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Nonlinear Dynamics in COMEX Gold Futures Basis: LSTAR-EGARCH

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Abstract Research on the basis nonlinearity has been widely concerned in recent papers. Better understandings of the basis nonlinearity would be significant for hedging strategies and arbitrage behaviors both practically and theoretically. This paper attempts to examine the nonlinear dynamics of COMEX gold futures basis using a modified LSTAR-EGARCH model with t-distributed error. Empirical results demonstrate the gold futures basis of COMEX have nonlinear properties, asymmetric changes, long memory of volatility, and asymmetric responses in volatility to positive and negative shocks.

Keywords Basis; STAR; EGARCH; Nonlinearity; COMEX gold futures

1 Introduction

Gold has historically served as both a legitimate hedge against inflation and as an integral part of a diversified investment portfolio. Investors generally buy gold as a hedge or safe haven against any economic, political, social or currency-based crises. These crises include investment market declines, burgeoning national debt, currency failure, inflation, war and social unrest. Such crises usually lead to volatile price changes imposing great risk on investors. Therefore, it is of great significance for investors to understand the evolution of the price change risk. Better understandings of risk can provide investors with more efficient hedge or even arbitrage strategies. In futures markets, a well accepted risk indicator is the futures basis. Basis measures the relations between the futures price and spot price, and provides an important foundation for perfect hedging. Therefore, research on the futures basis changes has become increasingly important both practically and theoretically.

Some existing literature has spawned to investigate futures basis change using linear methods and nonlinear models. Mackinlay, Ramaswamy(1988) and Yadav, Pope(1990) found that the basis change is significant of first-order autocorrelation. With transaction costs, the market produces the no-arbitrage bounds; the deviation from the equilibrium value of the basis may exhibit a random walk process. However, due to transaction costs and heterogeneous investors, the dynamic adjustment of the basis tends to be nonlinear. Some studies motivate the adoption of threshold-type

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models to empirically characterize the nonlinear behavior of the basis. Yadav, Pope, and Paudyal(1994) suggested the use of the self-exciting threshold autoregressive (SETAR) model for the price differential between FTSE100 futures and cash. Dwyer, Locke, and Yu (1996) used a nonlinear econometric model to examine the relationship between the S&P 500 futures and cash indexes. Brooks and Garrett (2001) employed a SETAR model to analyze the relationship between FTSE100 stock index and futures contract. Monoyios and Sarno (2002) proposed a smooth transition autoregressive (STAR) model in examining the S&P500 index futures market. They suggested that STAR model is more suitable than TAR model. In short, the basis researches mentioned above mainly focus on the exchange rate and stock index futures market, while relatively less about commodity futures. Therefore, in this paper we investigate the nonlinear dynamics of COMEX gold futures basis using the modified LSTAR-EGARCH model.

The remainder of the paper is organized as follows. Section 2 formulates STAR-EGARCH model. Section 3 discusses the data and preliminary analysis. Nonlinearity test and the model estimation results are provided in Section 4. Section 5 presents the conclusions of the paper.

2 Model Specification

Nonlinear time series models have become very popular in recent years. As regime switching models are particularly popular in the class of non-linear models, it is of interest to investigate regime switching models with GARCH errors, with an emphasis on Smooth Transition Autoregressive (STAR) models.

Tong (1978) and Tong and Lim (1980) proposed the Threshold Autoregressive (TAR) model. To allow for smooth transition behavior, Granger and Teräsvirta(1993) and Teräsvirta(1994) extended the TAR model and proposed the Smooth Transition Autoregressive (STAR) model:

$$y_t = \phi_{10} + \sum_{j=1}^{p} \phi_{1j} y_{t-j} + \left(\phi_{20} + \sum_{j=1}^{p} \phi_{2j} y_{t-j} \right) G\left(s_t;\gamma,c\right) + a_t \quad (1)$$

where $\{y_t\}$ is a stationary and ergodic process, $a_t \sim iid(o,\sigma^2)$, $G\left(s_t;\gamma,c\right)$ is the transition function, assumed to be at least twice differentiable, ranging from 0 to 1. s_t is the threshold variable, which is not only an exogenous variable $(s_t = z_t)$,but also an endogenous lagged variable $(s_t = y_{t-d})$. γ is the transition rate, which effectively determines the speed and smoothness of switching from one regime to another, and the parameter c can be interpreted as the threshold, as in TAR models, which may be seen as the equilibrium level of $\{y_t\}$.

Several transition functions are available, with the most popular being the first-order Logistic STAR (LSTAR) function in the empirical studies:

$$G\left(s_t;\gamma,c\right) = (1+\exp(-\gamma(s_t - c)))^{-1}, r > 0 \quad (2)$$

with the following properties: $\lim_{s_t \to -\infty} G\left(s_t;\gamma,c\right) \to 0$, $\lim_{s_t \to \infty} G\left(s_t;\gamma,c\right) \to 1$,

$G(s_t;0,c)=0.5$, $\lim_{r\to-\infty} G(s_t;\gamma,c)\to 0$, $\lim_{r\to\infty} G(s_t;\gamma,c)\to 1$. As $r\to\infty$, $G(s_t;\gamma,c)$ becomes a Heaviside function: $G(s_t;\gamma,c)=0$, $s_t\le c$; $G(s_t;\gamma,c)=1$, $s_t\ge c$, and reduces to a $TAR(p)$ model. When $r=0$, equation (1) becomes a linear $AR(p)$ model.

Although the logistic function is frequently used, another popular choice is the Exponential STAR (ESTAR) model given by:

$$G(s_t;\gamma,c)=1-\exp(-\gamma(s_t-c)^2), r>0 \tag{3}$$

The parameters in (3) change symmetrically about c with s_t. The exponential transition function is bounded between zero and unity, has the properties: $\lim_{s_t\to-\infty} G(s_t;\gamma,c)\to 1$, $\lim_{s_t\to\infty} G(s_t;\gamma,c)\to 1$, $\lim_{s_t\to c} G(s_t;\gamma,c)\to 0$. When $r=0$, equation (1) becomes a linear $AR(p)$ model.

Many empirical studies show that financial return series have some important features: time-varying, heavy tails, volatility clustering and asymmetry. However, the models based on the normal distribution usually can not fully take into account these features. EGARCH model with t-distributed error introduced by Nelson (1991) can fit better effect than GARCH model by Bollerslev(1986). An alternative form for the t-EGARCH(1,1) model specifies as:

$$\begin{aligned} a_t &= \sigma_t\varepsilon_t \\ \ln(\sigma_t^2) &= \alpha_0+\alpha_1\frac{(|a_{t-1}|+\theta a_{t-1})}{\sigma_{t-1}}+\beta\ln(\sigma_{t-1}^2) \end{aligned} \tag{4}$$

where $\varepsilon_t \sim IIDt(0,1,\upsilon)$. A positive a_{t-1} contributes $\alpha_1(1+\theta)|\varepsilon_{t-1}|$ to the log volatility, whereas a negative a_{t-1} gives $\alpha_1(1-\theta)|\varepsilon_{t-1}|$, where $\varepsilon_{t-1}={a_{t-1}}/{\sigma_{t-1}}$.The θ parameter signifies the leverage effect of a_{t-1}. Again, we expect θ to be negative in real applications.

Therefore, STAR-EGARCH(1,1) model is specified as:

$$\begin{aligned} b_t &= \phi_{10}+\sum_{j=1}^{p}\phi_{1j}b_{t-j}+\left(\phi_{20}+\sum_{j=1}^{p}\phi_{2j}b_{t-j}\right)G(\gamma;b_{t-d}-c)+a_t \\ G(\gamma;b_{t-d}-c) &= (1+\exp(-\gamma(b_{t-d}-c)))^{-1}, r>0 \\ G(\gamma;b_{t-d}-c) &= 1-\exp(-\gamma(b_{t-d}-c)^2), r>0 \\ a_t &= \sigma_t\varepsilon_t, \varepsilon_t \sim IIDt(0,1,\upsilon) \\ \ln(\sigma_t^2) &= \omega+\alpha\frac{(|a_{t-1}|+\theta a_{t-1})}{\sigma_{t-1}}+\beta\ln(\sigma_{t-1}^2) \end{aligned} \tag{5}$$

3 Data and preliminary analysis

To construct a continuous series of futures prices, the paper uses daily closing prices of futures contracts with 1 months to maturity for COMEX gold futures contracts. These futures and spot prices are obtained from Bloomberg. The sample period covers from 1/4/2000 to 4/30/2010.The number of observations is 2589. Given the spot and futures prices, basis is defined as the difference in the natural logarithms of spot and futures prices, as follows: $b_t = \log(s_t) - \log(f_t)$, where s_t denotes the spot price at time t, f_t denotes futures price at time t.

Table 1 presents some summary statistics for $\log(s_t)$, $\log(f_t)$, and b_t. During the whole sample period, the standard deviation of basis is smaller than that log-level of spot prices and futures prices, indicating that basis are much lower volatile relative to log-level of spot and futures prices. The level and difference of basis series tend to be negatively skewed and substantial excess kurtosis, implying significant higher peaks and fatter tails. Jarque–Bera test statistic indicates basis series to be significantly non-normal. ADF test results indicate a rejection the unit root null hypothesis applied to b_t in the level and difference, suggesting stationarity of the basis and possibly the existence of a cointegrating relationship between the spot prices and futures prices.

Table 1 Descriptive statistics of spot prices, futures prices and basis

statistics	$\log(s_t)$	$\log(f_t)$	b_t	Δb_t
Mean	6.187244	6.188271	-0.000817	1.62E-06
Std. Dev	0.465277	0.46592	0.004205	0.005303
Skewness	0.301062	0.301515	-0.174254	-0.341884
Kurtosis	1.723638	1.722521	28.06532	48.23476
Jarque-Bera	214.7666	215.1918	67761.56	220697.20
ADF	0.226856	0.215405	-9.819364**	-27.15203**

Notes: Δ is the first-difference operator, "**" indicates significance at 1% level.

4 Empirical results

4.1 Test for STAR Nonlinearity

In empirical applications, determining the order of the linear AR model is usually the first step toward carrying out the linearity test. A common technique is to use an order selection criterion like AIC and SIC to select a proper subset of lags. Table 2 gives the values of AIC and BIC. The comparative results indicate that the AR(7) model seems better to describe the linear part of STAR model.

Next, we test for the existence of STAR-type nonlinearity in the residual from the chosen AR model. According to Granger and Teräsvirta (1993) and Teräsvirta (1994), we use a sequence of linearity tests to artificial regressions which can be interpreted as second or third-order Taylor series expansions of (1). The artificial regression is specified as:

$$y_t = \beta_{00} + \sum_{j=1}^{p} (\beta_{0j} b_{t-j} + \beta_{1j} b_{t-j} b_{t-d} + \beta_{2j} b_{t-j} b_{t-d}^2 + \beta_{3j} b_{t-j} b_{t-d}^3) + \varepsilon_t \quad (6)$$

Hence, testing the null hypothesis of a linear AR model against a nonlinear STAR model is equivalent to testing the null hypothesis $H_0 : \beta_{1j} = \beta_{2j} = \beta_{3j} = 0$, $\forall j \in \{1, 2, ..., p\}$ in the above auxiliary regressions. This linearity test assumes constant conditional variance, and is therefore not robust against conditional heteroskedasticity. Davidson and MacKinnon (1985), Granger and Teräsvirta (1993) revised the LM test for nonlinearity, which is robust toward heteroskedastic errors. The results are presented in Table 3. The null hypothesis of linearity is strongly rejected at a 5% significance level since the p-values from both the χ^2 test and F test are less than 0.05 for d =1. Following the Teräsvirta Rule, an STAR model with $p = 7$ and $d = 1$ is selected for the futures basis.

Table 2 choosing the order of the autoregression

P	1	2	3	4	5	6	7	8	9	10
AIC	-8.15	-8.20	-8.22	-8.26	-8.26	-8.27	-8.28	-8.28	-8.28	-8.28
SIC	-8.14	-8.19	-8.22	-8.25	-8.25	-8.26	-8.26	-8.26	-8.26	-8.26
LL	10544	10605	10639	10679	10680	10689	10692	10692	10690	10688

Table 3 nonlinearity test results

d	1	2	3	4	5	6
F	1.7300	1.2352	1.5154	1.1190	1.0548	0.8562
P	0.0207**	0.2101	0.0621*	0.3190	0.3917	0.6500
χ^2	36.2266	25.9695	31.7884	23.5498	22.2094	18.0575
P	0.0206**	0.2076	0.0615 *	0.3154	0.3875	0.6454

Notes: “**” and “*” indicates significance at 5% and 10% level, respectively.

4.2 Choosing between LSTAR and ESTAR

Following Teräsvirta (1994), the LM test based on (6) can also be used to discriminate between LSTAR and ESTAR, since third-order terms disappear in the Taylor series expansion of the ESTAR formulation. Thus, after fixing the delay parameter, LSTAR and ESTAR model are selected by testing a sequence of nested hypotheses. The sequence is defined as follows:

$$H_{03} : \beta_{3j} = 0, j = 1, 2, 3, ..., p$$
$$H_{02} : \beta_{2j} = 0 \big| \beta_{3j} = 0, j = 1, 2, 3, ..., p \quad (7)$$
$$H_{01} : \beta_{1j} = 0 \big| \beta_{2j} = \beta_{3j} = 0, j = 1, 2, 3, ..., p$$

According to decision rules proposed by Teräsvirta (1994), if the p-values of H_{02} test is the smallest of the three, select an ESTAR model ; if not, choose a

LSTAR model. As reported in Table 4, H_{03} has least p -values, therefore fit LSTAR model for basis series.

Table 4 Sequence of F tests on basis series

hypothesis	F-value	p-value	Desicion
H_{03}	8.36917	1.74E-24***	Reject
H_{02}	5.82165	1.37E-10***	Reject
H_{01}	6.31039	7.05E-07***	Reject

Notes: "***" indicates significance at 1% level.

4.3 Estimation of the STAR-EGARCH Model

After specifying the family of models and determining the AR order and delay parameter, LSTAR-EGARCH model with $p=7$ and $d=1$ is estimated by two-stage method. The estimated results are reported in Table 5.

From Table 5, we can see that the transition parameter γ is significantly different from zero at the 1% significance level, indicating nonlinear property for the gold basis of COMEX. Based on the decision rules of Teräsvirta (1994), an LSTAR model is more appropriate to describe the COMEX gold basis changes. The transition function $G(\gamma;b_{t-1}-c)$ is shown in Figure 1, which is a monotonic increasing function of the transition variable b_{t-1}, showing the smooth transition between regimes and asymmetric adjustment of COMEX gold futures basis for deviation from the equilibrium level. This is mainly because of existing market frictions in the commodities futures markets, such as transaction costs, heterogeneous investors. The parameter β is significant at the 1% level, indicating the long memory of volatility in the gold basis changes. As expected, the output shows that the estimated leverage effect θ is negative and statistically significant at the 10% level with t-ratio (-1.332), highlighting the asymmetric responses in volatility to positive and negative shocks. Therefore, a positive shock contributes $0.223|\varepsilon_{t-1}|$ to the log volatility, whereas a negative shock gives $0.273|\varepsilon_{t-1}|$, suggesting that the impact of a negative shock in volatility is larger than that of a positive shock of the same size. According to Table 3, the delay parameter d equals to 1, which implies the optimum delay order of COMEX gold futures basis is 1. When the basis deviates from the equilibrium value, there are some arbitrage opportunities in the futures markets. The average response time of investors to deviations of basis from the equilibrium is one day, which implies the arbitrage trading is active and effective.

Table 5 Estimation of LSTAR-EGARCH model

Parameters	Value	Std. Error	T value
ϕ_{10}	-0.005	0.002	-2.500***
ϕ_{11}	-0.168	0.081	-2.074**
ϕ_{12}	0.110	0.033	3.333***

ϕ_{13}	0.111	0.032	3.469***
ϕ_{14}	0.133	0.035	3.800***
ϕ_{15}	0.125	0.038	3.289***
ϕ_{16}	0.156	0.037	4.216***
ϕ_{17}	0.035	0.035	1.000
ϕ_{20}	0.004	0.001	4.000***
ϕ_{21}	-0.418	0.074	-5.649***
ϕ_{22}	0.051	0.044	1.159
ϕ_{23}	0.038	0.051	0.745
ϕ_{24}	0.099	0.046	2.152**
ϕ_{25}	-0.125	0.052	-2.404***
ϕ_{26}	-0.028	0.051	-0.549
ϕ_{27}	0.063	0.047	1.340*
γ	1.129	0.242	4.665***
c	-0.001	0.001	-1.000
ω	-0.541	0.0873	-6.203***
α	0.248	0.0335	7.418***
β	0.966	0.0066	146.169***
θ	-0.099	0.074	-1.332*

Notes: "***","**"and "*" indicate significance at 1% ,5% and 10% level, respectively.

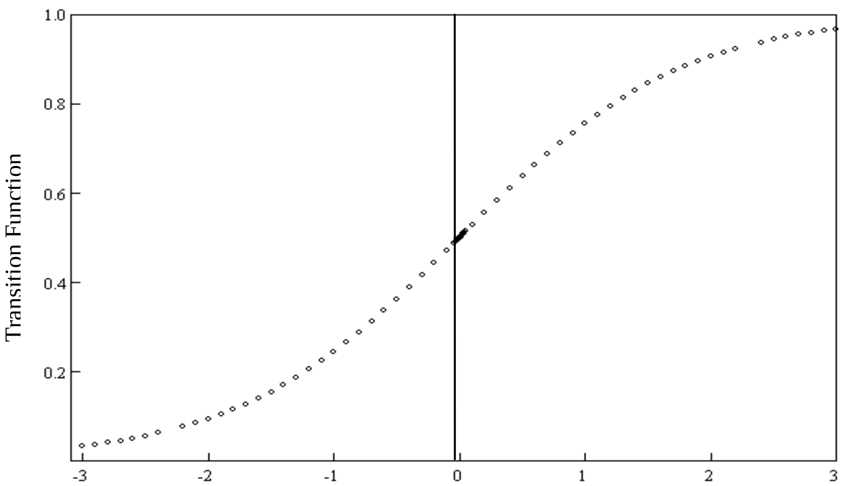

Figure 1. Estimated transition function

5 Conclusion

This paper discusses the nonlinear behavior of COMEX gold futures basis using LSTAR-EGARCH model. First, the AR order and delay parameter are determined by nonlinearity test. $p = 7$ and $d = 1$ are selected for STAR model. Then, LSTAR and ESTAR model are discriminated by the LM test. LSTAR model is found to be appropriate to gold basis. Finally, the estimation of LSTAR-EGARCH model is given in this paper. Empirical results reveal that gold futures basis of COMEX have nonlinear properties, asymmetric changes, long memory of volatility, and asymmetric responses in volatility to positive and negative shocks.

In conclusion, futures basis plays a very important role in the hedging decisions and arbitrage behaviors. Therefore, studying the nonlinear dynamic properties of commodities futures basis provides traders more information about the market structure.

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