Scheduling Problem with Discretely Compressible Release Dates

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Abstract In this paper, we address the scheduling model with discretely compressible release dates, where processing any job with a compressed release date incurs a corresponding compression cost. We consider the following problem: scheduling with discretely compressible release dates to minimize the sum of makespan plus total compression cost. We show its NP-hardness, and design an approximation algorithm with worst-case performance ratio 2.

Keywords Scheduling; Discretely compressible release dates; NP-hardness; Approximation algorithm

1 Introduction

In this paper, we study the scheduling problem with discretely compressible release dates. In the classical scheduling, it is always assumed that the parameters of a job, say its processing time, its release date etc. are all fixed. However, in the real world, the processing of jobs is not only determined by the machine speed, but also by other resources such as labor, funds etc., therefore the parameters of a job may not be fixed. For example, we can compress the original release dates. Of course, processing any job with a compressed release date incurs a compression cost.

The scheduling problem with compressible release dates has its deep root in the real world. It commonly arises in manufacturing systems where the preprocessing of the jobs depends on a common resource such as fuel, catalyzer, raw materials, etc. Real-life examples of such problems are given in Janiak ([1], [2], [3]) in the context of steel production which involves preheating of iron ingots ([4]).

Let \( J = \{1, 2, \cdots, n\} \) denote a list of given jobs. We write SCR as an abbreviation of the scheduling problem with compressible release dates. We denote by TRC the total compression cost in SCR. There are two variants for SCR, the continuous one and the discrete one, which are denoted by SCCR and SDCR, respectively. In SCCR, any job \( J_j \) can be processed with a release date \( r_j \in [l_j, u_j] \) and a corresponding compression cost \( c_j(u_j - r_j) \) is incurred, where \( c_j \) is the cost coefficient. And in SDCR, the value of \( r_j \) can

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be selected from among \( \{ r_{j1}, r_{j2}, \ldots, r_{jk} \} \), and the corresponding compression costs are \( e_{j1}, e_{j2}, \ldots, e_{jk} \).

There are the following four models for SCR:

(P1) To minimize \( F_1 + F_2 \);
(P2) To minimize \( F_1 \) subject to \( F_2 \leq a \);
(P3) To minimize \( F_2 \) subject to \( F_1 \leq b \);
(P4) To identify the set of Pareto-optimal points for \( (F_1, F_2) \).

Where \( F_1 \) is the original objective function, and \( F_2 \) is TRC. In the objective function field of the notation of Graham et al. ([5]), we write the above four model as \( F_{SDCR} \), respectively.

Nowicki and Zdrzalka ([6]) showed that \( 1\mid r_j, cr\mid C_{\text{max}} + \text{TRC} \) is strongly NP-hard and designed an approximation algorithm with worst-case ratio 2 which is the best possible. Sun ([7]) concentrated on the problems \( 1\mid r_j, p_j = 1, cr\mid (C_{\text{max}}, \text{TRC}) \), \( 1\mid r_j, p_j = 1, c_j = 1, cr\mid (C_{\text{max}}, \text{TRC}) \) and \( 1\mid r_j, p_j = 1, c_j = 1, cr\mid (\sum C_j, \text{TRC}) \). Sun and R.J. Kibet ([7]) also studied the problem \( 1\mid r_j, p_j = 1, cr\mid (\sum C_j, \text{TRC}) \). Cheng and Shakhlevich ([8]) studied the problems \( 1\mid r_j, p_j = 1, cr\mid C_{\text{max}} + \text{TRC} \), \( 1\mid r_j, p_j = 1, cr\mid C_{\text{max}} / \text{TRC} \), \( 1\mid r_j, p_j = 1, cr\mid C_{\text{max}} / \text{TRC} \) and the cases with integer compression amounts for the above three problems. They also discussed the problems \( 1\mid r_j, p_j = 1, c_j = 1, cr\mid C_{\text{max}} + \text{TRC} \), \( 1\mid r_j, p_j = 1, c_j = 1, cr\mid C_{\text{max}} / \text{TRC} \) and \( 1\mid r_j, p_j = 1, c_j = 1, cr\mid C_{\text{max}} / \text{TRC} \). And furthermore, they showed that \( 1\mid r_j, p_j = 1, cr\mid \sum w_j C_j + \text{TRC} \), \( 1\mid r_j, p_j = 1, cr\mid \sum w_j C_j / \text{TRC} \) and \( 1\mid r_j, p_j = 1, cr\mid \sum w_j C_j / \text{TRC} \) are all NP-hard in the ordinary sense.

The major work in the area of scheduling with compressible parameters considered either compressible processing times and fixed release dates ([11, 12, 14]), or compressible release dates and fixed processing times ([6, 7, 8, 10, 15, 16]). Cheng, Kovalyov and Shakhlevich ([13]) examined the problem of optimal scheduling the jobs on a single-machine when job processing times and release dates are compressible parameters and the objective is to minimize the makespan together with the linear compression cost function. They constructed a reduction to the assignment problem for the case of equal release date compression costs and develop an \( O(n^2) \) algorithm for the case of equal release date compression costs and the equal processing time compression costs.

In this paper, we address the P1 model for SDCR, \( 1\mid r_j, dr\mid C_{\text{max}} + \text{TRC} \) following the notation of Graham et al. The rest of this paper is organized as follows. Some preliminaries are given in Section 2. In section 3, we show it is strongly NP-hard. In section 4, we present an approximation algorithm with worst-case ratio 2. Conclusion and remarks are given in Section 5.

## 2 Preliminaries

An algorithm \( A \) is a \( \rho \)-approximation algorithm for a minimization problem if it produces a solution which is at most \( \rho \) times the optimal one, with a running time bounded by a polynomial in the input size. We also say \( \rho \) is the worst-case ratio of algorithm \( A \). The worst-case ratio is the most frequently used measure for the quality of an approximation algorithm for a scheduling problem: the smaller the ratio is, the better the approximation algorithm is.
In the following, we introduce some basic notations and formulate the problem considered in this paper. Each job $j \in J$ is characterized by a processing time $p_j$, a set of $k$ ($k \geq 1$ and is given) potential release dates: $r_{j1}, r_{j2}, \ldots, r_{jk}$ with $r_{j1} > r_{j2} > \cdots > r_{jk}$, where $r_{ji}$ is called the normal release date and $r_{jk}$ the minimum possible release date, and a set of $k$ potential compression costs: $e_{j1}, e_{j2}, \ldots, e_{jk}$. Whenever $r_{ji}, j \in J, 1 \leq i \leq k$, is selected as the actual release date of job $j$, a corresponding compression cost $e_{ji}$ is paid. Our work is to select a release date for each job and schedule these jobs such that the objective value $C_{\text{max}} + \text{TRC}$ is minimized. It is also assumed that $0 = e_{j1} < e_{j2} < \cdots < e_{jk}$. This assumption is sound as the more release dates we compress, the more cost we should pay.

Let $r = (r_1, r_2, \ldots, r_n)$ be a vector of actual release dates where $r_j \in \{r_{j1}, r_{j2}, \ldots, r_{jk}\}$, and $\pi$, a schedule of $J$ defining the job processing order. We denote by $R$ the set of all feasible actual release dates, $R = \{r : r_j \in (r_{j1}, r_{j2}, \ldots, r_{jk})\}$, and by $\Pi$ the set of all schedules of $J$. We assume that a pair $(r, \pi)$ uniquely determines a completion time $C_j(r, \pi)$ of each job $j \in J$. Two general performance measures are considered in our problem in this paper. The first one is based on the completion time $C_j(r, \pi)$ and therefore it will be called a completion cost. Given $r$ and $\pi$, the completion cost, which is denoted by $F_1(r, \pi)$, is the maximum completion time of all the jobs, i.e. $F_1(r, \pi) = \max\{C_j(r, \pi) : 1 \leq j \leq n\}$. The second measure is the total cost of compressions $F_2(r) = \sum_{i=1}^n e_{ji} (1 \leq i \leq k)$. $F_2(r)$ will be called a compression cost. For a pair $(r, \pi)$, a total scheduling cost $M(r, \pi)$ is defined by $M(r, \pi) = F_1(r, \pi) + F_2(r)$. Denote by $\pi^*(j, r)$ the actual release date of job $j$ in $\pi$, where $\pi(j)$ is the $j$th processed job in $\pi$ and $\pi(j, r)$ the index of its release time, $1 \leq \sigma(j, r) \leq k$. The problem considered in this paper is formulated as follows. Find $\pi^* \in \Pi$ and $r^* \in R$ minimizing

$$M(r, \pi) = \max_{1 \leq j \leq n} \left( r_{\pi(j)}, \sigma(j, \pi) + \sum_{i=1}^{n} p_{\pi(i)} \right) + \sum_{j=1}^{n} e_{\pi(j), \sigma(j, \pi)}$$

subject to $r \in R$ and $\pi \in \Pi$.

## 3 Proof of NP-hardness

Nowicki and Zdrzalka ([6]) showed that the problem $1|r_j, cr|C_{\text{max}} + \text{TRC}$ is strongly NP-hard by a reduction to the problem $1|\sum \omega_j T_j$, which is strongly NP-hard. We remark that their proof also applies to the special case that all the parameters are nonnegative integers. Next, we will prove by reduction to this case that $1|r_j, dr|C_{\text{max}} + \text{TRC}$ is also strongly NP-hard.

**Theorem 1.** $1|r_j, dr|C_{\text{max}} + \text{TRC}$ is strongly NP-hard.

**Proof.** For any integral instance of $1|r_j, cr|C_{\text{max}} + \text{TRC}$, $I = (p_1, p_2, \ldots, p_n; l_1, l_2, \ldots, l_n; u_1, u_2, \ldots, u_n, c_1, c_2, \ldots, c_n)$, where all the numbers are nonnegative integers, construct an instance $I'$ of $1|r_j, dr|C_{\text{max}} + \text{TRC}$ as follows: there are also $n$ jobs in total, for each job $j$, $1 \leq j \leq n$, its release time has $u_j - l_j + 1$ choices, $r_{j1} = u_j, r_{j2} = u_j - 1, \ldots, r_{j,u_j-l_j+1} = l_j$, and the corresponding costs are $e_{j1} = 0, e_{j2} = c_j, \ldots, e_{j,u_j-l_j+1} = (u_j - l_j)c_j$.

Obviously $I$ and $I'$ have exactly the same optimal objective values as well as the same optimal schedules. Since the construction can be done in pseudo-polynomial time of the size of $I$, the proof is done.
4 An approximation algorithm

In what follows, we discuss approximation algorithms for our problem. First note that compressing the release date may decrease the completion times of the jobs but incurs additional costs, so we here assume without loss of generality that
\[ e_j < r_{j1} - r_{j2}, e_{j3} < r_{j1} - r_{j3}, \cdots, e_{jk} < r_{j1} - r_{jk} \quad \text{for any job } j \in J. \]

**Algorithm G (General approximation algorithm)**

Step 1. Choose a schedule \( \pi^G \).

Step 2. Determine \( r^G \) minimizing \( M(r, \pi^G) \) subject to \( r \in R \).

Suppose that \( \pi^G \) is chosen according to one of the following rules: arbitrary schedule \( (G0) \), nondecreasing \( r_{j1} \) \( (G1) \), nondecreasing \( r_{jk} \) \( (G2) \), nondecreasing \( r_{jk} + e_{jk} \) \( (G3) \), nondecreasing \( r_{j1} + p_j \) \( (G4) \), nondecreasing \( r_{jk} + p_j \) \( (G5) \), nondecreasing \( r_{j1} + p_j \) \( (G6) \), nonincreasing \( p_j \) \( (G7) \), nonincreasing \( r_{j1} + p_j \) \( (G8) \), nondecreasing \( r_{jk} + e_{jk} + p_j \) \( (G9) \).

If there is a choice in \( G1-G3 \), then take the job with the largest processing time.

For the minimizing problem in Step 2, we design an optimal algorithm OMC. Before describing it, we state the following lemma which plays an important role in design and analysis of the algorithm OMC.

**Lemma 2.** There always exists an optimal schedule with the following properties:
(1) There is no idle time between adjacent jobs.
(2) There exists at least one job whose release time is exactly its start time (we call this job critical job).

**Proof sketch.** If there exists some idle time between two adjacent blocks, here block means a largest possible set of jobs in the optimal schedule with the property that the completion time of the former job is just the start time of the latter one, we can just postpone the start time of the former block such that its completion time is exactly the start time of the latter block, and so that there is no idle time between these two blocks. Obviously this will not change the objective function value, thus property (1) holds. Property (2) is straightforward.

Obviously, after determining the critical job and its release date, we can easily calculate the objective value of the schedule with no idle time between adjacent jobs, which is also the main idea of our optimal algorithm. To simplify our discussion, we may assume without loss of generality that the jobs are re-indexed so that \( \pi^G = (1, 2, \cdots, n) \) after choosing \( \pi^G \) in Step 1. Suppose job \( J_{j_0} \) is the critical job we determine, and its actual release date is denoted by \( r_{j_0,i_0} \).

For \( 1 \leq x < j_0 \), we define
\[ r(j_0, i_0, x) = \max \{ r_{ix} : 1 \leq i \leq k, r_{ix} \leq r_{j_0,i_0} - \sum_{t=x}^{j_0-1} p_t \} \]

For \( j_0 < y \leq n \), we define
\[ R(j_0, i_0, y) = \max \{ r_{iy} : 1 \leq i \leq k, r_{iy} \leq r_{j_0,i_0} + \sum_{t=j_0}^{y-1} p_t \} \]
Denote \( c(j_0,i_0,x) \) and \( c(j_0,i_0,y) \) as the corresponding indexes of release dates, namely 
\[
r(j_0,i_0,x) = r_{x,c(j_0,i_0,x)}, R(j_0,i_0,y) = r_{y,c(j_0,i_0,y)}.
\]

Algorithm OMC (Optimal minimum cost)

Input an instance \( \mathcal{J} \).
Let \( H_{j_0} := \infty, B_{j_0} := \infty \)
For \( j_0 = 1 \) to \( n \) do
  For \( i_0 = 1 \) to \( k \) do
    Calculate \( c(j_0,i_0,1), \ldots, c(j_0,i_0,j_0-1), c(j_0,i_0,j_0+1), \ldots, c(j_0,i_0,n) \)
    If all \( c(j_0,i_0,h)(h \in [1,j_0-1] \cup [j_0+1,n]) \) exist then let 
      \[
        H_{j_0} := \min \{ H_{j_0}, \sum_{x=1}^{j_0-1} e_{x,c(j_0,i_0,x)} \},
        B_{j_0} := \min \{ B_{j_0}, r_{j_0i_0} + \sum_{t=j_0}^{n} p_t + e_{j_0} + \sum_{y=j_0+1}^{n} e_{y,c(j_0,i_0,y)} \}
      \]
  EndFor
EndFor

Output \( \min \{ H_i + B_i : 1 \leq i \leq nk \} \) and the corresponding schedule.

It's not hard to calculate that the running time is \( O(n^2k) \).

In what follows we show results of the worst-case analysis for various variants of Algorithm \( G \), obtained from the general scheme by applying various rules for choosing \( \pi^G \) in Step 1. The rules \( G_3, G_6, G_9 \) are justified by the following lower bound. By (1), we get
\[
M(r^*, \pi^*) \geq \min_{\pi \in \Pi} \max \left\{ \min_{1 \leq j \leq n} \left( \sum_{j=1}^{n} p_j + \sum_{j=1}^{n} e_{j,c(j)} \right) \right\}
\]
\[
\geq \min_{\pi \in \Pi} \max \left\{ r_{j_k} + e_{j_k} + \sum_{t=j_k}^{n} p_t \right\}
\]
Thus we can get the lower bound on \( M^* \) by scheduling the jobs in the order of nondecreasing \( r_{j_k} = r_{j_k} + e_{j_k} \), in the classical problem \( 1|r_{j_k}|C_{\text{max}} \).

Theorem 3. The worst-case ratio of Algorithm \( G \) is 2.

**Proof.** Denote \( C_{\text{max}}(r, \pi) = \max_{1 \leq j \leq n} \left( r_{j,\sigma(j)} + \sum_{t=j}^{n} p_t \right) \). Let \( \bar{\pi}(r) \) be a schedule minimizing \( C_{\text{max}}(r, \pi) \) over \( \pi \in \Pi \). For any \( \pi \in \Pi \), \( C_{\text{max}}(r, \bar{\pi}) \) \leq \max_{1 \leq j \leq n} r_{j,\sigma(j)} + \sum_{j=1}^{n} p_j \), and \( C_{\text{max}}(r, \bar{\pi}(r)) \geq \max_{1 \leq j \leq n} r_{j,\sigma(j)} \), and \( C_{\text{max}}(r, \bar{\pi}(r)) \geq \sum_{j=1}^{n} p_j \). We have the following observation, i.e., for any \( \pi \in \Pi \),
\[
C_{\text{max}}(r, \pi) \leq 2C_{\text{max}}(r, \bar{\pi}(r)) \quad (2)
\]
Since $M(r^*, \pi^*) = M(r^*, \tilde{\pi}(r^*))$, we thus get from (2) that

$$M^G \leq M(r^*, \pi^G) = C_{\text{max}}(r^*, \pi^G) + \sum_{j=1}^{n} e^*_j, \sigma(j)$$

$$\leq 2C_{\text{max}}(r^*, \tilde{\pi}(r^*)) + \sum_{j=1}^{n} e^*_j, \sigma(j)$$

$$\leq 2M(r^*, \pi^*) = 2M^r \quad \Box$$

5 Conclusion and remarks

In this paper, we have discussed the scheduling problem with discretely compressible release times. This model is of interest both in the real world and in the sense of theory and it has attracted relatively little attention compared with traditional scheduling problems. We address the P1 model for scheduling with discretely compressible release times to minimize makespan. We show that it is strongly NP-hard, and present an approximation algorithm with worst-case ratio 2.

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