

# Nonlinear Dynamic Analysis of an *Enteromorpha proliferata* Population Model

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**Abstract** Considering biochemistry mechanism of biology growth and the complementarity of the nutrition which is caused by the animalcule decomposition of the propagation reliquiae, as well as its losing caused by physical therapies, we set up a nonlinear dynamical Nutrition- *E. proliferata*(N-E) model. The model consists of two coupled ordinary differential equations, which explicitly simulates the concentrations of nutrient and *E. proliferata* in the oceanic mixed layer. Then, using analytical techniques, we focus on the existence and nature of steady states. Specially we analyze the dynamical stability of the steady state of the model. With the results, several measures are given for controlling the *E. proliferata* disaster.

**Keywords** *Enteromorpha proliferata*; Nonlinear dynamics; Stability; *Enteromorpha proliferata* disaster

## 1 Introduction

The sunlit surface waters of the world's oceans are populated by tiny plankton. Plankton is a general term used to describe freely-floating and weakly-swimming marine and freshwater organisms. Plankton may be broadly divided into two groups. Phytoplankton are the plants and are mostly microscopic in size and unicellular; they are consumed by zooplankton, the animals, which in turn are eaten by larger organisms. In addition to their role at the base of the food chain, phytoplankton influence the global carbon cycle, with consequences for climate change that are at present undetermined[1]. Direct measurement of plankton biomass is difficult and expensive, and so the modeling of plankton populations is an essential tool to improve our understanding of the physical and biological processes which affect the population dynamics.

Phytoplankton are the source of almost all energy passing through aquatic food webs and comprise some 40 percents of the total fixed global primary productivity[2]. A large component of this productivity can be attributed to the occurrence of both seasonal and sporadic algae blooms that form as patches over the ocean's surface in areas of localized nutrient enrichment[3, 4, 5, 6]. Once triggered, bloom events lead to rapid rates of increase in phytoplankton growth. Biomass can sometimes increase by several orders of magnitude, only to decrease or crash as suddenly as the bloom mysteriously appeared. As phytoplankton sink out of the water column, they transport large quantities of carbon in a manner that intimately connects these primary producers with the earth's global carbon

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cycle. Hence phytoplankton have the capability of directly affecting large-scale global processes such as ocean-atmosphere dynamics and climate change. In freshwater lakes, rivers, and reservoirs, phytoplankton communities can have a major impact on ecosystem dynamics. Here, the appearance of algae blooms are often a signal of dangerous eutrophication and may result in major water-quality problems.

*E. proliferata*, distributing worldwide in the inter-tidal zone of sea, is one of the most common fouling green algae which is one kind of phytoplankton. It is one of the dominant seaweeds in the littoral zone of China. It distributes in a wide variety of coastal water, such as brackish-waters of inner bays and estuaries and so on and greatly affects the carbon cycle and recovery of the contaminant water body to border on the sea and often contributes to the formation of the so-called "green tide", which causes ecological and indirect economic damages [7]. For example, in early July 2008, *E. proliferata* bloom threatened the upcoming Olympic sailing events in Qingdao, China.

Until recently, there has been little research describing the generic mathematical mechanisms that underlie the dynamics of *E. proliferata* succession and blooms. So it is necessary to study the population dynamics from mathematical review and provide some measures for controlling the *E. proliferata* disaster. In this paper, we give a Nutrition-*E. proliferata*(N-E) model firstly. Then we analyze the existence and stability of the steady state. At last, with the results, several measures are given for controlling the *E. proliferata* disaster.

## 2 Nonlinear dynamical model of *E. proliferata*

Here, we present a model that illustrates the important dynamic properties of *E. proliferata* population. Although the model is of a very simple structure, it is an extremely useful one, and its dynamics are inherently generic to more complex models. The model consists of only two kinds of variables: nutrients levels,  $N_i$ , and *E. proliferata* biomass,  $E$ . It is assumed that small levels of nutrients enter the system at a slow but constant rate. *E. proliferata* biomass,  $E$ , rely on nutrient "uptake" for growth and are removed from the water column through mortality and sinking. In this model, we consider the regeneration of nutrient due to bacterial decomposition of the dead biomass. The importance of nutrient recycling has been well documented[8] and extensively investigated for closed ecological systems. Nutrient recycling in many of these studies is usually assumed to be instantaneous. In other words, the time that is required to regenerate nutrient from dead plankton via bacterial decomposition is neglected in the model formulation. This gives the following system:

$$\begin{cases} \frac{dE}{dt} = \text{uptake} - (\text{death} + \text{sinking}), \\ \frac{dN_i}{dt} = \text{input} - \text{uptake} - \text{loss} + \text{recycling}. \end{cases} \quad (1)$$

The functional forms used are

$$\begin{cases} \frac{dE}{dt} = \sum_{i=1}^n f_i(N_i, E) - \gamma_E E, \\ \frac{dN_i}{dt} = N_{f_i} - f_i(N_i, E) - \gamma_{N_i} N_i + \varepsilon_i E, \end{cases} \quad (2)$$

where  $E$  and  $N_i$  are the concentrations of *E. proliferata* and all kinds of nutrients such as phosphorus(P), nitrogen(N) and so on, respectively.  $\gamma_E$  is the sum of mortality and

sinking rates of *E. prolifera*, and  $\gamma_{N_i}$  are the loss rates of nutritions.  $f_i(N_i, P)$  is a function describing the nutrient uptake *E. prolifera*.  $N_{f_i}$  are constant nutrient input rates.  $\varepsilon_i$  is *E. prolifera* recycling rate, with  $0 < \varepsilon_i < \gamma_E$ .

It is not difficult to describe the process from mathematical view. But, if we blindly pursue the biodiversity and the ecological integrity, it is not propitious for us to understand the essential character of ecosystem dynamics. So, based on ecodynamics, all the nutritions are attributed to an overall which is denoted by one variable. For example, we can use  $N$  to denote the total concentration of all the nutritions. The model of *E. prolifera* can be rewritten as follows.

$$\begin{cases} \frac{dE}{dt} = f(N, E) - \gamma_E E, \\ \frac{dN}{dt} = N_f - f(N, E) - \gamma_N N + \varepsilon_E E, \end{cases} \quad (3)$$

where  $f(N, E)$  is expressed by Lotka-Volterra Equation[9]. Then the model can be written as follows:

$$\begin{cases} \frac{dE}{dt} = \alpha_m EN - \gamma_E E, \\ \frac{dN}{dt} = N_f - \alpha_m EN - \gamma_N N + \varepsilon_E E. \end{cases} \quad (4)$$

There are five parameters in this model. Nutrient inputs flow into the system at a constant rate,  $N_f$ , and nutrient uptake rate of *E. prolifera* is determined by parameter  $\alpha_m$ . In equations (4), the bilinear Lotka-Volterra interaction,  $NE$ , implies that the probability of a *E. prolifera* utilizing a nutrient is determined by the product of their relative abundances (or proportional probabilities). The parameters  $\gamma_E$  and  $\gamma_N$  are the percapita- mortality/loss rates of *E. prolifera* and nutritions.  $\varepsilon_E$  is the *E. prolifera* recycling rate. All the parameters are positive, with  $0 < \varepsilon_E < \gamma_E$ .

The interactions between these components are illustrated in Fig.1. The arrows indicate flows of matter through the system.

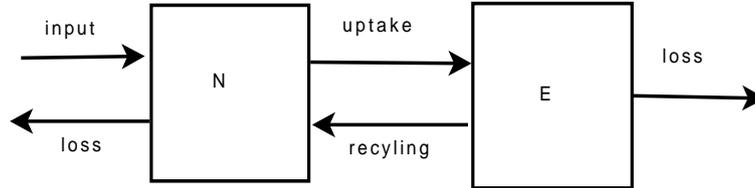


Figure 1: Interactions between nutrients(N) and *E. prolifera*(E). Arrows indicate flows of matter through the system. Arrows not starting or not finishing at a compartment indicate input to and losses from the system. "uptake" means *E. prolifera* uptaking nutrients, and "recycling" meaning regeneration of nutrient due to bacterial decomposition of the dead *E. prolifera*.

### 3 Nonlinear dynamics of *E. prolifera* model

In this section we will analyze the stability of the equilibrium points of Eq.(4). Let

$$\begin{cases} \frac{dE}{dt} = 0, \\ \frac{dN}{dt} = 0, \end{cases} \quad (5)$$

one can get the equilibrium points of Eq.(4):

$$E_1^* = 0, N_1^* = \frac{N_f}{\gamma_N}$$

and

$$E_2^* = \frac{\alpha_m N_f - \gamma_N \gamma_E}{(\gamma_E - \varepsilon_E) \alpha_m}, N_2^* = \frac{\gamma_E}{\alpha_m}$$

In order to study the stability of these two equilibrium points, we take coordinate conversion. Let  $e = E - E^*, n = N - N^*$  where  $(E^*, N^*)$  are the coordinates of equilibrium points of Eq.(4), then from Eq.(4), one can get

$$\begin{cases} \frac{de}{dt} = \alpha_m(e + E^*)(n + N^*) - \gamma_E(e + E^*), \\ \frac{dn}{dt} = N_f - \alpha_m(e + E^*)(n + N^*) - \gamma_N(n + N^*) + \varepsilon_E(e + E^*). \end{cases} \quad (6)$$

The Jacobin matrix of Eq.(6) at the equilibrium point  $(0, 0)$  is

$$J = \begin{bmatrix} \alpha_m N^* - \gamma_E & \alpha_m E^* \\ -\alpha_m N^* + \varepsilon_E & -\alpha_m E^* - \gamma_N \end{bmatrix}. \quad (7)$$

For the first equilibrium point  $(E_1^*, N_1^*) = (0, \frac{N_f}{\gamma_N})$ ,

$$J_1 = \begin{bmatrix} \frac{\alpha_m N_f}{\gamma_N} - \gamma_E & 0 \\ -\frac{\alpha_m N_f}{\gamma_N} + \varepsilon_E & -\gamma_N \end{bmatrix}, \quad (8)$$

The eigenvalues of  $J_1$  are  $\lambda_1 = -\gamma_N, \lambda_2 = \frac{\alpha_m N_f}{\gamma_N} - \gamma_E$ . Thus, according to the theory of stability, we get the following conclusion.

**Conclusion 1.** When  $\gamma_E > \frac{\alpha_m N_f}{\gamma_N}$ ,  $\lambda_1 < 0$  and  $\lambda_2 < 0$ . That is to say, the equilibrium point  $(E_1^*, N_1^*) = (0, \frac{N_f}{\gamma_N})$  of model Eq.(4) is asymptotically stable, if  $\gamma_E > \frac{\alpha_m N_f}{\gamma_N}$ .

For the second equilibrium point  $(E_2^*, N_2^*) = (\frac{\alpha_m N_f - \gamma_N \gamma_E}{(\gamma_E - \varepsilon_E) \alpha_m}, \frac{\gamma_E}{\alpha_m})$ ,

$$J_2 = \begin{bmatrix} 0 & \frac{\alpha_m N_f - \gamma_N \gamma_E}{\gamma_E - \varepsilon_E} \\ \varepsilon_E - \gamma_E & -\frac{\alpha_m N_f - \gamma_N \varepsilon_E}{\gamma_E - \varepsilon_E} \end{bmatrix}, \quad (9)$$

The eigenvalues of  $J_2$  are  $\lambda_{1,2} = -\frac{\alpha_m N_f - \gamma_N \varepsilon_E}{\gamma_E - \varepsilon_E} \pm \sqrt{(\frac{\alpha_m N_f - \gamma_N \varepsilon_E}{\gamma_E - \varepsilon_E})^2 - 4(\alpha_m N_f - \gamma_N \gamma_E)}$ . According to the theory of stability, we get the second conclusion as follows.

**Conclusion 2.** The second equilibrium point  $(E_2^*, N_2^*) = (\frac{\alpha_m N_f - \gamma_N \gamma_E}{(\gamma_E - \varepsilon_E) \alpha_m}, \frac{\gamma_E}{\alpha_m})$  of model Eq.(4) is asymptotically stable, if  $\gamma_E < \frac{\alpha_m N_f}{\gamma_N}$ ; Otherwise, the equilibrium point  $(E_2^*, N_2^*)$  is not stable.

**Proof.** If  $\alpha_m N_f - \gamma_N \varepsilon_E = 0$ , considering  $\gamma_E > \varepsilon_E$ , then  $\alpha_m N_f - \gamma_N \gamma_E < 0$ , thus  $\lambda_{1,2} = \pm \sqrt{-(\alpha_m N_f - \gamma_N \gamma_E)}$ . One of the eigenvalues has positive real part;

If  $\alpha_m N_f - \gamma_N \varepsilon_E < 0$ , from the expression of  $\lambda_{1,2}$ , we can see that one of the eigenvalues is positive real number;

If  $\alpha_m N_f - \gamma_N \varepsilon_E > 0$  and  $\alpha_m N_f - \gamma_N \gamma_E < 0$ , then one of the eigenvalues is positive real number;

If  $\alpha_m N_f - \gamma_N \varepsilon_E > 0$  and  $\alpha_m N_f - \gamma_N \gamma_E > 0$ , the two eigenvalues both have negative real parts. Considering  $\gamma_E > \varepsilon_E$ , if  $\alpha_m N_f - \gamma_N \gamma_E > 0$ , then  $\alpha_m N_f - \gamma_N \varepsilon_E > 0$  holds. Simply, if  $\alpha_m N_f - \gamma_N \gamma_E > 0$ , the two eigenvalues both have negative real parts.

In conclusion, if and only if  $\alpha_m N_f - \gamma_N \gamma_E > 0$ , i.e.  $\gamma_E < \frac{\alpha_m N_f}{\gamma_N}$ , both of the eigenvalues of  $J_2$  have negative real parts, that is to say the equilibrium point  $(E_2^*, N_2^*)$  of model Eq.(4) is asymptotically stable; Otherwise, the equilibrium point  $(E_2^*, N_2^*)$  is not stable.

## 4 Conclusion and discussion

In this paper, we have shown a nonlinear dynamical model of *E. prolifera* population and analyzed the stability of the equilibrium points of this model. The conditions for each equilibrium point to be stable are given. From the conclusion we get above, we know that there are two approaches to control the *E. prolifera* disaster.

On one hand, we can adjust the parameters to satisfy  $\gamma_E > \frac{\alpha_m N_f}{\gamma_N}$  which guarantees the first equilibrium point  $(E_1^*, N_1^*) = (0, \frac{N_f}{\gamma_N})$  to be stable. That is to say, *E. prolifera* biomass,  $E$ , will asymptotically approaches zero after a long time. Thus the *E. prolifera* disaster disappears. In order to satisfy  $\gamma_E > \frac{\alpha_m N_f}{\gamma_N}$ , we can increase the mortality/loss rates  $\gamma_E$  and  $\gamma_N$  or decrease the input rate  $N_f$  of nutrient and the uptake rate  $\alpha_m$ . The most doable measure is to decrease the input rate  $N_f$  of nutrient. That is to say reduce nutrient inputs flow into the system.

On the other hand, we can adjust the parameters to make the first coordinate of the second equilibrium point  $E_2^* = \frac{\alpha_m N_f - \gamma_N \gamma_E}{(\gamma_E - \varepsilon_E) \alpha_m}$  be less than the threshold of *E. prolifera* biomass which cause the *E. prolifera* disaster takes place. Thus, after a long time *E. prolifera* biomass,  $E$ , asymptotically approaches a value which can not result in a *E. prolifera* disaster. The concrete measure is similar as above.

The model given in this paper is a simple one which does not consider the zooplankton graze *E. prolifera*. In our later paper, we will consider the impact of zooplankton.

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