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Performance Analysis of Sensor Nodes in a WSN With Sleep/Wakeup Protocol

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Abstract A Wireless Sensor Network (WSN) is an energy-constrained system. To conserve the energy of sensor nodes, a sleep/wakeup protocol is introduced in IEEE 802.15.4, where the sensor node will enter into the sleep mode whenever communication is not required, and will wake up as soon as a new data frame is ready. In this paper, a discrete-time multiple vacation queueing model with a setup is built to capture the working principle of the sleep/wakeup protocol by taking into account the switching procedure between the sleep mode and the active mode. The queueing model for the steady state is also derived. Correspondingly, we present the formulas for some performance measures in terms of the average latency of data frames and the average energy consumption. Finally, numerical results are presented to discuss the relationship between the system performance and the constellation size.

Keywords Wireless Sensor Network; Sleep/Wakeup; Performance Measures; Constellation Size; Queueing Model; Setup.

1 Introduction

With the development of microsensor and microelectronic technology in parallel with wireless communication, research into WSNs has attracted significant attention [1]. For the characteristics of self-organization, microsize, low-cost and flexibility, WSN are being applied in many fields, such as military, environmental science, medical and health, space exploration, and commerce [2]. A WSN is, however, an energy-constrained system.

The sensor node in WSN is a micro-embedded device, which typically carries only limited battery power [3]. Large numbers of sensor nodes are densely deployed over a wide region for monitoring complex environments. In some regions, replacement or recharging of the battery is impossible. This necessitates the designing of a low-power mechanism to minimize energy consumption. On the other hand, data latency is also an important factor in user QoS that can not be neglected when designing a practical system.

To improve the energy efficiency and also to lower the data latency for the sensor nodes in a WSN, an efficient modulation strategy and an accurate analytical method must be provided. With this in mind, some mathematical models that faithfully reproduce the behavior of the WSN have been constructed. However, most of these system models are based on continuous-time models.

In [4], a data communication and aggregation framework is presented, in which the degree of data aggregation is manipulated to maintain specified acceptable latency bounds on data delivery while attempting to minimize energy consumption. An analytic model is constructed to describe the relationships between timeliness, energy consumption and the degree of aggregation. In [5], for satisfying a given throughput and delay requirement, considering the delay and peak-power constraints, a new modulation strategy is given to minimize the total energy consumption. In [6], the performance measures in terms of the average power consumption and the data delay are given based on a continuous-time queueing model. Unfortunately, the switching procedure from the sleep mode to the active mode is neglected to simplify the process of analysis.

Considering the digital nature and the setup procedure of the Phase-Locked Loop (PLL) from the sleep mode to the active mode, we build a discrete-time multiple vacation queueing model with a setup to describe the working principle of the sleep/wakeup protocol in this paper. We analyze the relationships between the constellation size and the performance measures. Moreover, numerical results are given to demonstrate the influence of the constellation size on the system performance. Finally, we also determine the optimal value of the constellation size for minimizing the energy consumption under some conditions.

2 The Sleep/Wakeup Protocol and System Model

In order to minimize the energy consumption of sensor nodes, a sleep/wakeup protocol is offered in IEEE 802.15.4, in which each sensor node manages its state independently. The communicated sensor nodes are required to keep synchronous with the sleep/wakeup scheme so as to reduce the data latency. The implementation of such a sleep/wakeup scheme typically requires two different channels, namely, a data channel for normal data communication and a wakeup channel for awaking nodes when needed. A sender (source node) will communicate with a neighboring node (receiver). The sensor node sends a series of signals on the wakeup channel, and once the receiver realizes that there is a pending signal, the receiver sends back a wakeup acknowledgement and turns on its data radio. The receiver wakes up periodically and listens for a short time to monitor whether there are data frames arriving. If a receiver does not detect any activity on the wakeup channel, the receiver will return to the sleep mode.

Based on the sleep/wakeup protocol, sensor nodes work in two modes: the active mode and the sleep mode. The active mode is also called the working state, in which the data frames are transmitted normally. The sleep mode includes two stages: the sleep stage and the listening stage. In the sleep stage, the sensor node will turn off the wireless communication module so data frames are not received or sent; if data frames are detected in the listening stage, the sensor node will enter into the active mode, i.e., the system will switch to the active mode. Otherwise the sensor node will return back to the sleep stage.

Consider the transmission from the source node to the destination node as a research object. Let the distance between the source node and the destination node be d. Pulse Amplitude Modulation (PAM) is used and the channel between the source node and the destination node is supposed to be a Gaussian channel. In addition, we suppose that the error-correcting codes are not used in the communication. A square wave pulse is used in the system. Let B be the bandwidth, and k $(k = 1, 2, \ldots)$ be the modulation constellation size. We assume that the length of a data frame follows a general distribution with an average of L bits. So the number of bits transmitted per second is kB and the transmission time of a data frame follows a general distribution with an average of $L/(kB)$.

The sleep stage of the sleep mode is regarded as a vacation denoted by V_S, the listening stage of the sleep mode is regarded as another vacation denoted by V_L. Obviously, the system vacation V is composed of V_S and V_L, namely, $V = V_S + V_L$. We assume that V_S and V_L have the fixed length of T_{V_S} and T_{V_L}, respectively, and V has the length of T_V. Then, we have that $T_V = T_{V_S} + T_{V_L}$. The time period for transmitting data frames continuously is considered as a busy period denoted by Θ. Moreover, the setup procedure of the Phase Lock Loop (PLL) in the frequency synthesizer when the system switches from the sleep mode to the active mode is regarded as a setup period denoted by U. Therefore, we build a multiple vacation queueing model with a setup.

3 Model Analysis

Considering the memoryless property of users initiated data frame arrivals, the queueing model presented in this paper is a discrete-time multiple vacation Geom/G/1 queueing model with a setup. Suppose the data frame arrival interval follows a geometric distribution with arrival ratio p $(0 < p < 1)$. The probability that no arrival occurs is $\bar{p} = 1 - p$. The transmission time of a data frame is assumed to be an independent and identically distributed random variable denoted by S. The data frames are transmitted on a First-In First-Out (FIFO) discipline.

We can give the probability distribution, the Probability Generating Function (PGF) $S(z)$ and the average $E[S]$ of S as follows:

$$P\{S = j\} = s_j, \quad S(z) = \sum_{j=1}^{\infty} z^j s_j, \quad j = 1, 2, \ldots, \quad E[S] = \frac{L}{kB}. \tag{1}$$

Let A_j^V be the number of data frames arrived in a system vacation period V, the PGF $A_V(z)$ of A_j^V is given as follows:

$$A_V(z) = \sum_{j=0}^{T_V} z^j \binom{T_V}{j} p^j \bar{p}^{T_V - j} = (pz + \bar{p})^{T_V}, \quad T_V > 0. \tag{2}$$

Let the setup period U (in slots) follow a general distribution. The probability distribution, the PGF $U(z)$ and the average $E[U]$ of U are given as follows:

$$P\{U = j\} = u_j, \quad U(z) = \sum_{j=1}^{\infty} u_j z^j, \quad E[U] = \sum_{j=1}^{\infty} j z^j, \quad j = 1, 2, \ldots. \tag{3}$$

Let A_j^U be the number of data frames arriving during a setup period U, the probability distribution and the PGF $A_U(z)$ of A_j^U are given as follows:

$$P\{A_j^U = j\} = \sum_{i=j}^{\infty} u_i \binom{i}{j} p^j \bar{p}^{i-j}, \quad j = 0, 1, 2, \ldots, \tag{4}$$

$$A_U(z) = \sum_{j=0}^{\infty} P\{A_j^U = j\} z^j = U(pz + \bar{p}). \tag{5}$$

Let Q_B be the number of data frames at the beginning instant of a busy period Θ. The probability distribution b_j and the PGF $Q_B(z)$ of Q_B are given as follows:

$$b_j = P\{Q_B = j\} = \begin{cases} \dfrac{1}{1-\bar{p}^{T_V}} \displaystyle\sum_{i=1}^{j} A_i^V A_{j-i}^U, & j = 1, 2, ..., T_V \\ \dfrac{1}{1-\bar{p}^{T_V}} \displaystyle\sum_{i=1}^{T_V} A_i^V A_{j-i}^U, & j \geq T_V + 1, \end{cases} \tag{6}$$

$$Q_B(z) = \sum_{j=1}^{\infty} b_j z^j = \frac{1}{1-\bar{p}^{T_V}} \left((pz+\bar{p})^{T_V} - \bar{p}^{T_V} \right) U(pz+\bar{p}). \tag{7}$$

Differentiating Eq. (7) with respect to z at $z = 1$, we can obtain the average $E[Q_B]$ of Q_B as follows:

$$E[Q_B] = \frac{T_V p}{1-\bar{p}^{T_V}} + pE[U]. \tag{8}$$

When the system load $\rho = pE[S] < 1$, the system will arrive at a state of equilibrium.

3.1 Queueing Length and Waiting Time

Let L^+ be the queueing length at the transmission completion instant. Obviously, we have $L^+ = L_0 + L_d$, where L_0 is the queueing length for the classical Geom/G/1 queueing model, and L_d is the additional queueing length introduced by the setup period and multiple vacations introduced in the system model presented in this paper. Referencing to [7], we can get the PGF $L_0(z)$ and the average $E[L_0]$ of L_0 as follows:

$$L_0(z) = \frac{(1-\rho)(1-z)S(pz+\bar{p})}{S(pz+\bar{p}) - z}, \tag{9}$$

$$E[L_0] = \rho + \frac{p^2 E[S(S-1)]}{2(1-\rho)}. \tag{10}$$

Using the boundary state variable theory in [7], the PGF $L_d(z)$ and the average $E[L_d]$ of L_d are given as follows:

$$L_d(z) = \frac{1-Q_B(z)}{E[Q_B](1-z)} = \frac{1-\bar{p}^{T_V} - \left((pz+\bar{p})^{T_V} - \bar{p}^{T_V} \right) U(pz+\bar{p})}{(T_V p + (1-\bar{p}^{T_V}) pE[U])(1-z)}, \tag{11}$$

$$E[L_d] = \frac{T_V(T_V-1)p + 2T_V pE[U] + p\left(1-\bar{p}^{T_V}\right) E[U(U-1)]}{2\left(T_V + (1-\bar{p}^{T_V})E[U]\right)}. \tag{12}$$

Combining Eqs. (10) and (12), we can get the the average $E[L^+]$ of L^+ by

$$\begin{aligned} E[L^+] &= E[L_0] + E[L_d] \\ &= \rho + \frac{p^2 E[S(S-1)]}{2(1-\rho)} + \frac{T_V\left(T_V-1\right)p + 2T_V pE[U] + p\left(1-\bar{p}^{T_V}\right) E[U(U-1)]}{2\left(T_V + (1-\bar{p}^{T_V}) E[U]\right)}. \end{aligned} \tag{13}$$

The stationary waiting time W can be decomposed into the sum of two independent random variables, i.e., $W = W_0 + W_d$, where W_0 is the waiting time of a data frame for a classical Geom/G/1 queueing model, and W_d is the additional waiting time of a data frame due to the setup procedure and multiple vacations. Therefore, the PGF $W_0(z)$ of W_0 is given as follows:

$$W_0(z) = \frac{(1-\rho)(1-z)}{(1-z)-\rho(1-S(z))}. \tag{14}$$

Differentiating Eq. (14) with respect to z at $z = 1$, the average $E[W_0]$ of W_0 is given by

$$E[W_0] = \frac{pE[S(S-1)]}{2(1-\rho)}. \tag{15}$$

By applying a similar method used when deriving the queueing length, we can get the average additional waiting time $E[W_d]$ of W_d in this system as follows:

$$E[W_d] = \frac{T_V(T_V-1)+2T_VE[U]+(1-\bar{p}^{T_V})E[U(U-1)]}{2(T_V+(1-\bar{p}^{T_V})E[U])}. \tag{16}$$

Combining Eqs. (15) and (16), we can obtain the average $E[W]$ of W as follows:

$$\begin{aligned} E[W] &= E[W_0]+E[W_d] \\ &= \frac{pE[S(S-1)]}{2(1-\rho)} + \frac{T_V(T_V-1)+2T_VE[U]+\left(1-\bar{p}^{T_V}\right)E[U(U-1)]}{2\left(T_V+(1-\bar{p}^{T_V})E[U]\right)}. \end{aligned} \tag{17}$$

3.2 Busy Cycle

The busy cycle R is defined as a time period from the instant in which a busy period finishes, to the instant in which the next busy period ends. The busy cycle R consists of one or more vacation periods V, a setup period U and a busy period Θ.

Let θ be the busy period for the classical Geom/G/1 queueing model, and the average $E[\theta]$ of θ is $E[\theta] = E[S]/(1-\rho)$. Then the average $E[\Theta]$ of Θ can be obtained as follows:

$$E[\Theta] = E[Q_B]E[\theta] = \frac{\rho}{1-\rho}\left(\frac{T_V}{1-\bar{p}^{T_V}} + E[U]\right). \tag{18}$$

Let N_V be the number of switches between the sleep mode and the active mode in a busy cycle. We then can get the probability distribution and the average $E[N_V]$ of N_V as follows:

$$P\{N_V = j\} = \left(\bar{p}^{T_V}\right)^{j-1}\left(1-\bar{p}^{T_V}\right), \quad j \geq 1, \tag{19}$$

$$E[N_V] = \frac{1}{1-\bar{p}^{T_V}}. \tag{20}$$

Conclusively, we can give the average $E[R]$ of R as follows:

$$E[R] = E[\Theta]+E[U]+E[N_V]T_V = \frac{1}{1-\rho}\left(\frac{T_V}{1-\bar{p}^{T_V}} + E[U]\right). \tag{21}$$

Let p_b, p_v, p_u be the probabilities for the system being in the busy period, the vacation period and the setup period. They then follow that

$$p_b = \frac{E[\Theta]}{E[R]} = \rho, \tag{22}$$

$$p_v = \frac{E[N_V]T_V}{E[R]} = \frac{T_V\,(1-\rho)}{T_V + (1-\bar{p}^{T_V})\,E[U]}, \tag{23}$$

$$p_u = \frac{E[U]}{E[R]} = \frac{(1-\rho)(1-\bar{p}^{T_V})E[U]}{T_V + (1-\bar{p}^{T_V})E[U]}. \tag{24}$$

4 Performance Measures

In this section, we present some important performance measures for the system proposed in this paper.

The average latency σ of data frames is the time period in slots elapsed from the arrival instant of a data frame to the end instant of the transmission of this data frame. The average latency σ is the sum of the waiting time and the transmission time of a data frame, so we have that

$$\begin{aligned}\sigma &= E[W] + E[S] \\ &= \frac{pE[S(S-1)]}{2(1-\rho)} + \frac{T_V(T_V-1) + 2T_V E[U] + (1-\bar{p}^{T_V})E[U(U-1)]}{2\,(T_V + (1-\bar{p}^{T_V})E[U])} + E[S].\end{aligned} \tag{25}$$

The average energy consumption $E[P]$ is defined as the sum of the energy consumed by two adjacent sensor nodes per slot. Note that the energy is mainly consumed by a circuit and an amplifier. Let P_{CS} be the circuit power consumption in the sleep mode, P_{CA} be the circuit power consumption in the active mode, P_{amp} be the power consumption for the amplifier, and P_{SA} be the power consumption when the system switches from the sleep mode to the active mode. We then get the average energy consumption $E[P]$ as follows:

$$E[P] = P_{CS}p_v + (P_{CA} + P_{amp})p_b + P_{SA}\frac{1}{E[R]}. \tag{26}$$

As set out in [6], the minimum power consumption of the amplifier P_{amp} is given as

$$P_{amp} = 8(\mathrm{M}^2-1)\pi^2 d^2 B\mathrm{N}_0 \left(Q^{-1}(T_0)\right)^2 (3G\Gamma^2)^{-1} \tag{27}$$

where $Q(x) = (2\pi)^{-1/2}\int_x^{\infty} e^{-t^2/2}dt,\ x \geq 0,$ Γ is the carrier wavelength, T_0 is the Bit Error Rate (BER), M=2^k, k is the constellation size, f_c is the carrier frequency, and G is a constant defined by the antenna gain and other system parameters.

Substituting Eq. (27) with Eq. (26), then differentiating Eq. (31) with respect to the constellation size k, we can get the derivative function of $E[P]$ as follows:

$$\begin{aligned}\frac{\partial E[P]}{\partial k} &= \frac{pL}{k^2 B} \times (P_{CS} \times A_0 + P_{SA} \times C_0 - P_{CA}) \\ &+ \frac{pL}{k^2 B} \times 8\pi^2 d^2 B\mathrm{N}_0 \left(Q^{-1}(T_0)\right)^2 (3G\Gamma^2)^{-1}(4^k k \ln 4 - 4^k + 1)\end{aligned} \tag{28}$$

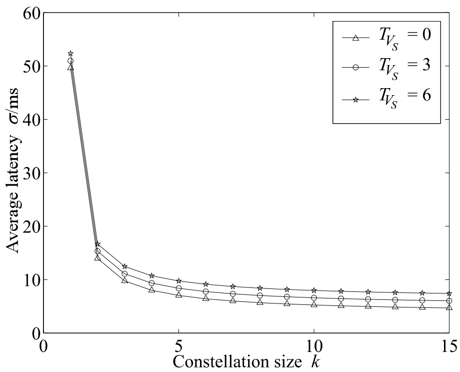

Figure 1: The average latency σ vs. constellation size k.

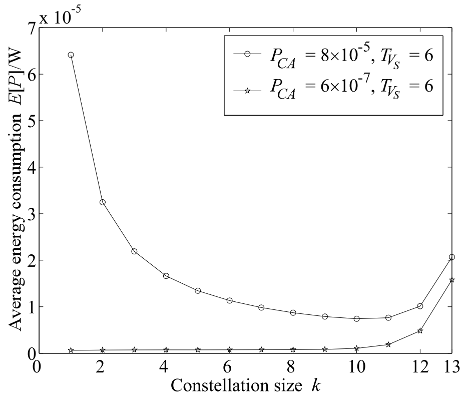

Figure 2: The average energy consumption $E[P]$ vs. constellation size k.

where

$$A_0 = \frac{T_V}{T_V + (1-\bar{p}^{T_V})E[U]}, \quad C_0 = \frac{1-\bar{p}^{T_V}}{T_V + (1-\bar{p}^{T_V})E[U]}.$$

It can be found that $\dfrac{\partial E[P]}{\partial k}$ is always greater than zero under the condition that $P_{CA} \leq (P_{CS} \times A_0 + P_{SA} \times C_0)$, so $E[P]$ is a monotone increasing function in this case. Therefore, we can derive the minimum value of $E[P]$ at $k = 1$. On the other hand, when $P_{CA} > (P_{CS} \times A_0 + P_{SA} \times C_0)$, there is a minimum value of $E[P]$ when the constellation size k is set to an optimal value k^*.

5 Numerical Results

We numerically evaluate the system performance in this section. Referencing to calculations in [6], we set the system parameters as follows: a slot is 1 ms, $p = 0.05$, $d = 30$ m, $T_0 = 10^{-4}$, $G = 2$, $L = 16$ kb, $f_c = 10^8$ Hz, $B = 1$ MHz, $P_{CS} = 10^{-7}$ W, $P_{SA} = 5 \times 10^{-5}$ W, $N_0 = 2 \times 10^{-16}$ W/Hz. Let the average setup period $E[U] = 3$ ms. The time length of the listening period $T_{V_L} = 3$ ms. The dependency relationships between the performance measures and the system parameters are shown in Figs. 1 and 2.

Figure 1 shows that how the average latency σ of data frames changes with the constellation size k for different the time length T_{V_S} of the sleep window.

It is observed that for the same constellation size k, the average latency σ of data frames will increase with the time length T_{V_S} of the sleep window. The reason is that the longer the time length of the sleep window is, the longer the waiting time of data frames will be, so the greater the average latency will be. On the other hand, the average latency σ of data frames will decrease as the constellation size k increases for all time lengths T_{V_S} of the sleep window. This is because the larger the constellation size is, the shorter the transmission time of a data frame is, so the less the average latency of data frames will be.

The influence of the constellation size k on the average energy consumption $E[P]$ is plotted in Fig. 2.

When $T_{V_S} = 6$ ms, we can conclude that the average energy consumption $E[P]$ will increase as the constellation size k increases with $P_{CA} = 6 \times 10^{-7}$ W. So we can obtain the minimum average energy consumption at $k = 1$ for this case. On the other hand, the average energy consumption $E[P]$ experiences two stages as the constellation size k increases with $P_{CA} = 8 \times 10^{-5}$ W. When $1 \leq k \leq 10$, the average energy consumption $E[P]$ will decrease as the constellation size k increases, when $k > 10$, the average energy consumption $E[P]$ will increase as the constellation size k increases. Therefore, there is an optimal value k^* for a minimum energy consumption for this case.

6 Conclusions

In a wireless sensor network, data latency and energy consumption are both important performance measures which need to be considered. However, there is a tradeoff between these two performance measures. We built a discrete-time multiple vacation queueing model with a setup to capture the working principle of the sleep/wakeup protocol offered in IEEE 802.15.4. We analyzed the system model in the steady state, and proposed the formulas for the system performance measures in terms of the average latency of data frames and the average energy consumption. Moreover, we presented numerical results to investigate the nature of the dependency relationships between the system parameters and the performance measures.

This paper provides a theoretical basis for the optimal setting of the system parameters in a wireless sensor network, and has potential applications in solving other energy conservation related problems.

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References

[1] J. Yick, B. Mukherjee and D. Ghosal, "Wireless sensor network survey," *Computer Networks*, vol. 52, no. 12, pp. 2292-2330, 2008.

[2] G. Anastasi, M. Conti, M. Francesco and A. Passarella, "Energy conservation in wireless sensor networks: a survey," *Ad Hoc Networks*, vol. 7, no. 3, pp. 537-568, 2009.

[3] C. Chiasserini and M. Garetto, "An analytical model for wireless sensor networks with sleeping nodes," *IEEE Trans. on Mobile Computing*, vol. 5, no. 12, pp. 1282-1291, 2006.

[4] T. Abdelzaher, T. He and J. Stankovic, "Feedback control of data aggregation in sensor networks," *Proc. of the 43rd IEEE Conference on Decision and Control*, pp. 1490-1495, 2004.

[5] S. Cui, A. Goldsmith and A. Bahai, "Modulation optimization under energy constraints," *Proc. of ICC'03*, pp. 2805-2811, 2003.

[6] J. Guo, H. Yu and X. Xu, "Performance research of wireless sensor networks based on queue model," *Computer Engineering and Applications*, vol. 42, no. 8, pp. 20-22, 25, 2006 (in Chinese).

[7] N. Tian, X. Xu and Z. Ma, *Discrete Time Queueing Theory, Science Publishers*, Beijing, 2008 (in Chinese).