

A Solution Method for Manpower Scheduling Problems by RCPSP/ τ

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Abstract In the resource-constrained project scheduling problem (RCPSP), the available maximum amount of the resource in each time is restricted, and the total of the resource consumption of the activities in each time cannot exceed this value. This problem can deal with many types of scheduling problems by considering various kinds of attributes on the activities to be resource constraints. By contrast the manpower scheduling problem (it is called a staff-scheduling problem or a shift-scheduling problem, also) have peculiar shift constraints, such as the number of necessary staffs at each time, the interval between duties, the maximum consecutive duties of each staff, and the prohibition of the some staff's pair. It becomes difficult to solve within the frame of standard RCPSP by those constraints. In this research, we show that the manpower scheduling problems can be solved by RCPSP if we use the extended RCPSP (it is called RCPSP/ τ) and use some dummy resources. We evaluate the method by a sample data of the nurse-scheduling problem that is a typical shift scheduling problem. The data is for the three shifts nurse scheduling problem that we made by slightly modifying a real problem. Using our method, we could solve the shift scheduling problem in 445 second by a standard PC. To solve the problem, we use our RCPSP engine by transforming the constraints to the resource constraints, and we did not change a programming code of the engine.

Keywords Scheduling; RCPSP; Manpower Scheduling; Shift Timetabling;

1 Introduction

The project scheduling problem, where each activity has the precedence constraints with the other activities and it needs fixed resources, is called RCPSP (Resource Constrained Project Scheduling Problem). In a number of relevant research papers many types of solution algorithms and extended models of which various constraints were relaxed or added have been proposed [1]. However to get the optimal solution for a large scale RCPSP is very difficult since it is NP-hard [1]. On the other hand, RCPSP has a merit that it can deal with many scheduling problems [2,3] because the various

types of scheduling constraints (such as the number of machines or workers, installation spaces, capability of a machine per unit time, various disjunction constraints, etc.) can be regarded as resource constraints of the RCPSP. Because of this merit, many scheduling problems have been discussed by using the framework of RCPSP [4,5].

We have developed a general project scheduling solver SEES (Scheduling Expert Engine System) and have applied it to many actual problems [5,6]. SEES has adopted RCPSP/ τ model of which the resource consumption of the activity within its duration time varies by unit time [7].

On the other hand, in the scheduling problems that make the shift plans of staffs (we call the issue a manpower scheduling problems in this paper) such as the nurse scheduling problem, there are many different types of constraints. These are not only the total number of staffs who can participate but also the number of minimum staffs needed, the interval between a service and the next service, and the order constraints between two services [8]. Generally, we cannot transform these constraints only by normal RCPSP.

In this research, we show that the manpower scheduling problems can be solved by RCPSP, if we introduce RCPSP/ τ and utilize dummy resources. We evaluate the method by a sample data of 3 shifts nurse scheduling problem (3-shift NSP) of which the data was made based on a real problem.

In remainder of this paper, we describe SEES, including the realization technique in Section 2. In Section 3, the constraints of the manpower scheduling problem and the NSP are stated. We discuss the methods transforming the manpower scheduling constraints within the framework of the RCPSP in Section 4, and in Section 5, we introduce the nurse scheduling data used in this research and show the result. Lastly, in Section 6, we discuss the conclusions and future work.

2 The Processing Outline of SEES

2.1 The Expression Methods of Hard Constraints

Two types of constraints exist in a scheduling problem. There are restrictions that must be satisfied absolutely, which are called hard constraints. There are also restrictions that should be satisfied to the greatest extent possible, which are called soft constraints. We calculate the periods that the resource constraints (and some hard constraints which can be calculated as resource constraints) satisfy by the inclusive relation of two rectangular patterns called a consumption pattern and an available pattern. Such constraints are called pattern constraints in this paper. Assuming R pattern constraints exist in an activity, the activity has R pairs which consist of an available pattern and consumption pattern. An available pattern has a horizontal time axis from 0 as the scheduling start time and a vertical axis expressing the maximum available quantity of hard restrictions in a unit time (Fig. 1).

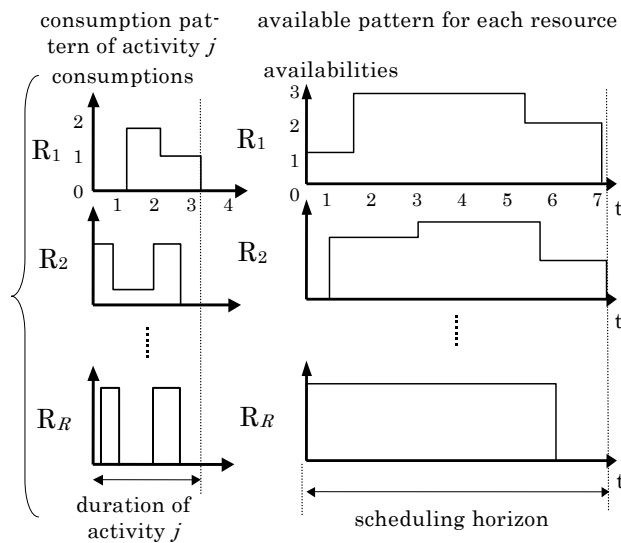


Figure 1: Examples of the pattern expression for amount of consumption and availability

2.2 Calculating Methods for Hard Constraints

SEES calculates all periods which satisfy pattern constraints. We call this method the packing method. Using the packing method, pattern constraints can be added or removed easily without changing a line of code[5,6]. The packing in the case of allocating activity j is shown below (Fig. 2).

2.3 Calculation of the Allocation Time

In the consideration of the precedence constraints of the activities, we find all new time periods which satisfy all hard constraints within the packing periods of 2.2. Furthermore, using the evaluation formula of the soft constraints that are defined according to each problem, the start time of the activity is decided within these periods. The activity is allocated at the specified time. We recalculate the new resource environment by subtracting the resource consumption pattern of the activity from the resource pattern which was available before scheduling the activity (Fig. 3). SEES repeats the processing until all activities are allocated using backtracking procedure.

2.4 The Search Space and Search Method

After an activity is allocated, the scheduler calculates the set of eligible activities which should be processed at the next allocation, where the scheduler holds these activities as a search list. Using the priority rule defined according to each problem, the next processing activity is chosen from the list, and is allocated by the packing method. The same allocating order for the activities is not chosen more than once since the selected activity is deleted from the search list. This process is repeated until all activities are allocated.

The scheduler backtracks using the list when an activity cannot be allo-

cated or the expected evaluation value cannot be obtained. In the backtracking, SEES cuts the edge which is chosen by defined cutting rules. The scheduler selects a new node within the tree to restart the schedule.

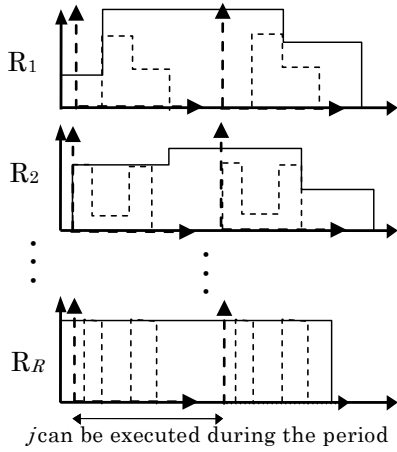


Figure 2 : Calculated time periods

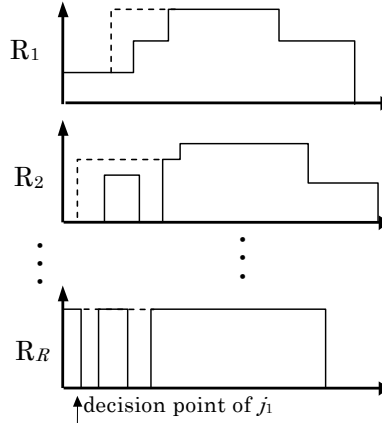


Figure 3: Calculation of the new environment

3 The Constraints of Manpower Scheduling

3.1 About a Duty Condition to Consider

1: Attendance, 0:Rest

time \ staff	1	2	3	T	
m ₁	1	0			1	Order con- straints
m ₂	0	0			1	
m ₃	1	1			0	
:	:	:			:	
:	:	:			:	

Lineup constrains

Figure 4: The constraints of shift-table

In the manpower scheduling, a plan has to satisfy the condition of a staff's lineup on each time. Moreover the planner must consider the order of the services of each staff, too[8]. Because many attributes are given about the ability of the participating staff, the lineup should satisfy the constraints that the total of these attributes has to meet the predetermined condition to each working time. It can be said as the vertical constraints in the timetable of Fig.4. In the timetable, the scheduling-time is represented as the horizontal axis and the participating-staff on the vertical axis. On the other hand, the order constraints mean that the service pattern of each staff needs to satisfy

predetermined conditions, such as the restricted continuation times of night shift. In the same timetable it is represented as horizontal constraints

In this research, we adopt the following constraints as general expressions to the constraints about the staff's lineup composition at the time t .

1. Total number of participating staff should be n_t or less (constraints of maximum staffs)
2. Total number of the activities, of which attributes are P_{i_t} , should be x_t or more (the necessity minimum staffs constraints).
3. The activities with attribute P_j cannot be allocated at the same time (disjunctive constraints).

We defined following constraints as the order constraints for one staff.

4. Two activities with attribute P_k need the intervals more than y_k unit time (activities' interval constraints).
5. A staff should not participate to the service of the attribute P_l successive m times or more. If it exceed m , it is necessary to set the interval more than n for the next service (order constraints for services).

3.2 About NSP and the Experimental Data

NSP is one of typical manpower scheduling problem. The planner of NSP should generate service timetables for nurses in a hospital (or a section of hospital). Generally, the planner is a head nurse, chief, leader, or etc. On average, the scheduling period is one month and the number of participating nurses is dozens of staff. NSP includes many combinatorial constraints. In reality, the head nurse sometimes cannot make a timetable which satisfies all the constraints by hand, nonetheless she/he uses one day or more [9]. The planner feels a burden for the work because he/she has to service to original nursing duties. In Japan, the service shift system of two types such as 2-shift (day and night) and 3-shift (day, evening and night) exists. The nurse service condition in each Japanese hospital adopts 2-shift or 3-shift. In this research, we took a real example [10] and made the following 3-shift problem. In the data, a service timetable for 24 nurses must be generated for a 30 day schedule. The skill levels of nurses have three classifications called A1 (upper class), A2 (middle class), and A3 (beginners' class). Additionally, we consider three medical teams as T1, T2, and T3. Each nurse has the attributes as shown below (Table 1).

The adopted constraints are:

in each day shift

→ 8 nurses are needed, A1 (upper class) $\cong 2$, A3 (beginner class) $\cong 2$, and 2 or more nurses are needed from each medical team;

in each night and evening shift

→ 3 nurses are needed, A1 (upper class) $\cong 1$, A3 (beginner class) $\cong 1$, and 1 nurse is needed from each medical team;

the interval between service times is at least $\cong 2$ shifts, and

the maximum times of successive service times on a night shift $\cong 2$ times;

if the night shifts are not successive, 4 days or more are needed at interval

between two night shifts;
 each nurse can specify a day which is an off-day (the day is decided by a random value);
 it is forbidden that nurse 4 should be allocated on a night shift together with nurse 23 and nurse 24. The number of the duty for every ability attribute is as follows (Table 2).

Table 1: The data which are used on this

Team	ID	Classification of ability	Team	ID	Classification of ability	Team	ID	Classification of ability
T1	1	A1	T2	9	A1	T3	17	A1
	2	A1		10	A1		18	A1
	3	A1		11	A1		19	A1
	4	A2		12	A2		20	A2
	5	A2		13	A2		21	A2
	6	A2		14	A2		22	A2
	7	A3		15	A3		23	A3
	8	A3		16	A3		24	A3

Table 2: The number of service times for each ability

	night shift	day shift	evening shift
A1 (9 nurses belong)	4	10	4
A2 (9 nurses belong)	4	10	4
A3 (6 nurses belong)	3	10	3

4 A Solution for Shift-scheduling by RCPSP

4.1 The Pattern Expression of Each Constraint

At first we consider the followings to treat NSP within the framework of RCPSP.

a. Definition of the scheduling activity

We define one duty of one nurse for a shift as a scheduling activity. The number of scheduling activities has to be given.

b. Correspondence of a date and a serial number

We number each shift of each day. For example, if a scheduling period is one month (30 days) and a day has three shifts, there are 90 allocating points t ($t = 0, 1, 2, \dots, 89$) in total.

Next, we express various constraints of manpower scheduling problem as a pattern.

(1) Constraints of maximum staffs

The constraints of the required number of nurses on each day are realized by carrying out the packing of two patterns. The two patterns are the consumption pattern which is assigned to an activity, and the available resource

pattern which is assigned to the total required number. An example of pattern expression to the dayshift activity of nurse 1 is shown in Fig. 5.

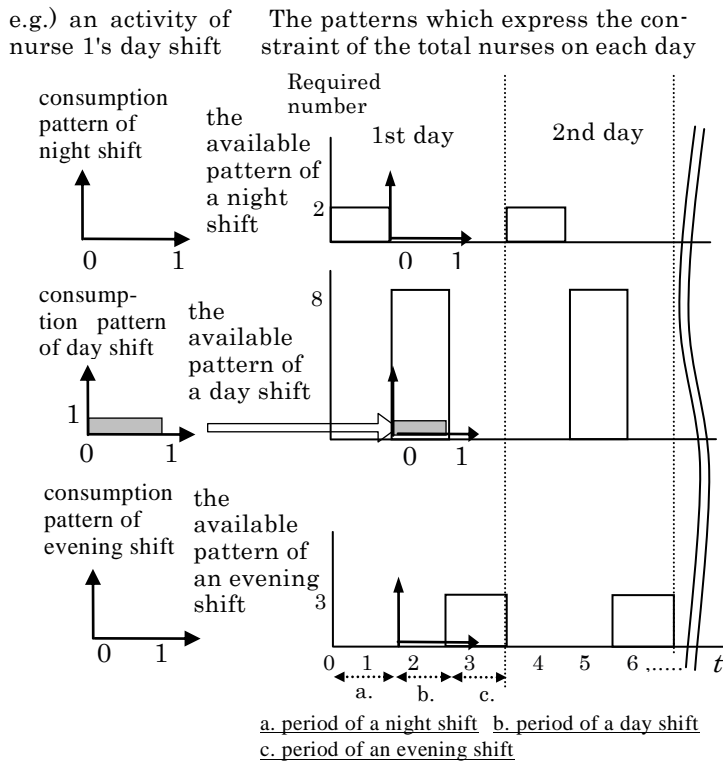


Figure 5: The pattern expression of the total participation constraints on each day

(2) The number of necessity minimum staffs

For each group, the maximum and the minimum participating number of nurses should be decided. In the RCPSP, the maximum available resource constraints can be treated directly, whereas the minimum resource constraints cannot be treated in the same way. So we propose a devised pattern expression. In order to present the expression of minimum constraints, we use the following example. Suppose there are three medical teams T1, T2 and T3. A night shift or an evening shift is carried out by the total of three nurses and requires at least one nurse from each team. Moreover, a day shift requires 10 nurses and includes two or more nurses from each team. In this case, maximum and minimum number of nurses from each team on the day shift can be stated as

$$2 \leq \text{number of nurses from T1} \leq 6, \quad 2 \leq \text{number of nurses from T2} \leq 6, \quad \text{and} \quad 2 \leq \text{number of nurses from T3} \leq 6.$$

As a result, on the day shift, it is assumed that at most six nurses (be-

longing to T1) and eight nurses (not from T1) can participate. In this case, the following negative patterns are prepared to realize the minimum constraints (Fig. 6).

In this example, the activity which is serviced by nurse 1 who belongs to T1 on a day shift gets the following six consumption patterns such as T1, not T1, T2, not T2, T3 and not T3.

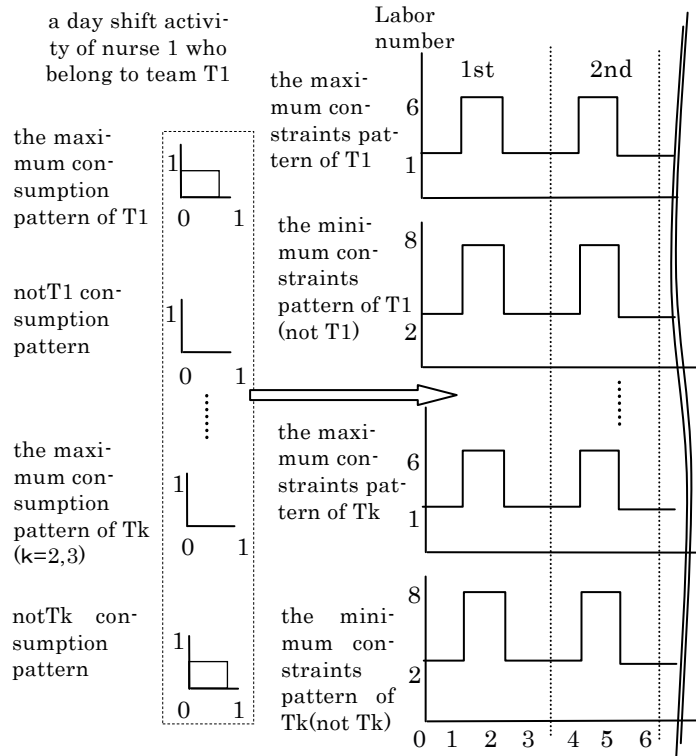


Figure 6: The pattern expression of the maximum and minimum constraints

Also to realize the other group constraints such as skill levels, similar patterns will be added to each activity.

(3) Disjunctive constraints

One nurse cannot work two or more shifts simultaneously. Restricted periods during which each nurse can (or cannot) work exist. These constraints can be realized using the following two patterns. One is an available pattern for which the height is one at the time point in which the nurse can work. Another is a consumption pattern which belongs to an activity, and consumes the available pattern. An example of the expression is shown in Fig.7.

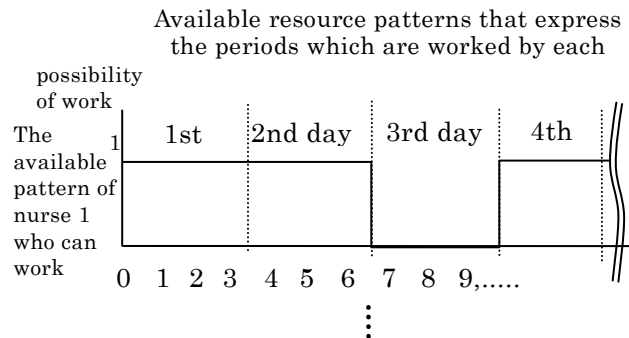


Figure 7: The pattern expression of the period in which each nurse cannot work

When we forbid specific nurses' night shift pair, a pattern which controls pair-prohibition with height one should be prepared. Then the consumer expenditure income pattern of height one, which only the pair prohibition nurse consumes, is added to each nurses' night shift activity (Fig.8).

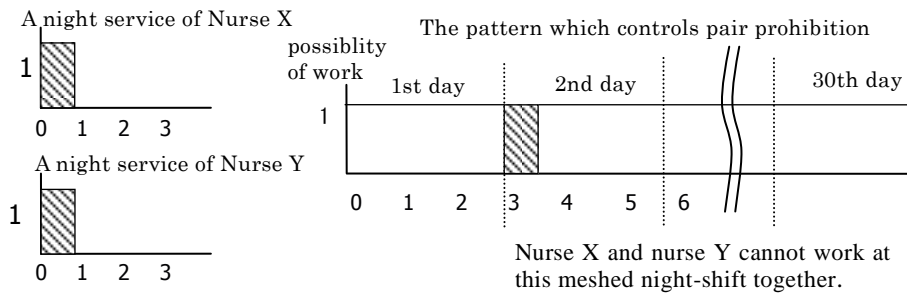


Figure 8: The pattern expression of the night pair prohibition between X and Y

(4) Activities' interval constraints

The constraints require two or more shifts of intervals between two successive activities for each nurse. In order to realize the constraints, we add the following patterns to the activities. A specified available resource pattern is prepared for each nurse activity. This pattern is a dummy pattern to manage the order constraints for the nurse. The consumption dummy patterns are prepared for each nurse activity. The consumption pattern has a length of 3 shifts (= 1 shift of working hours + 2 subsequent shifts). The rectangle is extracted from the dummy pattern. Therefore, additional shifts for the nurse cannot be allocated within the period of the following two shifts. An example of the expression is shown in Fig. 9 The activity of nurse 1 cannot be allo-

cated in $t = 3$ and $t = 4$.

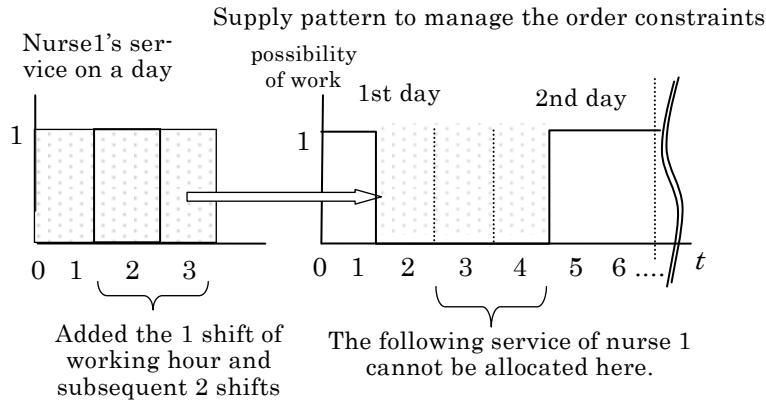
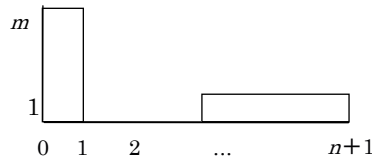


Figure 9: The pattern expression for the order constraints

(5) Order constraints for the night shift

The following consumption patterns of height m are added to each nightshift activity.



Packing ↓

The resource pattern for a night shift order constraints is made for every nurse.

m (the maximum number of successive night shift) n (a nightshift cannot be allocated at this period) a nightshift can be allocated

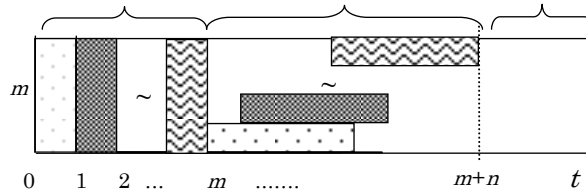


Figure 10 The pattern expression for the order constraints of nightshift (or evening shifts)

To satisfy the constraints regarding the maximum number of successive night shifts (or evening shifts), we also adapted a special expression of a pat-

6 Conclusions

This experiment shows the followings. Constraints which does not represented explicitly by resource constraints can be discussed within the framework of RCPSP by expressing the activity's attribute by a dummy pattern. In particular, RCPSP/ τ can deal with many types of constraints, since the consumption patterns of activities change according to the activity's duration time. Considering to pile up the consumption patterns of two activities like a key and a keyhole, the activity which can be started at the same time could also be limited. So more complicated allocation control can be realized.

We were able to get future possible extensions of SEES. In some manpower scheduling problems, we have to consider the range constraints of which the total number of participating staffs in one shift and total number of duty days for each staff are decided within the range from the minimum number to the maximum number. The proposed method for the minimum staffs' constraints cannot solve directly such conditions, since total number of all scheduling activities should be given in RCPSP. To satisfy such constraints, we have to decide the total duty days for each staff and the total participating staffs for each day before entering our algorithm. We call the decision work "planning process". It becomes important to also implement the process which can calculate and adjust such a value automatically to our solver, in order to improve convenience for users. If our solver adopts such function, this solver will be a very useful general-purpose scheduler.

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