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DEA with Weight Constraints for Evaluation of Work Safety Supervision

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Abstract In this paper, a data envelopment analysis (DEA) model with weight constraints is proposed to evaluate the work safety supervision in 18 districts. Conventional evaluation approaches of the work safety supervision only consider the simple data such as the number of accidents and the number of death in the accidents. There are two major shortcomings: the weights of criteria are difficult to determined, and the different inherent risky levels of districts are not considered. Therefore we introduced the DEA method to address these issues. The results show that the evaluation by using DEA model is more objective and reasonable than the conventional approaches.

Keywords Data Envelopment Analysis; Work safety; Relative efficiency; Weight constraints

1 Introduction

Work safety is all the safety issues related to work. For example, when the gas leak accident occurs in a coal field, the workers may be injured or killed, and the machines may be broken. Work safety supervision is one of the important tasks of the governments to enforce the enterprises and peoples to comply with the work safety related law and regulations, in order to reduce the loss of life and wealth in work related accidents.

The goal of evaluating work safety supervision is to compare the effects and efficiency of the work safety supervision departments in the local district governments, therefore help the departments to improve their work. Conventionally, the evaluation of work safety supervision is straightforward and only considers the output data such as the number of accidents and the number of death in the accidents. Since there are many criteria, the analytical hierarchy process (AHP) is applied to determine the weights of criteria. There are two major shortcomings in the conventional approach. Firstly, it is difficult to determine the weights of criteria accepted by everyone. Each district has its own preference based on its situation. Secondly, the inherent risky level is very different from district to district. For example, the district with more industrial activities has larger probability of

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work accidents than the district with more agricultural activities. And the average loss in one accident in the district with more industrial activities may be much higher than that in the district with more agricultural activities. Therefore, it is unfair to directly compare two districts of different types.

In order to evaluate the work safety supervision more objectively and reasonably, a data envelopment analysis (DEA) model with weight constraints is proposed in this paper. The paper is organized as follows. The DEA model is formulated in the next section. The results are shown in Section 3 and the conclusion is provided in the last section.

2 Method

DEA, occasionally called frontier analysis, is a nonparametric method in operations research and economics to measure the relative efficiencies of decision making units (DMUs). DEA was first put forward by Charnes, Cooper and Rhodes in 1978 [1, 2]. Their model is called CCR model which assumes constant returns to scale (CRS). In 1984, Banker, Charnes and Cooper proposed a new DEA called BCC model [3], which assumes variable returns to scale (VRS). From then on, many variants of DEA have been proposed and widely applied in many fields [4, 5]. For the evaluation of work safety supervision, it is unreasonable to assume the returns to scale is constant. Therefore, the BCC model is adapted in this study.

DEA is a multi-input multi-output performance measurement technique. In DEA, there are a number of DMUs for evaluation. The production process for each DMU is to take a set of inputs and produce a set of outputs. Each DMU has a varying level of inputs and gives a varying level of outputs. DEA attempts to determine which of the DMUs are most efficient, and to point out special inefficiencies of the other DMUs.

2.1 Data processing

As suggested by experts in the fields of work safety, six indexes are selected as the criteria for evaluation. These indexes belongs to two groups: the inputs and outputs. The inputs are the indexes related to the inherent properties of the district: inherent risk, supervision manpower, and economic situation. The outputs are the indexes related to the performance of the work safety supervision departments: accident, supervision productivity, and development potential. The six indexes are further divided into twenty three sub-indexes such as the number of production units, the number of inspectors, the GDP per capita, the number of accidents, the number of death in accidents, the inspection rate, and so on. The values of six indexes are the weighted sum of corresponding sub-indexes. The weights are determined by using AHP.

Before calculating the weighted sum, the sub-indexes need to be pre-precessed since the range of each sub-index is varying. Especially, some sub-indexes are reversed. Generally in DEA, the larger output and smaller input imply better efficiency. But some sub-indexes in this study are different. For example, the number of accidents in output (smaller value means better performance) and the number of production units in input (larger value means higher inherent risky level).

The following four pre-processing functions are used in this study:

- $f_1 : y_i = \frac{x_i}{\max_j x_j}$. This is linear normalization function for normal indexes such as supervision manpower.

- $f_2 : y_i = 1 - \frac{x_i}{\max_j x_j}$. This is linear normalization function for reversed indexes such as inherent risk and economic situation.
- $f_3 : y_i = \frac{\frac{x_i}{x_i + median_j x_j}}{\max_k \frac{x_k}{x_k + median_j x_j}}$. This is non-linear normalization function for normal indexes such as supervision productivity and development potential.
- $f_4 : y_i = \frac{1}{x_i + 1}$. This is non-linear normalization function for reversed indexes such as accident.

After pre-processing of the sub-indexes, the values of six indexes are computed by using the weights provided by AHP.

2.2 BCC model with weight constraints

The original BCC model does not weight constraints. That is, the weights of inputs and outputs can be arbitrarily chosen to maximize the benefit of current DMU. The result is that some important indexes may be ignored or underestimated when evaluating certain DMUs. In order to make the weights in DEA are consistent with the importance given by the decision makers, several weight constraints are imposed to the BCC model.

The DEA model for evaluating the work safety supervision is as follows:

$$
\begin{aligned}
\min \quad & z = \sum_{i=1}^{N_x} u_i x(i, j_0) + \mu \\
\text{s.t.} \quad & \sum_{i=1}^{N_y} w_i y(i, j_0) = 100, \\
& \sum_{i=1}^{N_x} u_i x(i, j_0) + \mu \geq \sum_{i=1}^{N_y} w_i y(i, j_0), \quad && j = 1, 2, \cdots, N \\
& w_j \geq M_1 \sum_{i=1}^{N_y} w_i, && j = 1, 2, \cdots, N_y \\
& u_i, w_j \geq \varepsilon, && i = 1, 2, \cdots, N_x,\ j = 1, 2, \cdots, N_y
\end{aligned}
\tag{1}
$$

where

- N is the number of DMUs, N_x is the number of inputs, N_y is the number of outputs.
- $x(i, j)$ is the ith input of jth DMU, $y(i, j)$ is the ith output of jth DMU.
- M_i is the lower bound of relative weight for ith output, which takes value from 0 to 1. The value of M_i is proportion to the importance grade of the ith output. If all M_i are zero, the model degenerates to the BCC model.
- ε is a small positive number, which takes value 0.001 in this study.

Suppose that z^* is the minimal objective value, the relative efficiency score of the j_0th DMU is defined as $E(j_0) = \frac{100}{z^*}$. If $E(j_0) = 1$, the j_0th DMU is technical efficient, i.e. it has done best in existing conditions. If $E(j_0) < 1$, the j_0th DMU is technical inefficient, i.e. it should do better in current conditions than it does now.

2.3 Target setting

After solving the optimization problem for all the DMUs, the decision makers can know which DMUs are relative efficient and which inefficient. Furthermore, the decision makers want to know how to improve the efficiency of an inefficient DMU. This can be done by solving the dual problem of model (1) as follows.

$$\begin{array}{lll}
\max & 100\theta + \varepsilon\left(\sum_{i=1}^{N_x} r_i + \sum_{i=1}^{N_y} s_i\right) & \\
\text{s.t.} & \sum_{j=1}^{N} x(i,j)\lambda_j + r_i = x(i,j_0), & i = 1,2,\cdots,N_x \\
& \theta y(i,j_0) - \sum_{j=1}^{N} y(i,j)\lambda_j + s_i + t_i - \sum_{j=1}^{N_y} M_j t_j = 0, & i = 1,2,\cdots,N_y \\
& \sum_{i=1}^{N} \lambda_i = 1, & \\
& \lambda_i \geq 0, & i = 1,2,\cdots,N \\
& r_i \geq 0, & i = 1,2,\cdots,N_x \\
& s_i, t_i \geq 0, & i = 1,2,\cdots,N_y
\end{array} \tag{2}$$

According to the theory of DEA, the optimal solution θ of dual problem represents the maximal growth rate of j_0th DMU from current level to the best level it could be. That it, if the j_0th DMU can operate efficiently in current conditions, it will reach a better level depicted by the solution. The better level is called the target of the DMU.

Suppose that the optimal solution of the dual problem (2) are: $(\theta^*, \lambda_i^*, r_i^*, s_i^*, t_i^*)$. The target of j_0th DMU is computed as follows:

$$\begin{aligned}
x'(i,j_0) &= \sum_{j=1}^{N} x(i,j)\lambda_j^* \\
y'(i,j_0) &= \sum_{j=1}^{N} y(i,j)\lambda_j^*
\end{aligned} \tag{3}$$

That is, if j_0 DMU want to become technical efficient, it should reduce its inputs to x' while improve outputs to y'. By comparing to the original values, the input redundancy rates and the output insufficiency rates can be computed to investigate which parts are most important for improvement of efficiency. Note that since the lower bounds are imposed to the weights of outputs, the target outputs may be smaller than the original values of j_0th DMU, i.e. the output insufficiency may be negative.

3 Results

The proposed DEA model with weight constraints is applied to evaluate the work safety supervision in 18 districts. First, the data of each district in 12 months of 2009 are collected. After pre-processing, the values of three inputs and three outputs are calculated for each DMU (the pair of district and month). Then two kinds of average DMUs are added: one is the average of 18 districts in each month, denoted as AveD, the other is the average of each district in 12 months, denoted as AveM. The former is used to measure the performance of whole city in each month, while the latter is used to measure the annual performance of each district. So there are totally $(18+1)\times(12+1) = 247$ DMUs.

First the relative efficiency scores of 247 DMUs are calculated by model (1). The results are shown in Table 1.

Table 1: Relative efficiency scores of 18 districts in 12 months and the average units.

District	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	AveM
D1	0.98	0.99	1.00	0.57	0.94	0.95	0.96	0.97	0.97	1.00	1.00	0.97	0.93
D2	0.97	0.98	0.96	0.98	1.00	0.74	0.97	0.97	1.00	0.92	0.93	0.93	0.93
D3	0.99	0.98	1.00	0.98	0.98	0.66	0.90	0.91	0.68	0.86	0.86	0.90	0.89
D4	0.98	0.97	1.00	0.98	0.98	0.98	0.61	0.88	0.66	0.96	0.62	0.87	0.86
D5	0.94	0.81	0.81	0.87	0.95	0.96	0.82	0.95	0.94	0.81	0.74	1.00	0.87
D6	1.00	1.00	0.74	0.72	0.73	0.68	0.65	0.56	0.68	0.61	0.55	0.64	0.70
D7	0.69	0.96	0.69	0.99	0.74	0.66	0.92	0.66	0.93	0.64	0.67	0.68	0.75
D8	0.73	0.98	1.00	0.76	0.76	0.68	0.74	0.74	0.97	0.95	0.94	0.98	0.85
D9	0.99	0.99	1.00	0.68	0.74	0.75	0.93	0.71	0.93	0.63	0.92	0.63	0.82
D10	0.98	1.00	1.00	0.76	0.94	0.75	0.61	0.86	0.87	0.86	0.87	0.86	0.86
D11	0.95	0.95	0.69	0.73	0.87	0.90	0.72	0.94	0.96	0.83	0.86	0.84	0.84
D12	0.94	0.51	0.81	0.90	0.57	0.87	0.64	0.85	0.88	0.80	0.81	0.81	0.76
D13	0.95	0.95	0.95	0.65	0.65	0.84	0.68	0.95	0.67	0.66	0.62	0.93	0.78
D14	0.96	0.95	0.96	0.68	0.88	0.67	0.87	0.64	0.85	0.83	0.63	0.87	0.80
D15	0.98	0.98	0.65	0.90	0.70	0.93	0.64	0.93	0.94	0.83	0.56	0.78	0.81
D16	0.97	0.96	1.00	0.99	0.99	0.99	0.98	0.98	0.99	0.96	0.64	0.85	0.93
D17	0.96	0.97	0.97	0.56	0.86	0.80	0.59	0.84	0.85	0.84	0.84	0.83	0.81
D18	0.93	0.93	0.92	0.92	0.92	0.57	0.82	0.84	0.84	0.63	0.82	0.84	0.82
AveD	0.88	0.89	0.84	0.74	0.78	0.74	0.72	0.78	0.81	0.74	0.71	0.78	0.78

Each element in the table is the relative efficiency score of one DMU. Each row represents the relative efficiency scores of a district in months, while each column lists the relative efficiency scores of districts in a month. The last row is the relative efficiency score of whole city (the average of all districts). The last column is the annual relative efficiency scores of each districts.

With the results we can analyze the work safety supervision of all districts during the year. First we compared the relative efficiency among districts in the same month (see Figure 1). We can easily find the districts whose work safety supervision is excellent in current conditions (e.g. D2 and D16), and point out some districts which did not operate efficiently and should improve the efficiency (e.g. D6).

Then we analyzed the trends of the each district in different months (see Figure 2). The work safety situation in January and February of 2009 is better than other months, and the efficiencies of many districts decrease after March. One of the possible reasons is that the first two months are close to Spring Festival, the most important holiday in China, so that governments and people pay more attention to safety. The economic and industrial activities are also less than other months. The supervision of work safely was a little loosen in April, July and November.

After obtaining the efficiencies of all DMUs, we further calculate the input redun-

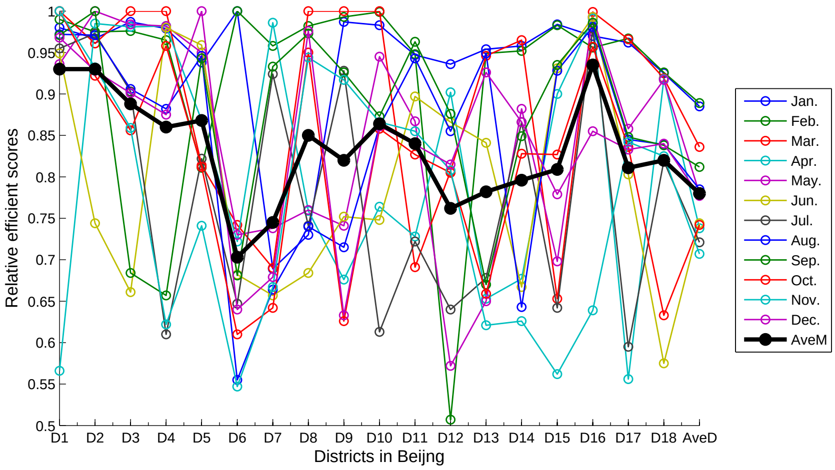

Figure 1: The relative efficiency of months. Each curve represents the relative efficiency of all districts in a month. The black curve is the annual relative efficiency of all the districts.

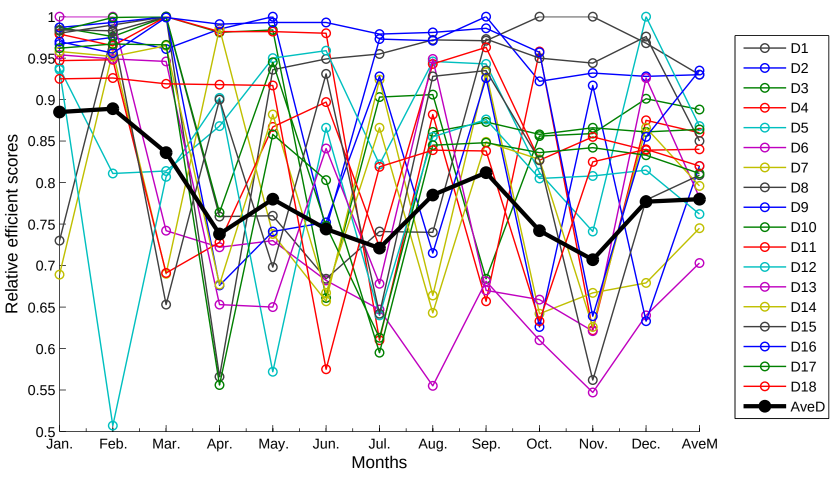

Figure 2: The relative efficiency of districts. Each curve represents the relative efficiency of a district. The black curve is the relative efficiency of the average of all districts.

dancy rates and the output insufficiency rates for each technical inefficient DMU. The results are not show here due to space limit. From the input redundancy rate and the output insufficiency rates, the decision makers can know why the DMU is inefficient and which aspects need to be improved. For example, the annual efficiency score of district D6 is 0.70, and the output insufficiency rates are 0.77, 0.09, 0.00 for accident, supervision productivity and development potential respectively. That is, district D6 should pay more attention to the accident control while its supervision productivity and development potential are rather good. And for another example, in November many districts have poor efficiencies, and the efficiency score of whole city in this month is 0.71. The output insufficiency rates are 0.46, 0.29, 0.58 for accident, supervision productivity and development potential respectively, from which one can see that all three outputs are relatively poor than other months. Therefore, we need an overall improve of performance.

4 Conclusion

In this paper, we develop a DEA model with weight constraints for evaluation of the work safety supervision in 18 districts. First, AHP method is applied to combine 23 sub-indexes to 6 indexes. Several weight constraints are used to ensure the efficient DMUs have balanced outputs. The lower bound of each weight represents the importance grade of each output given by the decision makers. The input redundancy rates and the output insufficiency rates are calculated to help decision makers to find the fields that need improve. The new approach is used to evaluate the 18 districts in 12 months of 2009. Compared with the conventional AHP based approach, this AHP+DEA approach can make more objective evaluation, which is confirmed by the experts.

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