new choice of k broken edges. After iterations enough to attain sufficient accuracy, we estimate F_k, z_k, and r_k. Here we give detailed algorithm for the expected maximum flow z_k. The coverage rate r_k immediately follows the expected maximum flow by simple calculation. The estimate for the distribution function F_k can be obtained through the empirical distribution of sample maximum flows $z_k^{(j)}$ in that algorithm.

Input: Network $G = (V, E)$. Number of iteration M. Edge deletion rate k.
Output: Estimation of Expected maxflow.
Initialization: Let $n = |V|$ and $m = |E|$. Assign a number to edges such as $E = \{1, 2, \ldots, m\}$.
Method:

Set $j = 1$ and $Z_k = 0$.
While $j \leq M$:
- Let E_k be the set of edges randomly chosen from E.
- Set the value of A_{ij} to 1 if $(i, j) \in E_k$, or to 0 otherwise.
- Solve the problem (2) and obtain sample maximum flow $z_k^{(j)}$.
- Let $Z_k = Z_k + z_k^{(j)}$.
- Let $j = j + 1$.

Compute Z_k/M for z_k.

In addition to the total demand coverage rate r_k we also consider a regional demand coverage rate. Let the sink source set T be partitioned into L subsets: $T = T_1 + \ldots + T_L$. In j-th Monte Calro iteration we calculate the flow into each subset T_l, $l \in \{1, \ldots, L\}$

$$z_k^l(j) = \sum_{i \in T_l} \left(\sum_j x_{ji} - \sum_j x_{ij} \right). \tag{6}$$

At the end of Monte Carlo method we calculate the average of $z_k^l(j)$:

$$z_k^l = \frac{1}{M} \sum_{j=1}^{M} z_k^l(j), \tag{7}$$

and the regional demand coverage rate for T_l is calculated by

$$r_k^l = \frac{z_k^l}{\sum_{i \in T_l} t_i}. \tag{8}$$

We use this regional demand coverage rate as a measure for the robustness of the regional demand area corresponding to the subset T_l.

3 Numerical Experiments

3.1 Assumptions and Input Data

In this study, we try to measure the robustness of the water supply network system by calculating the "coverage rate" of supplying water for meeting the final demand described in Sec. 2. This rate implies the degree of meeting the water demand in the network some of whose edges were broken randomly. We consider that the higher the coverage rate of the system is, the more robust the water supply network is. If the supplied water satisfies the water demand fully, the coverage rate is 1.0. The coverage rate depends on the edge broken rate. We concern the variation of the water coverage rate for not only whole Tokyo area but also regional areas in Tokyo. Tokyo is usually divided into three regions called by Joto, Yamate, and Tama, respectively. Each region has purification plants and final demand area, they are interconnected by water pipelines. We also concern the variation of the water coverage rate for each region, Joto, Yamate, and Tama.

Joto region, which is located in the east end region in Tokyo, is flat and low altitude. There are two major purification plants and the water demand in this region is 1,000,000 cubic meter per day. Purification plants supply usually enough water all over this region and send water into Yamate region. The network between Joto and Yamate region is able to covey water each other. Yamate region is almost tableland and the southern part is low. There are six major purification plants and the water demand is 2,500,000 cubic meter per day. The water demand in this region is planned to be supplied by purification plants in all regions in Tokyo. The network between three regions is able to send water into adjacent regions each other. Tama Region is also a tableland, and the west end part is hilly. There are three major plants and the demand is 1,000,000 cubic meter per day. They usually supply enough water all over this region and deliver water into Yamate region. Since the west end part is hilly, it is difficult to send water from lower part.

In the monthly water supply plan, purification plants take water from three rivers called by Tone, Tama, and Edo, respectively. In order to investigate the effect of purification plants damage to the water supply system, we consider the following three cases, according to the degree of damages the system receives.

- Case 1 (Normal): All purification plants take the water from rivers and supply it as planned.
- Case 2 (Tone damaged): Purification plants in the Yamate region cannot obtain water from Tone River while other plants can take it as planned. Higashimurayama plant in the Tama region delivers water to the Asaka plant in the Yamate region, then Asaka plant supplies it.
- Case 3 (Tama damaged): Purification plants in the Tama region cannot obtain water from Tama River while other plants can take it as planned. Asaka plant delivers water to the Higashimurayama plant, then supplies water.

Table 4: Water supply (unit : $10^3 m^3$/day)

Purification Plant Name	case 1	case 2	case 3
Kanamachi	939	1500	1500
Misato	900	1100	1100
Asaka	950	0	1700
Misono	280	280	300
Higashimurayama	843	1265	820
Ozaku	150	280	0
Sakai	120	300	0
Kinuta	0	114	0
Kinuta-simo	34	70	0
Nagasawa	230	230	230
Suginami	4	4	4

Table 4 shows the quantity of water supply from all purification plants in the system in each case. These water supply data are used as the input data s_i at the source nodes i ($i \in S$) in the graph G.

With respect to the capacities of the edges corresponding to the various pipes we consider the following 2 cases.

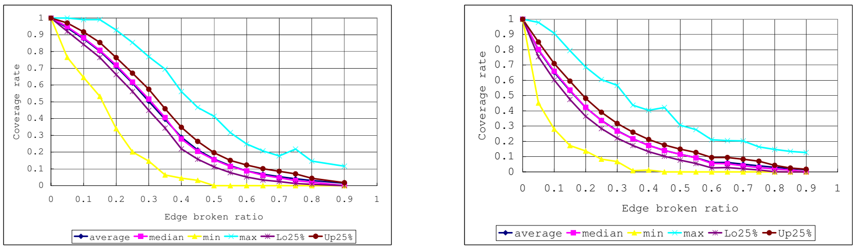
- Case A: Capacity u_{ij} is infinite.
- Case B: Capacity u_{ij} is defined equal to twice the planned one.

In case A, we try to measure the highest coverage rate of supplying water for meeting the final demand, which implies the highest connectivity of the network according to the damaged network. In case B, this assumption is considered to approximate an actual network system which we try to measure the robustness of the water supply network system by calculating the coverage rate of supplying water for meeting the final demand in the damaged network.

We apply Monte-Carlo Method to cases 1, 2 and 3 for each of case A and B. The rate of edge deletion $\frac{k}{m}$ is set to 5%, 10%, 15%, ..., 90% of m on calculation. The iteration of calculation M is taken to be 10,000. We try to obtain the approximate expected coverage rate for the whole system and also for the regional subsystem of the water supply network system by applying Monte Carlo method.

3.2 Numerical Results

Figures 3(a) and (b) show the numerical results of r_k corresponding to the whole Tokyo area of the water supply network model for case 1. These figures show the relation between the edge deletion ratio $\frac{k}{m}$ and the coverage rate r_k for the water demand for the cases A and B, respectively. Curves in these figures include average, median, maximum, minimum, lower 25% and upper 25%, respectively, obtained from Monte Carlo method.



(a) Flow capacity is infinite

(b) Flow capacity is regulated

Figure 3: Relation between $\frac{k}{m}$ and r_k for the whole Tokyo area

In both Figs. 3(a) and (b), we find that the average curve and the median curve mostly coincide with each other. The lower and upper 25% curves are located very close to the average curve. On the other hand, maximum and minimum curves are located a little far from the average curve. The minimum curve in Figure 3(a) shows 0.65 when $\frac{k}{m}$ is equal to 0.1, 0.35 when it is 0.2 and 0.15 when 0.3. This rapid decrease of minimal curve can be explained as follows. When many edges connected to source nodes or sink nodes happen to be deleted, flows can neither get into the network nor reach sink nodes. Thus we can see that minimum curve is located far below the average curve. On the other hand, as long as one route from source node to sink node exists, the coverage rate is 1.0. Hence maximum curve is located far above the average curve.

Comparing Figs. 3(a) and (b) we find that the average curve in (a) is generally located higher than that in (b). In the range where the $\frac{k}{m}$ is less than 0.5, the gap between two average curves of (a) and (b) is rather large. In Fig. 3(a), the maximum curve takes the value 1.0 until when $\frac{k}{m}$ is equal to 0.15 and the average is almost 0.85. In Fig. 3(b), the maximum r_k is 0.8 and the average r_k is about 0.55 when $\frac{k}{m}$ is equal to0.15.

From Figs. 3(a) and (b), when $\frac{k}{m}$ is less than 0.4, the decreasing rate of the average curve in Fig. 3(b) is 1.5 times larger than that in Fig. 3(a). In the range where the $\frac{k}{m}$ is larger than 0.5, the average curves for Fig. 3(a) and (b) vary from 0.15 to 0 and from 0.1 to 0, respectively. Both coverage rate curves are almost similar. It seems that when many edges are broken and deleted from the network, connectivity of the network system is getting worse similarly regardless of the edge capacities and both average curves of Figs. 3(a) and (b) approach zero similarly.

In case A, which assumes all the edge capacities are infinite, it is possible to send the flow to sink nodes, as long as there exists a path connecting the source and sink node in the network regardless edge capacities on the path. Therefore case A shows us the connectivity of the network. In Fig. 3(a), the maximum curve stays 1.0

when the $\frac{k}{m}$ is in the range 0 to 0.15. In this range, since there are many routes from source nodes to sink nodes in spite of the fact that 15% edges are deleted, it is thought that the network has the good connectivity and conveys water to final comsume areas, as long as the deleted edges are not located at critical points. Our main concern in this analysis is on the situation that the whole network is kept connected but some of edges are broken. Since it is thought that as long as the $\frac{k}{m}$ is less than 0.15, the connectivity of the network is high enough. We consider that the estimated coverage rate r_k is reasonable in the range where the $\frac{k}{m}$ is between 0.0 and 0.15.

With reflecting practical situation, the transition pipelines with large diameter, have sufficient strength against large earthquakes. The small diameter pipes don't have sufficient strength against the earthquake. When the large earthquake occurs, there should be a lot of small pipes destroyed in a wide area and water supplying service should be cut off in many places. The edge deletion rate between 0.0 and 0.15 is considered to simulate these situations.

The simulation results show that the water coverage rate r_k is linearly decreasing with respect to the $\frac{k}{m}$ when $\frac{k}{m}$ is between 0.0 and 0.4. In this range the coverage rate r_k decreases drastically from 1.0 to 0.3 in Fig. 3(a) and from 0.0 to 0.15 in Fig. 3(b). The slope of r_k in Fig. 3(a) is approximately -1.5 and in 3(b) it is approximately 2. It seems that the deleting edges decreases the r_k and the connectivity of the models about twice as much.

3.3 Regional Characteristics of the Robustness

The estimated coverage rates r_k in three regions are shown in Figs. 4(a), (b), (c), respectively. We execute these simulations for case B with the condition that edge capacities are regulated less than or equal to twice as planned capacities. So this condition is considered to be close to actual water supply in emergency. We focus on the water supply condition in case that facilities are damaged. We measure the regional robustness by comparing the variation of the $\frac{k}{m}$ in case 1, 2 and 3.

The water demand in Yamate region is 2,500,000 cubic meter per day. It is planned that all the purification plants in Tokyo supply water to this region. It is impossible to meet final demand of water only by supplying from plants in Yamate region.

In case 2, the facilities including water gates and channels in the Tone River are damaged and purification plants cannot take water from the Tone River. Thus Asaka purification plant cannot supply water while Misono plant can supply 210,000 cubic meters of water. The quantity of insufficient water is 1,000,000 cubic meter a day. In this case, purification plants in other regions increase water supply and deliver water to Yamate region for covering the insufficient water supply. Despite of this back-up, coverage rate in case 2 is less than that in case 1 by 0.1. In this case, it is necessary to carry water far from purification plants in other regions, thus as the distance for carrying water becomes long, the damaging possibility of the network is getting larger. It is thought that carrying water through long distance to the final demand area is very difficult. In case 3 the facilities in the Tama River are damaged and the purification plants in the region cannot take water from the Tama River, thus the quantity of insufficient water is 1,200,000 cubic meter a day. Purification plants in both Joto and Yamate regions need to increase water supply carrying to Tama region. Asaka plant pump up the river water to the Higashimurayama plant in Tama region, then purifies this water and send it to Tama region. The value of r_k in Yamate region is almost as same as in case 1. It is assumed that as long as the plants in Yamate region operate usually, the water demand in Yamate can be met.

The water demand in Joto region is 1,000,000 cubic meter per a day. The purification plants in Joto region take water from the Edo River. These plants usually supply water in this region and carry it to Yamate region. The water demand in this region can be met by these plants.

Comparing cases 1, 2 and 3, we find that these are almost similar. This means that the plants in Joto region increase supply water, thus enable them back up the insufficient water demand in Yamate region. In Joto region, we find that the damage of facilities in the Tone River doesn't decrease r_k so much.

Water demand in Tama region is 1,000,000 cubic meter per a day. The purification plants in Tama region take water from the Tama River. These plants usually supply water in this region and carry to Yamate region. Water demand in this region can be met by these plants.

In case 2, the value of r_k is almost similar to case 1. This means that the plants in Tama increase, then back up the insufficient water in Yamate region. In case 3, the r_k decreases by 0.1 compared with case 1. This means that it is difficult to meet the water demand in hilly area, i.e., the western part of Tama region. Since the plants in other regions are located at lower altitude, it is difficult to send water to the higher hilly area.

4 Summary and Conclusions

In this study, we tried to measure the variation of the water supply coverage rate r_k with respect to edge deletion rate$\frac{k}{m}$. To this end we executed numerical experiments simulating the various supply network conditions with respect to the edge deletion ratio $\frac{k}{m}$ for every 5%. Thus we evaluated the robustness of the water supply network by calculating r_k.

It is shown in the results of the simulation that the robustness of the model is linearly decreases with respect to the $\frac{k}{m}$. It is necessary to keep the $\frac{k}{m}$ less than 0.15 for keeping the robustness at appropriate status. As long as the $\frac{k}{m}$ is less than 0.15, the connectivity of the model is good enough for meeting the flows demand.

It is shown that the water demand in Yamate region and Tama region is difficult to be supplied perfectly. This is thought to result from carrying the water through the long distance or pumping it up to the high altitude area. It is thought necessary to devise the methods for improving the robustness, for example to construct a new pipeline, to have redundancy in important parts in the network, to reinforce pipes and connections, to replace old connections to the seismic proof type connection. These methods are thought to be effective to strengthen the connectedness between regions and reduce the damage in the network.

In this study, we assume that the probability of edges broken is same in any pipelines. We think it is necessary that the definition of the probability is set alike to the realistic condition, the large pipes as strong and the small pipes as weak. Besides, by further simulation, we need to find the weak points in the network and have simulation with the deleted conjunctions model.

References

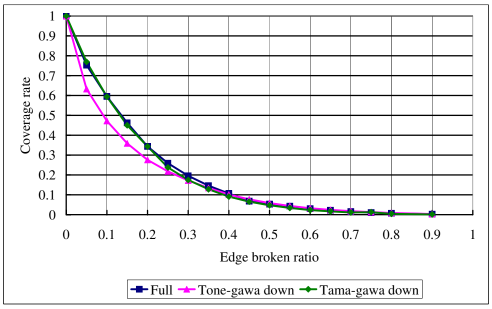
[1] M. Iri, S. Fujishige and T. Oyama. *Graph, Network and Matroid*, Shangyo-Tosho Publishing, 1986. (in Japanese)

[2] G. S. Fishiman. *Monte Carlo — Concepts, Algorithms, and Applications*, Springer, 1996.

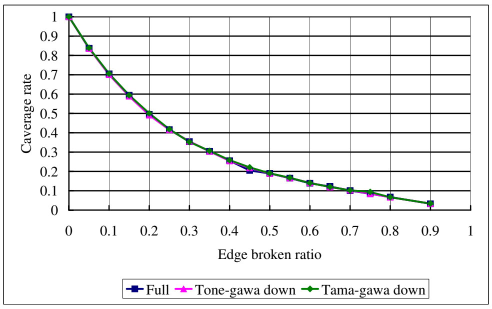
[3] T. Oyama. Weight of shortest path analyses for the optimal location problem. *Journal of the Operations Research Society of Japan*, 43, 176–196, 2000.

[4] T. Oyama and H. Morohosi. Applying the shortest-path-counting problem to evaluate the importance of city road segments and the connectedness of the network-structured system. *International Transactions in Operations Research*, 11, 555–573, 2004.

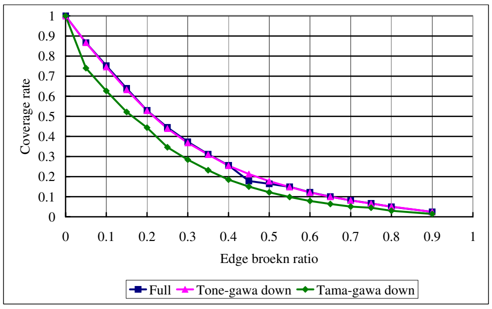
[5] T. Oyama and A. Taguchi. Application of the shortest path counting problem to evaluate the importance of city road segments in Japan. *Perspectives of Advanced Technology Society 3: Urban Life and Traffic*, Y. Matsuda and M. Fushimi (eds.), Maruzen Planet Co., Tokyo, Japan, 3–20, 1996.



(a) Yamate Region



(b) Joto Region



(c) Tama Region

Figure 4: The variation of the r_k in Yamate region, the Joto region and Tama region. $\square$: Full, $\triangle$: Tone down, $\Diamond$: Tame down.