Queueing Model and Performance Analysis for Discrete Time Switch Virtual Channels Systems*

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Abstract *In this paper, we establish a discrete time Geom/G/1 queue with setup-close delay-close down based on operation mechanisms with Switch Virtual Channels (SVC). Different from conventional intricate and time consuming imbedded Markov renewal processes, we use the factorization principle of discrete time queues with general vacation to analyze the response time, connection establishment ratio, operating efficiency and idle ratio of SVCs. Furthermore, we confirm the dependency relationships between these performance measures and the close delay timer's setting, which have potential applications in system optimal design and network control.*

Keywords Switch virtual channel, Discrete time Geom/G/1 queue, Performance analysis, Factorization principle, General vacation, Optimal design and network control

1 Introduction

Kobayashi and Konheim [1], Takagi [2] indicated that it would be more accurate and efficient using discrete time models than continuous counterparts when analyzing and designing digital transmitting systems. The classical discrete time queue analysis was given by [2] and [3]. The analyses of discrete time queue with vacation and setup strategy can be found in Zhang and Tian [4], Tian and Zhang [5]. Jin and Tian [6] built a discrete time vacation queue model, analyzed some

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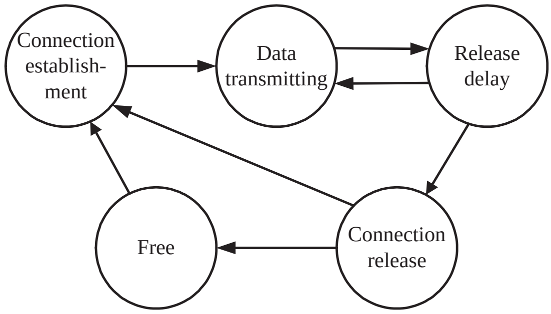

Figure 1: State transition of SVC with setup close-delay close-down.

SVC performance measures. The conventional discrete queues method starts with the analysis of the intricate and time consuming imbedded Markov renewal process at the departure epoch.

In this paper, we model the system with switch virtual channel as a discrete time Geom/G/1 queue with a setup-close delay-close down mechanism. We derive some important system performance measures using the factorization principle of discrete time queue with general vacation and derive the performance measure formulae of the packet response time, virtual channel setup (release) ratio, idle ratio etc.

2 Analysis of SVC Operating Mechanism

In the model of SVC, the end to end user must establish the virtual channel (VC) connection before transmitting, and release this connection after transmitting. Connection establishment is processed by sending specified signaling, and this period is called setup period U. Similarly the connection release is processed by sending specified signaling. This period is called close down period C.

In order to increase system efficiency, a close delay timer is set. When the system completes a transmission, and there are no other packets to be transmitted for the moment, the system would not release the VC immediately but hold the VC for a while. This period is called close delay period D. If there is data arriving within D, the primary SVC can be used directly. Only when no data needs transmitting within this period, will the SVC be released. The system state transition is shown in Fig. 1. The key point is what length of time should the close delay timer D be set for.

The key point is what length of time should the close delay timer D be set for. Obviously, if D is too long, the VC would have no data to be transmitted for a long time, and would cause great waste of communication resource, if D is too short, the virtual connect would be setup and released too frequently, that would lose Close delay timer setting meaning.

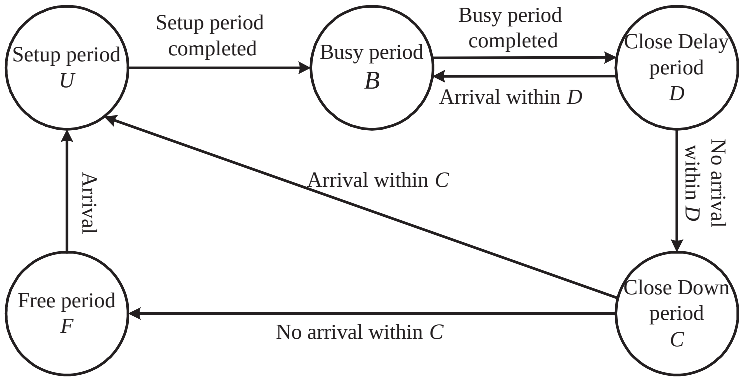

Figure 2: State transition of SVC with setup close-delay close-down.

3 Queue Model

We introduce the setup-close delay-close down mechanism to the classical discrete time Gemo/G/1 queues. Namely, the system enters into the close delay period D once the server is idle, and if there is packet arrival during the close delay period, the system will return to a busy period. Otherwise, the system will enter into the close-down period C after the close delay period is over. If there is at least one packet arrival during the close-down period before the close-down period is over, the first arriving packet will trigger a setup period U. Otherwise, the system will enter into the free period F after the close down period is over. The packet arriving during the free period will finish the free period F, and the system will enter into the setup period U. Finally, the system will enter into the busy period B after the setup period is over.

In a discrete-time network, the network is operating on the basis of time slotting. Suppose that S is the transmission time of a packet in slots, while U, D, F, S and B are all stochastic variables. This mechanism can be built as a discrete time queue with vacation shown in Fig. 2. We notice that D, U, C and S are independent mutually, the distribution and the generation function (G.F.) of the length of D, U, C and S are as follows:

$$\begin{aligned} P\{D=k\} &= d_k, \quad k \geq 1, \qquad D(z)=\sum_{k=1}^{\infty} d_k z^k, \\ P\{C=k\} &= c_k, \quad k \geq 1, \qquad C(z)=\sum_{k=1}^{\infty} c_k z^k, \\ P\{S=k\} &= s_k, \quad k \geq 1, \qquad S(z)=\sum_{k=1}^{\infty} s_k z^k, \end{aligned}$$

$$P\{U=k\} \quad = \quad u_k, \quad k\geq 1, \qquad U(z)=\sum_{k=1}^{\infty}u_k z^k$$

where d_k, c_k, s_k and u_k are the probabilities that the lengths of the close delay period D, the close-down period C, the time S of a packet transmission and the setup period U are k slots, respectively.

4 Performance Analysis

We assume that the time axis is divided into slots of a fixed length. In a time slot, an arrival occurs with probability p and no arrival occurs with probability $q=1-p$.

Then the distribution and the G.F. of packet arrival numbers within the transmission time S are given by

$$P\{A_s=j\}=\sum_{k=j}^{\infty}s_k\binom{k}{j}p^j q^{k-j},\ A_s(z)=S(q+pz).$$

The distribution and the G.F. of packet arrival numbers within the setup period U are given by

$$P\{A_u=j\}=\sum_{k=j}^{\infty}u_k\binom{k}{j}p^j q^{k-j},\ A_u(z)=U(q+pz).$$

The distribution and the G.F. of packet arrival numbers within the close delay period C are given by

$$P\{A_c=j\}=\sum_{k=j}^{\infty}c_k\binom{k}{j}p^j q^{k-j},\ A_c(z)=C(q+pz).$$

Obviously, when $\rho=pE[S]<1$, the system will arrive at a state of equilibrium. Now we analyze some steady-state parameters.

When paralleling with the factorization principle of continuous time BMAP/G/1 queue system with general vacation, which is put forward by Chang, et al. [7], the principle is in existence in the discrete time queue system with general vacation (the detailed proof can be seen at author's another paper), so we have theorems as follows:

The relationship between the G.F. $Y(z)$ of system queue length at anytime and the G.F. $Y_I(z)$ of queue length at anytime given idle state is as follows:

$$\boldsymbol{Y}(z)(z\boldsymbol{I}-\boldsymbol{A}(z))=(1-\rho)(z-1)\boldsymbol{Y}_I(z)\boldsymbol{A}(z)$$

where $\boldsymbol{I}$ is an identity matrix. In Geom/G/1 queue system with general vacation, $\boldsymbol{A}(z)=S(1-p+pz)$, then we can obtain that

$$Y(z) \quad = \quad \frac{(1-\rho)(1-z)S(1-p+pz)}{z-S(1-p+pz)}Y_I(z). \tag{1}$$

The relationship between the G.F. $Y^{SC}(z)$ of system queue length after the departure and the G.F. $Y_I(z)$ of queue length at anytime given idle state is as follows:

$$\boldsymbol{Y}^{SC}(z)(z\boldsymbol{I} - \boldsymbol{A}(z)) = \frac{(1-\rho)}{\lambda}(\boldsymbol{D}(z) - \boldsymbol{I})\boldsymbol{Y}_I(z)\boldsymbol{A}(z).$$

The same, in Geom/G/1 queue system with general vacation, $\boldsymbol{A}(z) = S(1-p+pz)$ and $\boldsymbol{D}(z) = 1-p+pz$, then we have

$$Y^{SC}(z) = \frac{p(1-\rho)(1-z)S(1-p+pz)}{\lambda(S(1-p+pz)-z)}Y_I(z). \tag{2}$$

We define the joint G.F. $Y_B(z,\theta)$ that the queue length and the remaining transmission time of the being served packet and the G.F. $Y_I(z)$ of queue length at anytime given idle state to be as follows:

$$Y_B(z,\theta) = \sum_{k=1}^{\infty}\sum_{l=1}^{\infty} \Pr\left\{Y = k, X^+ = l\right\} z^k\theta^l$$

where Y is the queue length with packets, X^+ is the remaining transmission time (slots) of the being served packet. It is given by

$$Y_B(z,\theta)(\boldsymbol{D}(z) - \theta\boldsymbol{I}) = \frac{z\theta}{\rho}(1-\rho)\boldsymbol{Y}_I(z)(\boldsymbol{D}(z)-\boldsymbol{I})(z\boldsymbol{I}-\boldsymbol{A}(z))^{-1}(\boldsymbol{A}(z) - S(\theta)\boldsymbol{I}).$$

Again, using $\boldsymbol{A}(z) = S(1-p+pz)$ and $\boldsymbol{D}(z) = 1-p+pz$, then we get

$$Y_B(z,\theta) \quad = \quad \frac{zp\theta(1-\rho)(1-z)(S(1-p+pz)-S(\theta))}{\rho(S(1-p+pz)-z)(1-p+pz-\theta)}Y_I(z). \tag{3}$$

Let the length of the close delay period be T_D, the probability and G.F. of T_D are given by

$$P\{T_D = k\} = d_k q^k + q^{k-1}p\sum_{j=k}^{\infty} d_j, \quad k \geq 1,$$

$$T_D(z) = \frac{pz + (1-z)D(qz)}{1-qz}.$$

Then we first discuss the average lengths of the close delay period $E[T_D]$, close down period $E[T_C]$, free period $E[T_F]$, setup period $E[T_U]$ and the whole idle period $E[T_I]$ of the system. Obviously, $D(q)$ and $C(q)$ are the probabilities that no arrivals occur during D and C. So we can consider $D(q)$ and $C(q)$ as the system parameters.

By differentiating the above equation at $z = 1$, we can derive the average length of close delay period D as follows:

$$E[T_D] = \frac{1 - D(q)}{p}.$$

There is a free period F only when no arrival occurs during $D + C$, and its length is a residual inter arrival time. The average length of the free period F is given by

$$E[T_F] = \frac{1}{p} D(q)C(q).$$

Suppose that the average length of C under the condition that the event closing down occurs is $E[C]$, the average length of the close down period C is given by

$$E[T_C] = D(q)E[C].$$

Suppose that the average length of C under the condition that the event setting up occurs is $E[U]$, the average length of the setup period U is given by

$$E[T_U] = D(q)E[U].$$

$E[C]$ and $E[U]$ can be constants, which are appropriate to SVC's building and releasing.

The whole idle period I of the system is composed of U, D, F and C, the average length of I is given by

$$E[T_I] = \frac{1 - D(q) + D(q)C(q)}{p} + D(q)(E[U] + E[C]).$$

Now we introduce the following notation:

$$H = 1 - D(q) + D(q)C(q) + pD(q)(E[U] + E[C]),$$

then we obtain $E[T_I]$ and $Y_I(z)$ as follows:

$$E[T_I] = \frac{H}{p}. \tag{4}$$

It can be seemed that $Y_I(z)$ equals 1 with probability $(1 - D(q) + D(q)C(q))/H$, equals $(1 - C(q + pz))/pE[C](1 - z)$ with probability $pD(q)E[C]/H$, and equals $C(q)(U(q + pz) - 1)/pE[U]$ with probability $pD(q)E[U]/H$.

$$Y_I(z) = \frac{1 - D(q) + D(q)C(q)}{H} + \frac{D(q)}{H}\left(\frac{1 - C(q + pz)}{1 - z} + C(q)\left(U(q + pz) - 1\right)\right). \tag{5}$$

From the Eq. (1), we can derive the G.F. $Y(z)$ of queue length at anytime as follows:

$$Y(z) = \frac{(1 - \rho)(1 - z)S(1 - p + pz)}{z - S(1 - p + pz)}\left(\frac{1 - D(q) + D(q)C(q)}{H} + \frac{D(q)}{H} \cdot \left(\frac{1 - C(q + pz)}{1 - z} + C(q)(U(q + pz) - 1)\right)\right). \tag{6}$$

With the Eq. (2), we can derive the G.F. $Y^{SC}(z)$ of queue length at any data packet departure epoch as follows:

$$\begin{aligned} Y^{SC}(z) &= \frac{p(1-\rho)(1-z)S(q+pz)}{\lambda(S(q+pz)-z)}\Bigg(\frac{1-D(q)+D(q)C(q)}{H}+\frac{D(q)}{H} \\ &\quad\cdot\left(\frac{1-C(q+pz)}{1-z}+C(q)(U(q+pz)-1)\right)\Bigg). \end{aligned} \tag{7}$$

The waiting time W can be derived from the joint G.F. of queue length and the remaining transmission time of the being served packet, and can also be derived directly by queues length at the departure epoch, namely,

$$Y^{SC}(z) = W(q+pz)S(q+pz).$$

Analog to $S(q+pz)$, $W(q+pz)$ is the G.F. of packet arrival within the waiting time W given by

$$\begin{aligned} W(z) &= \frac{(1-\rho)(1-z)}{pS(z)-z+q}\cdot\frac{1}{H(1-z)}\big((1-z)+D(q)(z-q) \\ &\quad -D(q)U(z)(pC(z)-C(q)(1-z))\big). \end{aligned} \tag{8}$$

The period that the server continuously serves packets is named the busy period B, and we define the busy cycle R as the period from the moment at which a busy period is over to the moment at which the next busy period is over.

The average length of the idle period has already been obtained from the Eq.(4). Now we will discuss the average length of the busy period.

The G.F. of the number of packets at the beginning of the busy period is as follows:

$$\begin{aligned} Q_b(z) &= (1-D(q))z+D(q)C(q)zU(q+zp)+D(q) \\ &\quad\cdot(C(q+zp)-C(q))U(q+zp). \end{aligned} \tag{9}$$

Let B_0 denote the length of the busy period in classical Geom/G/1, its average is as follows:

$$E[B_0] = \frac{E[S]}{1-\rho}. \tag{10}$$

With the Eq. (9) and Eq. (10), we can obtain the G.F. $B(z)$ of queue length during the busy period by

$$\begin{aligned} B(z) &= (1-D(q))B_0(z)+D(q)C(q)B_0(z)U(q+pB_0(z)) \\ &\quad +D(q)(C(q+pB_0(z))-C(q))U(q+pB_0(z)). \end{aligned}$$

We can derive the average length of the busy period B by differentiating the above equation at $z=1$ as follows:

$$E[T_B] = (1-D(q)+D(q)C(q)+pD(q)(E[U]+E[C]))\frac{E[S]}{1-\rho} = H\frac{E[S]}{1-\rho}. \tag{11}$$

The average length of busy circle R can be derived from the Eq. (4) and the Eq. (12) as follows:

$$E[T_R] = E[T_I] + E[T_B] = \frac{H}{p(1-\rho)}.$$

Let p_B, p_D, p_C, p_F and p_U denote, respectively, the probability that the system in a busy period B, close delay period D, close down period C, free period F and setup period U at a stationary state. Based on renewal reward theory, we can obtain that

$$\begin{aligned}
p_B &= \frac{E[T_B]}{E[T_R]} = \rho, \\
p_D &= \frac{E[T_D]}{E[T_R]} = \frac{(1-D(q))(1-\rho)}{H}, \\
p_C &= \frac{E[T_C]}{E[T_R]} = \frac{pD(q)(1-\rho)E[C]}{H}, \\
p_F &= \frac{E[T_I]}{E[T_R]} = \frac{D(q)C(q)(1-\rho)}{H}, \\
p_U &= \frac{E[T_U]}{E[T_R]} = \frac{pD(q)(1-\rho)E[U]}{H}.
\end{aligned}$$

5 Performance Measures

From above analysis, we can obtain the following performance measures in the system as follows:

Average packet response time $E[T]$. It is the sum of the average waiting time $E[W]$ and the average transmission time $E[S]$. We can derive the average length of the waiting time by differentiating the Eq. (8) at $z = 1$.

$$\begin{aligned}
E[T] &= E[S] + E[W] \\
&= E[S] + \frac{p}{2(1-\rho)}E[S(S-1)] + \frac{D(q)C(q)}{H}E[U] + \frac{1}{2H}\Big(pD(q) \\
&\quad \cdot (E[U(U-1)] + E[C(C-1)] + 2E[U]E[C])\Big).
\end{aligned} \tag{12}$$

We define setup (release) ratio γ to be the number of setups during a time slot.

$$\gamma = \frac{D(q)}{E[T_R]} = \frac{p(1-\rho)D(q)}{H}. \tag{13}$$

We define idle ratio θ to be the ratio of the delay period to the SVC existence period. The SVC exists during the busy period, close delay period, and close down period. The busy period is for transmitting data packets, the close down period is for interchanging some signals, and the delay period is inactive, therefore,

$$\theta = \frac{E[T_D]}{E[T_B] + E[T_D] + E[T_C]} = \frac{(1-\rho)(1-D(q))}{\rho H + (1-\rho)(1 - D(q) + pD(q)E[C])}. \tag{14}$$

We define operating efficiency φ to be the ratio of the transmission period to the SVC existence period.

$$\varphi = \frac{E[T_B]}{E[T_B] + E[T_D] + E[T_C]} = \frac{\rho H}{\rho H + (1-\rho)(1 - D(q) + pD(q)E[C])}. \qquad (15)$$

6 Numerical Results

As mentioned above Sections, SVC can be modeled as a discrete time queueing with setup close-delay close-down, where the virtual channel's setting and releasing is processed by a series of fixed length signaling, so we can assume U and C are, respectively, constants H_U and H_C, and give $D(q) = q^{H_U}$ and $C(q) = q^{H_C}$. When the data is transmitted at a constant rate, the transmission time S can also be assumed to be the constant H_S. $E[U]$ and $E[C]$ are given as constants, which are appropriate to SVC's building and releasing.

Using the parameters in reference [2] we consider a TELNET connection on ATM LAN. As a numerical example, the setup time is assumed as 50ms, release time is assumed as 30ms and transmitting rate is assumed as 4kbps.

The measures above can be shown in Figs. 3 - 6. We can see directly from the figures. the relationships between the measures such as average waiting time, setting ratio, idle ratio and the timer length, when ρ takes the value ranged from 0.25 to 0.75.

From Fig. 3, we conclude that at the same traffic intensity ρ, with the increase of the delay timer D, the average waiting time tends to a fixed value after a sharp decrease, but at the same delay timer D, different traffic intensity ρ will result in a very different average waiting time. In Fig. 4, we know, when the traffic intensity ρ takes the same value, with the increase of the delay timer D, the setup ratio will decrease, and tend to zero. On the other hand, at the same delay timer D, the setting up ratio is increasing as the traffic intensity ρ decreases. The idle ratio is demonstrated in Fig. 5, when the traffic intensity ρ is the same, with the increase of the delay timer D, idle ratio will tend to a fixed value after coming through a sharp increase with the increase of delay timer D. In Fig. 6, the relationship between the system efficiency and the delay timer D is shown, which is contrary to Fig. 5 to some extend. At the same traffic intensity ρ, with the increase of delay timer D, the system running efficiency tends to a fixed value after a sharp decrease. With the same delay timer D, larger traffic intensity ρ will result in greater running efficiency.

7 Conclusions

The SVC performance measures analyzing processes in this paper is very straightforward, without going through complicated and time consuming imbedded Markov renewal processes. This method has instructive meaning in analyzing discrete time queueing with general vacations. In the application of broadband network performance evaluation, relationships between SVC measures such as response time, connection establishment ratio, operating efficiency, idle ratio and the close delay timer's setting are also displayed using accurate equations and intuitive figures, which have potential applications in setting network adapters and optimizing the SVC performance.

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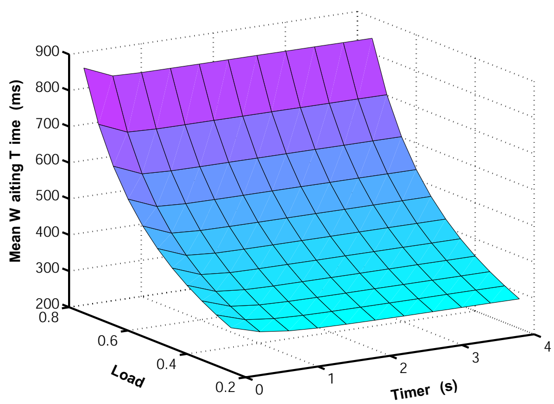

Figure 3: Average Waiting Time.

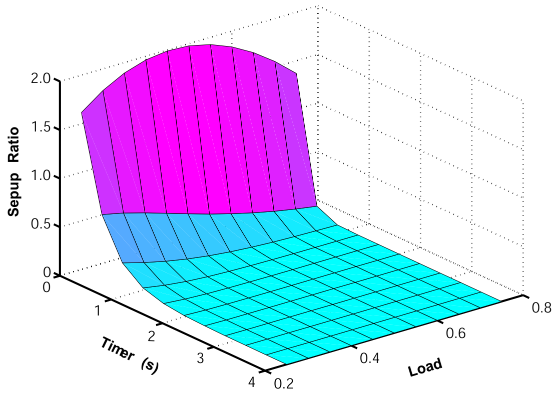

Figure 4: Setup Ratio.

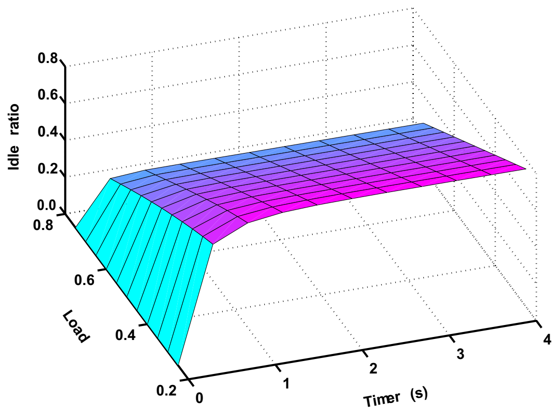

Figure 5: Idle Ratio.

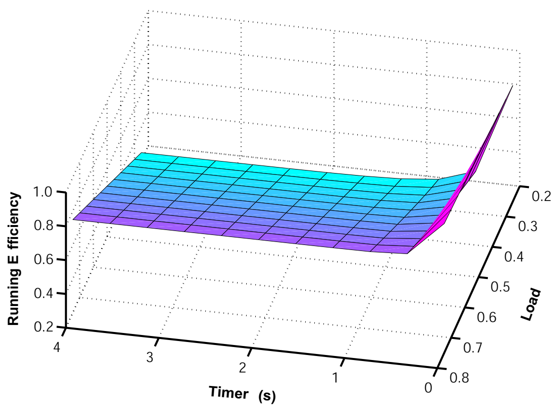

Figure 6: Running Efficiency.