Applying Path Counting Methods for Evaluating Edge and Node Deletion Connectivity Functions for the Water Supply Network System

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Abstract  There are many important and indispensable network-structured systems surrounding our daily lives, such as road networks for traffic road systems, electric power networks for electric transmission and distribution, city gas networks for city gas supply, water pipeline network for water supply, and so on. These networks are generally called ‘lifeline’ networks needed to maintain our daily lives. We aim at evaluating quantitatively reliability, stability, and strength of the connectivity of these network-structured systems. We propose a quantitative method for evaluating the stable connectedness of the network-structured system using path counting methods. Given a connected network \(N=(V,E)\) with the vertex set \(V\) of \(n\) nodes and the edge set \(E\) of \(m\) edges, there are \(n(n-1)\) paths connecting every pair of two distinct nodes. We define the stable connectedness of the network as the ratio between total number of paths connecting two different nodes and \(n(n-1)\) after deleting \(k\) edges out of \(m\) edges (edge deletion) or deleting \(p\) nodes out of \(n\) nodes (node deletion). We show the stable connectivity functions for many different types of graphs and some actual lifeline networks. Some theoretical results are also given for special types of graphs.