Dynamic Inventory Management with Financial Constraints

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Abstract *Traditional inventory models focus on operational decisions. In this research, we propose a general framework for incorporating financial states of an organization in multi-period inventory models with lost sales. This approach differs from the traditional ones in that operational decisions are correlated with and constrained by financial flows of the firm. We study the borrowing and and lending actions of the firm with or without loan limits. We characterize the optimal inventory management decisions and its dependence structure on the financial decisions.*

Keywords production/financing decisions, dynamic inventory management, loan limit, supply chain management

1 Introduction

Most traditional inventory models focus on operational decisions and inventory control, i.e., characterizing replenishment policies based on inventory level over a planning horizon. There is an extensive literature on inventory control in both deterministic and stochastic environments, which includes the well-known EOQ and newsvendor model, the (s, S) and (r, Q) multi-period stochastic inventory models, the single-echelon and multi-echelon inventory systems, and so on. (see Nahmias 2001; Zipkin 2000; Axsäter 2000; Porteus 2002 for most references; Clark and Scarf 1960; Scarf 1960; Lau 1980; Karlin 1960; Eeckhoudt et al. 1995; Axsäter and Rosling 1993; Agrawal and Seshadri 2000; Federgruen and Heching 1999). Almost all of the models assume that retailers can freely implement their optimal operational decisions that are based on their production information, and there exist no budget constraint on purchasing decisions.

In real applications, especially for start-up and growing firms, a firm is often short of capital, and as a result, the purchasing decisions have to be restrained by the availability of capital. Indeed, cash flow is a major reason for the bankruptcy of small- and medium- sized firms. However, only a few operational decision models consider budgetary constraints. By assuming the availability of market hedges, Birge (2000) adapts option pricing theory for incorporating risk into planning models by adjusting capacity and resource levels. Other models include those of Hadley and Whitin (1963), Sherbrooke (1968) and Rosenblatt (1981). However, these models assume that the budget in each period is fixed and will not be affected by inventory decisions. An exception is Rosenblatt and Rothblum (1990), who treat the capacity as a decision variable for studying multi-item inventory systems under a single resource capacity constraint. However, their model does not incorporate financial considerations.

Financial and operational decisions are usually studied separately. As one of the most fundamental results in corporate finance, Modigliani-Miller (MM) (Modigliani and Miller 1958) proposes that in perfect capital markets, the firm's capital structure and its financial decisions(e.g., the allocation between equity and debt) are independent of the firm's investment and its operational decisions(e.g., inputs and outputs, the levels of inventory and capital). However, real capital markets are imperfect: there are taxes, information asymmetry, accounting costs, bankruptcy costs, and so on. In many cases, start-up and growing firms with limited capital should seek help from banks or other lenders. However, the firm's operational decisions affect its borrowing capacity. For example, a lender loans money to a firm with the maximum amount of the loan partially based on the firm's inventory. Consequently, although the financial and operational decisions are usually made separately, there is an interplay between them.

Recently, several operations papers have recognized the importance of the interaction between financial and operational decisions, and tried to incorporate financial considerations into operational decisions. Li et al. (1997) consider a single-product firm that makes production decisions, borrowing decisions and dividend policies each period while facing uncertain demand. The firm maximizes the expected present value of the infinite-horizon flow of the dividends subject to loan size, production size, and liquidity constraints. The firm can obtain an unbounded single-period loan with a constant interest rate. The authors derive optimal policies, which turn out to be myopic, and study their properties.

Babich and Sobel (2004) coordinate financial decisions (loan size) and operational decisions (production and sales) to maximize the expected discounted proceeds from an initial public offering (IPO). They model the IPO event as a stopping time in an infinite-horizon discounted Markov decision process. Furthermore, they characterize an optimal capacity-expansion policy and sufficient conditions for a monotone threshold rule to yield an optimal IPO decision.

Archibald et al. (2002) hypothesize that start-up firms are more concerned with the probability of long-term survival than with profitability. They present a sequential decision model for a firm that faces an uncertain bounded demand and whose inventory ordering decisions are constrained by current assets.

The recent research by Buzacott and Zhang (2004) is closely related to ours. In their paper, they establish the link between the financial state of a firm and the amount the firm is able to borrow to finance its operations. In the deterministic environment, they use a mathematical programming model to maximize profit over a finite horizon. More interestingly, they analyze a leader-follower game between the bank and the retailer in a newsvendor inventory model. While the bank's decisions include the interest rate to charge and the loan limit, a retailer needs to decide the amount to borrow within the loan limit and the amount of inventory to order from suppliers. Both the bank and retailers maximize their expected returns. Buzacott and Zhang analyze the motivation of asset-based financing. However, their stochastic model is single-period.

In this paper, we propose a general framework for incorporating financial decisions into multi-period stochastic inventory models. Our goals in the paper are to give reasonable models that provide insight into the interaction between financial and operational decisions. That is, to show that (1) the amount of money the firm can borrow to finance inventory policies is affected by its production decisions and (2) the optimal inventory policies the firm wants to implement are constrained by the capital available. The joint consideration of operational and financial decisions gives this paper a feasible place in real business decision-making.

We mainly consider two cases as introduced above. The first one is the multi-period inventory model with financial budgetary constraints and assumption of no borrowing and lending. Instead of setting a fixed capacity constraint as most existing models suggest, we characterize the capital available in each period as a variable that is updated periodically according to production activities. In the other case, we allow borrowing and lending, and show that it gives the retailer an effective tool for reducing the influence of capital shortages. Furthermore, we introduce a loan limit and consider its impact on the firm for avoiding bankruptcy.

In addition, in our multi-period inventory model, instead of assuming a backlog in uncleared inventory, we assume lost sales for the unsold product. To the best of our knowledge, we refer readers to Cohen et al. (1988), De Kok (1985) and Ha (1997) for a thorough description of this consideration.

The rest of this paper is organized as follows. Section 2 presents the financial constraint model with the borrowing and lending prohibition. In Section 3, we allow depositing and borrowing and analyze dynamic inventory policies under financial decisions with or without loan limit. Conclusions are given in Section 4. All proofs are omitted in this presentation and details can be found in Chao, Chen, and Wang (2005).

2 Multi-Period Model with Capital Constraint

Start-up and growing firms usually get in trouble for being short of capital. If a firm has insufficient capital, he can no longer follow his inclinations for making operational decisions such as placing an order. That means, the financial state, denoted by S here, will affect the inventory policies. This section proposes a simple multi-period inventory model to characterize the influence of capital shortness to the optimal inventory policies.

We also consider a risk neutral retailer that makes replenishment decisions over a finite time horizon of N periods. For simplicity, the distribution of demands, selling price and linear ordering cost are assumed to be changeless over the total N periods, denote them by $F(\cdot)$, p and c respectively. Furthermore, we assume no holding cost and penalty cost. Let S_n be the accumulated capital level, here are two differences from the traditional model. One is the capital constraint: $c(y_n - x_n) \leq S_n, n = 1, 2, \ldots, N$, and the other one is the assumption of lost sales, which means the unsatisfied demand will be lost.

Therefore, the decision problem is to decide an ordering policy to maximize the final capital level, given an initial inventory level x_1 and a capital level S_1, subject to a capital constraint for each period. That is, the decision problem is:

$$\max_{y_1,\cdots,y_N} E[S_{N+1}], \tag{1}$$

subject to

$$0 \leq y_n - x_n \leq \frac{S_n}{c}, n = 1, 2, \ldots, N,$$

where $S_{n+1} = S_n + p\min\{y_n, D_n\} - c(y_n - x_n)$ and $x_{n+1} = (y_n - D_n)^+, n = 1, 2, \ldots, N$.

Denote by $V_n(x, S)$ the maximum achievable capital starting at the beginning of period n with an initial inventory level x and an accumulated capital S. We can apply a dynamic program for the decision problem as follows. Let

$$V_{N+1}(x, S) = S,$$

and

$$V_n(x, S) = \max_{x \leq y \leq x + \frac{S}{c}} E[V_{n+1}(x_+, S_+)], \tag{2}$$

where $x_+ = (y - D_n)^+$ and $S_+ = S + p\min\{y, D_n\} - c(y - x)$. Notice that we assume $V_{N+1}(x, S)$ is independent of x here, which implies zero salvage value. Finally, we have

$$\max_{y_1,\cdots,y_N} E[S_{N+1}] = V_1(x, S).$$

To solve the dynamic program problem, we introduce following lemmas firstly.

Lemma 1 *For any period n and fixed x, $V_n(x, S)$ is increasing in S.*

Lemma 2 *$V_n(A - z, B + pz)$ is increasing in z for fixed A and B.*

The proof follows from Lemma 1 and the following iterative formula:

$$\begin{aligned} & V_n(A - z, B + pz) \\ = & \max_{A - z \leq y \leq A + B/c + (p-c)z/c} E[V_{n+1}((y - D_n)^+, cA + B + (p - c)z + p\min\{y, D_n\} - cy)]. \end{aligned}$$

Then we can show that a capital dependent base stock inventory policy is optimal for decision problem (1).

Theorem 2.1 *$V_n(x, S)$ is jointly concave in x and S for any period n. Furthermore, a capital dependent base stock inventory policy is optimal.*

Since the optimal order-up-to quantity is a function of $S+cx$, we have following definition.

Definition 1 *Let $R = S + cx$ be the total wealth including capital S and inventory value cx.*

The following lemma makes sure that $R_n \geq 0$ for any period n. That is, in other words, the optimal inventory policy can be carried out for all periods.

Lemma 3 *$R_{n+1} \geq R_n$ for any $n = 1, 2, \ldots, N$.*

The proof follows directly from iterative formula $R_{n+1} = R_n + (p-c)\min\{y_n, D_n\}$.

Then giving non-negative initial capital S_1 and inventory x_1, the wealth R_n will be non-negative for any period.

Let $\tilde{V}_n(x, R) \triangleq \max_{x \leq y \leq R/c} V_{n+1}((y - D)^+, p\min\{y, D\} - cy + R) = V_n(x, S)$, with following lemmas and theorem we derive some properties of the optimal inventory policy.

Lemma 4 *(Topkis, 1998) Suppose that $g(y)$ is a real-valued function on $\mathbb{R}$, and $f(x) = g(a_1x_1 + a_2x_2)$ for x on $\mathbb{R}^2$, where $a_i > 0$ for i=1,2. If $g(y)$ is concave in y on $\mathbb{R}$, then $f(x)$ is submodular in x on $\mathbb{R}^2$.*

Lemma 5 *For any $n = 1, 2, \ldots, N$,*

(a) if $\tilde{V}_n(x, R)$ is submodular in (x, R), then $E[\tilde{V}_n((y - D_n)^+, R)]$ is submodular in (y, R),

(b) if $\tilde{V}_n(x, R)$ is concave in R, then $E[\tilde{V}_n(x, (p - c)\min\{y, D_n\} + R)]$ is submodular in (y, R),

(c) if $\tilde{V}_n(x, R)$ is submodular in (x, R) and concave in R, then $E[\tilde{V}_n((y-D_n)^+, (p-c)\min\{y, D_n\} + R)]$ is submodular in (y, R).

Theorem 2.2 *For any period n, (a) $\tilde{V}_n(x, R)$ is jointly concave in x and R, (b) $\tilde{V}_n(x, R)$ is submodular in (x, R) .*

Next we propose some properties of the optimal order-up-to quantity. From the theorem, we can see that if there is a capital constraint, the optimal inventory policy here is quite different from that for the traditional multi-period inventory model. In fact, the optimal order-up-to quantity in each period is less. That means, being short of capital will lead to a smaller ordering and hence a lower profit.

Theorem 2.3 *(a) For period n, the optimal order-up-to quantity $y^*_{n_c}(R)$ with initial total wealth R is as follows:*

$$y^*_{n_c}(R) = \begin{cases} R/c & , \text{ if } 0 \leq R \leq R_{n_c} \\ \hat{y}_{n_c}(R) & , \text{ if } R > R_{n_c} \end{cases} , \tag{3}$$

where R_{n_c} is the solution of equation $\hat{y}_{n_c}(R) = R/c$. Furthermore, $\hat{y}_{n_c}(R)$ is decreasing in R for $R > R_{n_c}$.

(b) For any period n, we have the following bound of the optimal policy:

$$F^{-1}(\frac{p-c}{p}) \leq \hat{y}_{n_c}(R) \leq U_D$$

where U_D is the maximum achievable demand.

The proof of this result is based on the following.

Lemma 6 *Let $f_n(y,R) = E[V_n((y-D_n)^+, p\min\{y, D_n\} - cy + R)] = E[\tilde{V}_n((y - D_n)^+, (p-c)\min\{y, D_n\} + R)]$. We have*

$$p(1-F(y)) - c \leq f_{n_1}(y,R) \leq (\frac{p}{c})^{N-n+1}(p-c)(1-F(y)),$$

and

$$1 \leq f_{n_2}(y,R) \leq (\frac{p}{c})^{N-n+1}.$$

where $f_{n_1}(y,R) = \frac{\partial f_n(y,R)}{\partial y}$ and $f_{n_2}(y,R) = \frac{\partial f_n(y,R)}{\partial R}$.

From the above theorem, the optimal retailer order-up-to quantity increases as R increases from 0 until R_{n_c} and eventually it decreases once $R > R_{n_c}$. In other words, the retailer will respond in the following ways for different R.

- Retailers with wealth level $R \leq R_{n_c}$ will have insufficient capital. Therefore they will use all the cash they have to finance their inventory but can not carry out their optimal inventory policy. Adding a little to R will increase the amount they order.
- Retailers with wealth level $R > R_{n_c}$ will have more than enough cash or inventory for their operations. Therefore they can order the amount they think is optimal. But as their wealth increases, to save up capital for further orders, they will order less!

Since the lack for capital, firms may want to borrow from the bank. In the following section, we will allow retailers to deposit and loan.

3 Multi-Period Model with Financial Decisions

The firms who are short of capital may seek help from the bank or other lenders. Once the borrowing and lending are allowed, retailers should decide not only the ordering quantity, but also the amount of loan or deposit. So they have to coordinate the operational and financial decisions to maximize their profit.

In §3.1, we introduce the basic consideration of incorporating financial decisions into inventory models and establish the basic inventory model. Then in §3.2, with the assumption that retailers can borrow with no loan limit, we analyze the multi-period inventory model based on financial decisions. Finally, in §3.3, we reconsider the inventory model in the environment where the bank sets a loan limit.

3.1 An Introduction to Financial Inventory Model

We establish the inventory model from the retailer's view point. At the beginning of each period, the retailer with initial capital S and inventory x places an order if necessary. If the capital is insufficient, the retailer can borrow from the bank at interest rate b. Conversely, if some capital remains after the order is placed, the retailer should deposit it in the bank at interest rate d. We assume that $b > d$. Furthermore, we assume $p > c(1+b)$ because no retailer will borrow otherwise. It is also clear that the retailer should use up all its cash before consider borrowing money from the bank. At the end of the period, the retailer will receive income from sales and repay the loan.

The retailer's objective is to decide an ordering policy and a financial decision to maximize the final capital level. Notice that given the inventory policy, we can derive the financial policy by simple deduction. For example, given inventory level x_0 and capital level S_0, if we want to order up to y_0, then the amount we need to borrow is $c(y_0-x_0)-S_0$, or the amount we will deposit is $S_0-c(y_0-x_0)$. Therefore, the decision objective is the same as that in the capital constraint model. That is, given an initial inventory level x_1 and a capital level S_1, the decision problem is:

$$\max_{y_1,\ldots,y_N} E[S_{N+1}], \tag{4}$$

where $x_{n+1} = (y_n - D_n)^+$ and $S_{n+1} = (1+d)[S_n - c(y_n - x_n)]^+ + p\min\{y_n, D_n\} - (1+b)[S_n - c(y_n - x_n)]^-, n = 1, 2, \ldots, N$.

In the following subsection, with the assumption of no loan limit, we will characterize the optimal inventory policy and study its properties.

3.2 Financial Inventory Model with No Loan Limit

Denote by $W_n(x, S)$ the maximum achievable capital starting at the beginning of period n with an initial inventory level x and an accumulated capital S. We can apply following dynamic program for solving decision problem (4). Let

$$W_{N+1}(x, S) = S,$$

and

$$W_n(x, S) = \max_{y \geq x} E[W_{n+1}(x_+, \bar{S}_+)], \tag{5}$$

where $x_+ = (y - D_n)^+$ and $\bar{S}_+ = (1+d)[S - c(y-x)]^+ + p\min\{y, D_n\} - (1+b)[S - c(y-x)]^-$.

The optimal inventory policy for dynamic program (5) is still a capital dependent base stock policy.

Theorem 3.1 *$W_n(x, S)$ is jointly concave in x and S for any period n. Furthermore, a capital dependent base stock inventory policy is optimal.*

Recall that in Lemma 3, we obtained $R_{n+1} \geq R_n$ and $R_n \geq 0, n = 1, 2, \ldots, N$ for the capital constraint model. However, for the deposit-loan model here, we do not have these inequalities. Defining a retailer as bankrupt for period n if $R_{n+1} < 0$, the following lemma shows the probability of retailer bankruptcy.

Theorem 3.2 *For period n, if the retailer borrows more than R_n/b, then the probability of retailer bankruptcy is $F\left(\frac{b(cy_n - R_n) - R_n}{p-c}\right)$.*

Because we do not add any constraint on the retailer's borrowing activity here, once the retailer is bankrupt for certain period we allow him to proceed with his inventory policies in the following periods. But if the retailer is bankrupt for period n, it will be certain that he will borrow in the next period.

Next we derive some properties of the optimal inventory policy. This differs from the capital constraint case. If depositing and borrowing are allowed, two possible situations can arise: (1) the retailer has insufficient initial wealth so he has to borrow from the bank, or (2) the retailer has enough wealth so he can deposit what remains. Then for dynamic program equation (5), function $E[W_{n+1}(x_+, \bar{S}_+)]$ is given by

$$E[W_n(x_+, \bar{S}_+)] = \begin{cases} E[W_n((y-D)^+, p\min\{y, D_n\} + (1+b)(R-cy))], & \text{if } R \leq cy, \\ E[W_n((y-D)^+, p\min\{y, D_n\} + (1+d)(R-cy))], & \text{if } R > cy. \end{cases} \quad (6)$$

From Theorem 3.1, we find that $E[W_n((y-D)^+, p\min\{y, D_n\} + (1+b)(R-cy))]$ and $E[W_n((y-D)^+, p\min\{y, D_n\} + (1+d)(R-cy))]$ both are concave in y. Therefore, let $\hat{y}_b(R)$ and $\hat{y}_d(R)$ be the optimal solutions to problems

$$\max_y E[W_n((y-D)^+, p\min\{y, D_n\} + (1+b)(R-cy))]$$

and

$$\max_y E[W_n((y-D)^+, p\min\{y, D_n\} + (1+d)(R-cy))]$$

respectively. The following theorem identifies the order-up-to quantity that a retailer with initial wealth R will choose.

Theorem 3.3 *For period n, the optimal order-up-to quantity $y^*(R)$ with initial total wealth R is as follows:*

$$y^*(R) = \begin{cases} \hat{y}_b(R), & \text{if } R \leq R_b \\ R/c\,, & \text{if } R_b < R < R_d \\ \hat{y}_d(R), & \text{if } R \geq R_d \end{cases}$$

where R_b, R_d are solutions of equations $\hat{y}_b(R) = R/c$ and $\hat{y}_d(R) = R/c$ respectively. Furthermore, $\hat{y}_b(R)$ is decreasing in R for $R \leq R_b$, and $\hat{y}_d(R)$ is decreasing in R for $R \geq R_d$.

From the above theorem, the optimal retailer order-up-to quantity decreases as R increases from $-\infty$ until R_b, then it increases until $R = R_d$ and eventually it decreases once $R > R_d$. In other words, the retailer will respond in the following ways for different R.

- Retailers with wealth level $R \le R_b$ will have insufficient capital, and even be indebted. Therefore they have to borrow from the bank to finance their inventory. However they will borrow less and then order less if their wealth increases.
- Retailers with wealth level $R_b < R < R_d$ will have enough capital and need not borrow. However, they have to use up all their cash.
- Retailers with wealth level $R \ge R_d$ will have more than enough cash or inventory. In fact, they have so much wealth that they can deposit the remaining cash. However, as their wealth increases, they may want to earn interest and hence they will order less.

3.3 Financial Inventory model with Loan Limit

In the real world, firms can not borrow an infinite amount and banks set loan limits most of the time. The loan limit is always a ratio of the firm's capital value, we assume it to be α. Banks have recently started to consider inventory as part of the firm's value. Therefore, if a firm has little capital but has a lot of inventory, the owner can also get a sufficiently-large loan from the bank. For simplicity, we let cx be the value of inventory, then the maximum loanable capital is $\alpha(S+cx)$.

Therefore, we have following dynamic program:

$$U_{N+1}(x,S) = S,$$

and

$$U_n(x,S) = \max_{x \le y \le (1+\alpha)(S+cx)/c} E[U_{n+1}(x_+, \bar{S}_+)], \tag{7}$$

where $x_+ = (y-D_n)^+$ and $\bar{S}_+ = (1+d)[S-c(y-x)]^+ + p\min\{y, D_n\} - (1+b)[S-c(y-x)]^-$.

The following theorem is also the same as that in the no loan limit case, with a few differences in the proof.

Theorem 3.4 *$U_n(x,S)$ is jointly concave in x and S for any period n. Furthermore, a capital dependent base stock inventory policy is optimal.*

Recall that in Theorem 3.2, under the condition that the bank does not set a loan limit, if the retailer borrows more than R_n/b for certain period he may be bankrupt with probability $F(\frac{b(cy_n-R_n)-R_n}{p-c})$. Then it will be risky for the bank to continue lending money to the retailer. Therefore, the bank will set a loan limit to avoid risk. The following theorem allows us to show that when there is not retailer bankruptcy.

Theorem 3.5 *If the loan limit rate α satisfies $\alpha < 1/b$, then the retailer will not be bankrupt when borrowing.*

Notice that $R_{n+1} = (p-c)\min\{y_n, D_n\} + (1+b)R_n - bcy_n$, so the proof follows from the loan limit $cy - R \leq \alpha R$.

Next we derive the property of the optimal inventory policy considering the loan limit. This is similar to the case of no loan limit. Let $\hat{y}_{lb}(R)$ and $\hat{y}_{ld}(R)$ be the optimal solutions to problems

$$\max_y E[U_n((y-D)^+, p\min\{y, D_n\} + (1+b)(R-cy))]$$

and

$$\max_y E[U_n((y-D)^+, p\min\{y, D_n\} + (1+d)(R-cy))]$$

respectively. We have following theorem.

Theorem 3.6 *For period n, the optimal order-up-to quantity $y_l^*(R)$ with initial total wealth R is as follows:*

$$y_l^*(R) = \begin{cases} (1+\alpha)R/c, & \text{if } 0 \leq R \leq R_\alpha \\ \hat{y}_{lb}(R) \quad , & \text{if } R_\alpha < R \leq R_{lb} \\ R/c \quad , & \text{if } R_{lb} < R < R_{ld} \\ \hat{y}_{ld}(R) \quad , & \text{if } R \geq R_{ld} \end{cases}$$

where R_α, R_{lb}, R_{ld} are solutions of equations $\hat{y}_{lb}(R) = (1+\alpha)R/c$, $\hat{y}_{lb}(R) = R/c$ and $\hat{y}_{ld}(R) = R/c$ respectively. Furthermore, $\hat{y}_{lb}(R)$ is decreasing in R for $R_\alpha \leq R \leq R_{lb}$, and $\hat{y}_{ld}(R)$ is decreasing in R for $R \geq R_{ld}$.

From the above theorem, the optimal retailer order-up-to quantity increases as R increases from 0 until R_α, then it decreases until $R = R_{lb}$, and then it increases until $R = R_{ld}$ and eventually it decreases once $R > R_{ld}$. And here the retailer will respond in the similar ways to that in the no loan limit case. The very difference is that

- Retailers with wealth level $R \leq R_\alpha$ will be short of cash or inventory and want to borrow. However, since their wealth is so small they can not borrow too much. Therefore they will borrow all they can to finance their inventory but can not carry out their optimal inventory policy.

By far we have characterized the optimal operational decisions with alternative financial decisions. We can see that if borrowing and lending are allowed, the structure of the optimal inventory policy is quite different with the traditional ones. In fact, the right to make financial decisions leads to more rational operational decisions and higher profit. In the following section we propose a simple computational result for a comparison of the three models.

4 Conclusions

In this paper, we propose a general framework for incorporating financial decisions into multi-period inventory models with lost sales. We consider the simple one-retailer and one-item case, and introduce three models. The first one is the capital constraint model, where the firm's operational decisions will be constrained by their limited capital. The latter two models are inventory models with financial decisions. One is with the assumption of no loan limit and in the other model banks set a loan limit.

For each model, we derive the optimal inventory policy, which turns out to be a capital dependent base stock policy. Furthermore, we study their properties (e.g. the dependence on the initial wealth). Our financial inventory models refer to the probability of firm bankruptcy as well.

With a simple computational result, we demonstrate that it is essential for retailers to take financial considerations into their operational decisions, especially for retailers who are short of capital. Retailers who make use of financial tools (e.g. borrowing and lending) will have their operational decisions more rational so as to make more profit.

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