The Ninth International Symposium on Operations Research and Its Applications (ISORA'10)
Chengdu-Jiuzhaigou, China, August 19–23, 2010


# A Study on Analyzing the Grid Road Network Patterns using Relative Neighborhood Graph

Daisuke Watanabe

Department of Logistics and Information Engineering,
Tokyo University of Marine Science and Technology, Tokyo, Japan

**Abstract**

The purpose of this paper is to clarify the properties of road network patterns by considering the relationship between neighborhood intersections. The Relative Neighborhood Graph (RNG), which is one of the proximity graph, is used to analyze the road network patterns of major cities in the United States as a network construction model. The grid-tree proportion (GTP) index and the crossing factor (CF) are used to compare road networks with regular road patterns. Each link in the proximity graphs using the point data of intersections on a digital map is compared with road links. From the result, the RNG links include most of the grid roads but some major streets cannot be described by proximity graphs. The matching ratio for roads increases as the GTP index increases.

**Keywords** road network pattern, intersection, relative neighborhood graph, digital map

## 1 Introduction

Road networks have evolved as the frameworks of cities reflecting their unique topographical and historical conditions. Orderly roads such as those arranged in square grids, typically seen in Western cities, are referred to as schematic road patterns in urban planning theory.

A road density index can be used to discuss the extent of road development but it disregards structural characteristics. For this reason, many studies have attempted to assess road patterns by using connections of points and lines. However, the pattern indexes suggested by these studies provide indexes for an area as a whole and make it possible to compare various areas. However, they do not permit the assessment of individual road segments within an area.

Okabe[2] introduces the theory of computational geometry to construct geometric graphs. The geometric graphs defined on the basis of proximity relations are called "proximity graphs". The proximity graphs have many applications in engineering, particularly to morphological problems, and are recently developed in the field of computational morphology like spatial and cluster analysis, computer vision, pattern recognition and computational perception. When it is given a set of points on the plane, it is desired to find some structure among the points in the form of edges connecting a subset of the pairs of points. Watanabe[5] analyzes the road network pattern of major cities in Japan using proximity graphs and find that the edges of the relative neighborhood graph include most of the grid road. Watanabe[6] evaluates the configuration and the travel efficiency on proximity graphs as transportation networks.

The objective of this study is to analize the road network patterns and extract areas with grid road patterns using a relative neighborhood graph, which is a type of proximity graph that can construct a square grid, based on the hypothesis that the relative closeness of intersections reflects road connections. Road network data for major American cites is used to create a relative neighborhood graph for each city, and graph edges matching actual road segments are extracted and studied. The structural ratio is defined as a percentage of matched lines and is used to compare cities, and confirm the relationship with pattern indexes.

# 2 Model Description

## 2.1 Definition of a graph

Graph theory is used to describe road networks in terms of connections of points and lines. When a road with a total length $L$, $n$ points, $m$ edges, and $p$ components is in an area $S$, an edge that composes a road is a line connecting these points which constitute a start point and an end point (usually road intersections).

A graph consists of a set of points $P = \{p_1, \dots p_n\}$ and a set of edges $E = \{e_1, \dots e_m\}$ connected to these points and is described as $G = (P, E)$. The total number of points is given as $|E| = m$, and the total number of lines is given as $|P| = n$. The Euclidean distance between two points $p_i$, $p_j$ is $d(p_i, pj)$ and when these points are connected by edges, it is described as $\overline{p_i p_j}$. A road network is described as a graph $G_{\mathrm{RN}} = (P, E_{\mathrm{RN}})$, consisting of road points $P$ and road edges $E_{\mathrm{RN}}$.

A graph defined by the proximity of points on a plane exemplified by Delaunay triangulation(DT) and a minimum spanning tree(MST) is generally called a proximity graph. By creating a proximity graph based on road points extracted from an existing road network and then comparing a graph edge to an actual road network, we can clarify the principles for creating a road segment. A relative neighborhood graph (RNG) $G_{\mathrm{RNG}} = (P, E_{\mathrm{RNG}})$ is a type of proximity graph. RNG is obtained by joining two points $p_i$, $p_j$ of $P$ with an edge if and only if lune does not contain any other points of P in its interior. Fig. 1 shows this lune as the gray region.

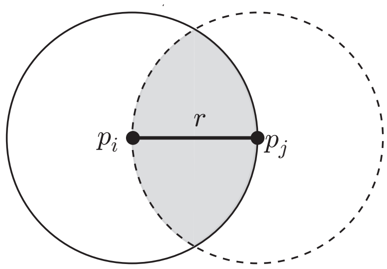


Figure 1: Search region for RNG

Fig. 3 shows these proximity graphs constructed with random 100 points. We can show that the edges of RNG consist of those of Delaunay triangulation, and the edges of a minimum spanning tree consist of those of RNG.

Fig. 3 shows the proximity graphs with regular points of square lattice and hexagonal lattice. RNG can construct typical grid road network on each lattice.

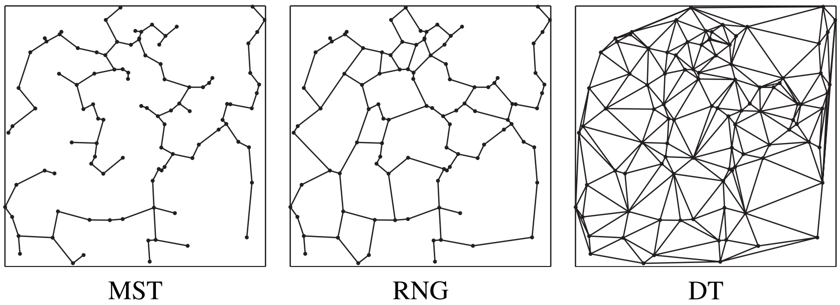


Figure 2: Proximity graphs on same random points

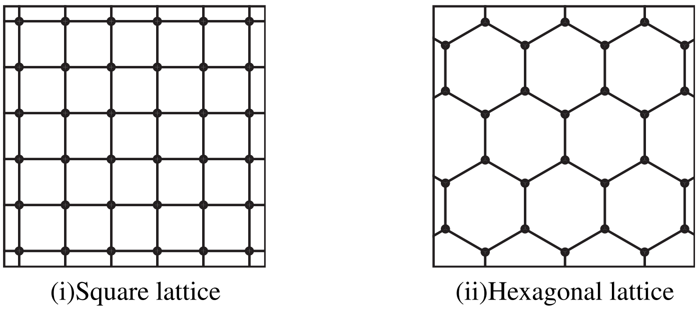


Figure 3: RNG with regular points

## 2.2 Data

The Census 2000 TIGER/Line is used as the data source for the inner city road networks of U.S. cities. The data are created by the U.S. Census Bureau and contain U.S. wide information equivalent to a map scale of 1:24000 and the network consists of line data which is made with nodes and edges.

Ten cities representing the east coast, inland, and west coast regions were chosen for this study and are shown in Fig.4. Zip codes were used to extract data for the central part of each city, which accommodates government offices and major transportation facilities, for analysis. Tab.1 shows detail data, the Num of points, edges and Road Density of each city.

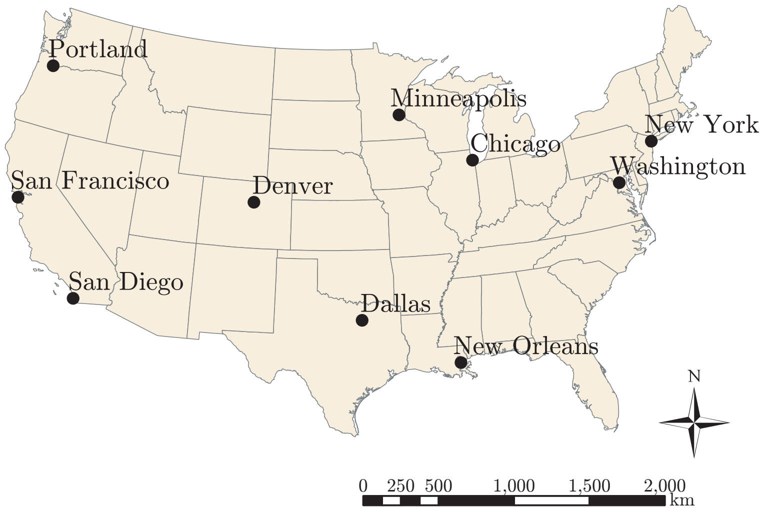


Figure 4: Selected Cities in United States

# 3 Comparative analysis of road networks in various cities

## 3.1 Assessment using pattern index

The grid-tree proportion (GTP)[1] index for road connectedness and the crossing factor (CF)[4] for road density are selected from existing pattern indexes. Various indexes describing the relationship between the number of roads and the number of intersections have been proposed in the field of quantitative geography[3]. An index for connectedness is expanded to make it possible to assess the relationship in a square grid road network, and is defined as the GTP index as

$$\mathrm{GTP} = \frac{m-n+1}{(\sqrt{n}-1)^2}. \tag{1}$$

The relationship between the total length of road and the number of intersections has been studied mainly by using a random line model based on integral geometry, in which random straight lines are treated as a road network. Based on this relationship, the CF is defined as

$$\mathrm{CF} = \frac{nS}{L^2}. \tag{2}$$

The total number and length edge of RNG with regular points of square lattice(square RNG) are respectively

$$m = 2n \tag{3}$$
$$L = 2\sqrt{nS} \tag{4}$$

Table 1: Result of GTP and CF

| City Name | Num of Points | Num of Edges | Road Density (m/ha) | GTP | CF |
|---|---|---|---|---|---|
| New York | 3,584 | 6,457 | 167 | 0.802 | 0.293 |
| Washington D.C. | 2,841 | 4,774 | 126 | 0.681 | 0.406 |
| Chicago | 4,458 | 7,265 | 135 | 0.630 | 0.366 |
| Minneapolis | 3,245 | 5,578 | 128 | 0.719 | 0.335 |
| New Orleans | 3,426 | 5,737 | 138 | 0.675 | 0.376 |
| Dallas | 3,840 | 6,228 | 111 | 0.622 | 0.486 |
| Denver | 3,170 | 5,406 | 135 | 0.706 | 0.334 |
| Portland | 4,630 | 7,801 | 157 | 0.685 | 0.398 |
| San Francisco | 4,473 | 7,185 | 167 | 0.607 | 0.364 |
| San Diego | 4,723 | 7,078 | 122 | 0.499 | 0.444 |

and those of hexagonal lattice(hexagonal RNG) are respectively

$$m = \frac{3}{2}n \tag{5}$$

$$L = 2\sqrt{\frac{3}{\sqrt{3}}}\sqrt{nS}. \tag{6}$$

The GTP index and CF for the area of concern are calculated as Tab.1 and plotted on a graph as shown in Fig.5.

Almost all of the cities fall in the middle range between a hexagonal RNG and a square RNG on a hexagonal grid. New York is the closest to being a square RNG as regards both the GTP index and CF. A larger GTP index implies greater connectedness. New York (GTP=0.802) has the largest GTP index and San Diego (GTP=0.499) has the smallest. As regards CF, a smaller figure implies a higher density. New York (CF=0.293) has the smallest figure and Dallas (CF=0.486) has the largest. Both indexes indicate that New York has the most advanced road network.

## 3.2 Comparison of relationship between RNG and road network

To assess the relationship between a road segment and a graph edge, the road edge ratio (RR)[5] is defined as

$$\mathrm{RR} = \frac{|E_{\mathrm{RN}} \cap E_{\mathrm{RNG}}|}{|E_{\mathrm{RN}}|}. \tag{7}$$

The index describes the percentage of matched road segments and graph edges to the total number of road segments within an area. When the relationship between RR and GTP is plotted on a graph, RR is directly proportional to the GTP index as shown in Fig.6; a larger RR correlates with a larger GTP index. In terms of the relationship between RR and CF, RR is indirectly proportional to CF as shown in Fig.7; a larger RR correlates with a smaller CF. A larger RR indicates that a higher percentage of edges created by the RNG

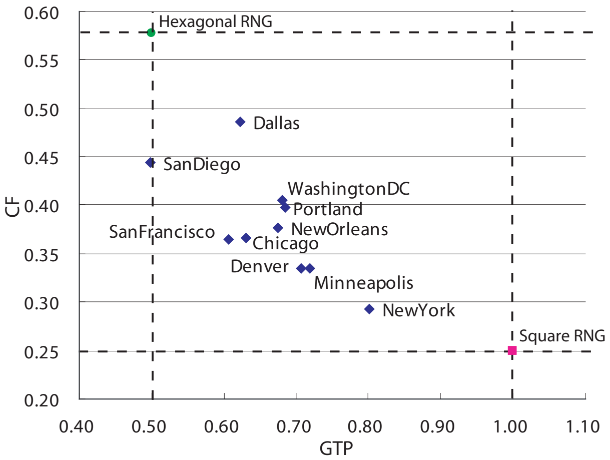


Figure 5: Comparison of GTP and CF

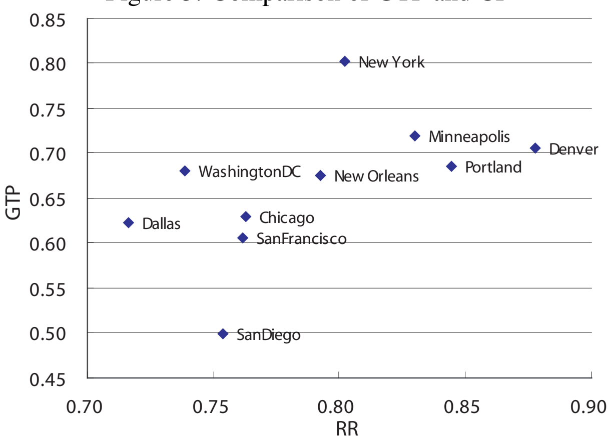


Figure 6: Comparison of RR and GTP

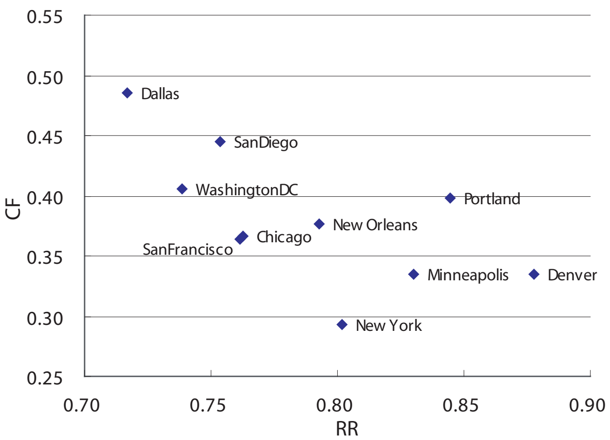


Figure 7: Comparison of RR and CF

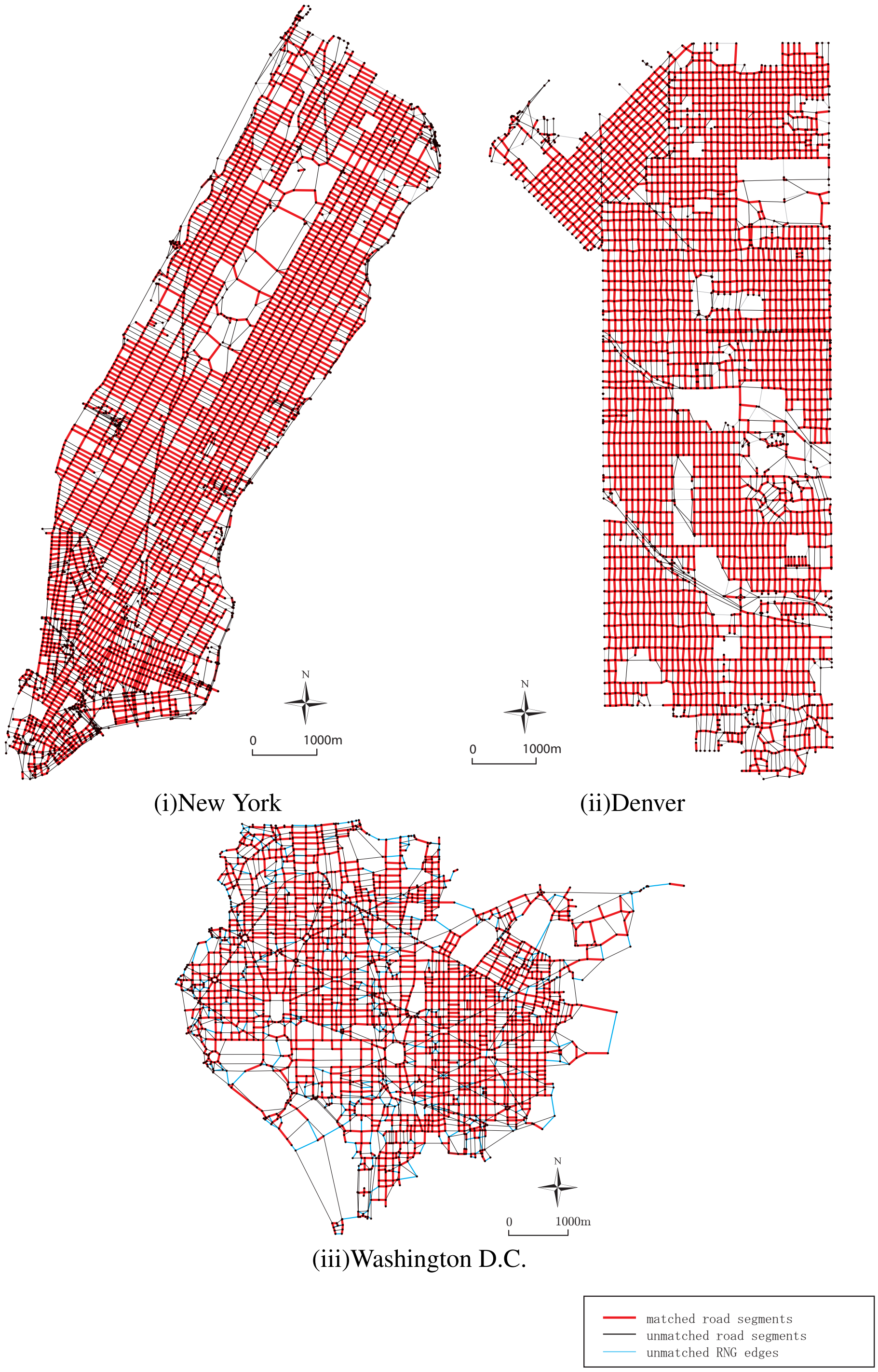


Figure 8: RNG on actual road network

match actual road segments. Denver (RR=0.88) has the highest RR and Dallas (RR=0.72) has the lowest.

Fig.8 compares the RNG with the actual road network and confirms the existence of grid roads in New York ,Denver and Washington D.C.. New York recorded the highest value according to the existing indexes GTP and CF described in the previous section, but uneven grid roads can be seen in areas such as Broadway due to slanted roads and the coastlines that surround the area. Denver recorded the highest value of RR and we can confirm almost all roads consists of grid road patterns. Washington D.C. is low value in RR, and we can confirm diagonal roads become unmatched road segments.

# 4 Conclusion

An analysis of inner city road networks using a relative neighborhood graph (RNG), which is a graph created based on the relative proximity of given points, suggested a method for extracting a grid road network from a complex road network within cities. The road networks within U.S. cities were analyzed using this method, which proved better able to evaluate grid road networks than existing indexes.

## Acknowledgements

This study was partially supported by Grand-in-Aid for Young Scientists (B) No.21710150 of the Ministry of Education, Culture, Sports, Science and Technology.

# References

[1] Noda, H.(1996), A Quantitative Analysis on the Patterns of Street Networks using Mesh Data System, City Planning Review, **202**, 64-72. (In Japanese)

[2] Okabe, A., Boots, B., Sugihara, K., and Chiu, S. N. (2000), *Spatial Tessellations: Concepts and Applications of Voronoi Diagrams, Second Edition*, 97-103, John Wiley and Sons.

[3] Taaffe, E. J., Gauthier, H. L. and O'Kelly, M. E. (1996), *Geography of Transportation, Second Edition*, 249-286, Prentice Hall.

[4] Vaughan, R. J.(1987), *Urban Spatial Traffic Patterns*, 149-155, Pion Limited.

[5] Watanabe, D.(2005): A Study on Analyzing the Road Network Pattern using Proximity Graphs, *Journal of the City Planning Institute of Japan*, **40(3)**, 133-138. (In Japanese)

[6] Watanabe, D.(2008): Evaluating the configuration and the travel efficiency on proximity graphs as transportation networks, *FORMA*, **23(2)**, 81-87.