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Evaluation of Capacities of Refuges in Urban Areas by Using Dynamic Network Flows

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Abstract In evacuation situations, it is very important that we have enough refuges. Therefore, we have to check in advance if capacities of refuges in urban areas are sufficiently large. In this paper, we present an algorithm for this problem by using a dynamic network flow which is a branch of the network flow theory, and then we apply our method to a numerical example modeling Kyoto City in Japan.

Keywords Optimization; Dynamic network flows; Evacuation; Refuge

1 Introduction

In May 2008, the Sichuan earthquake occurred, and tragedy fell upon many people. Not only earthquakes but also diverse disasters occurred and caused serious damages in many countries. In evacuation situations, it is very important that we have enough refuges in urban areas. Therefore, we have to check in advance if capacities of refuges are suitable. In this paper, we will consider this task from the theoretical viewpoint and model it as an optimization problem defined on a graph.

A typical approach to this problem is to use Voronoi diagram (for the formal definition, see [5]). Regarding refuges as Voronoi points, we partition an urban area into Voronoi regions. Then, if the sum of people in a Voronoi region is less than or equal to the capacity of the refuge corresponding to Voronoi point of this region, the capacity of this refuse is sufficiently large. Otherwise, we can say that the capacity of this refuge is not enough. Since the assumption that people will evacuate to the nearest refuge is acceptable, this approach is very useful. However, there is a problem with this approach. In this approach, we can not take a topology of a road network into consideration. This motivates us to investigate another approach in which we can take a structure of a network into consideration. For this purpose, in this paper, we model our problem as the *evacuation problem* by using *dynamic network* [4]. In the evacuation problem, we are given a *dynamic network* $\mathscr{D} = (D, c, \tau, b, S)$, where $D = (V, A)$ is a directed graph that consists of the vertex set V and the arc set A, each $x \in V$ has the supply $b(x)$, each $a \in A$ has the capacity $c(a)$ and the transit time $\tau(a)$, and $S \subseteq V$ is a set of sinks (see Figure 1). Since we consider an urban evacuation, vertices model buildings, rooms, exits and so on, and arcs model pathways or roads. For each $x \in V$, the supply $b(x)$ represents the number of people which exist at x. For each $a \in A$, the capacity $c(a)$ represents the number of people which can enter a per unit time, and the transit time $\tau(a)$ represents the time required to traverse a. Then, the evacuation problem asks for finding the minimum time required to send all the supplies to the sinks of S as well as a *dynamic flow* which attains the optimal evacuation time. In this paper, we propose a method for evaluating capacities of refuges by using an optimal dynamic flow for the evacuation problem and apply the method to a numerical example modeling Kyoto city in Japan.

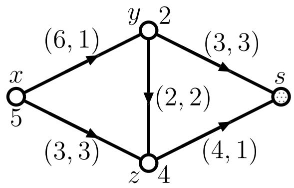

Figure 1: An example of a dynamic network. The number attached to each vertex represents its supply. The pair of numbers attached to each arc represents its capacity and transit time. The gray vertex is a sink.

Outline. The rest of this paper is organized as follows. In Section 2, we give necessary notations and propose our algorithm for evaluating capacities of refuges. Then, in Section 3, we show a numerical example modeling Kyoto city in Japan. Section 4 concludes this paper.

2 Our Model and Algorithm

We denote by $\mathbb{R}_+$ and $\mathbb{Z}_+$ the sets of nonnegative reals and nonnegative integers, respectively. For a finite set X, let $|X|$ be the number of elements of X.

2.1 A model

Let $D = (V,A)$ be a directed graph that consists of the vertex set V and the arc set A. In this paper, we assume without loss of generality that every directed graph has no parallel arcs. We denote by $a = xy$ an arc a with a tail x and a head y. It may be simply written as xy. For each $x \in V$, let $\delta_D(x)$ (resp., $\rho_D(x)$) be the set of arcs whose head (resp., tail) is x.

Let $\mathscr{D} = (D,c,\tau,b,S)$ be a *dynamic network* which consists of a directed graph D, a capacity function $c\colon A \to \mathbb{R}_+$, a transit time function $\tau\colon A \to \mathbb{Z}_+$, a supply function $b\colon V \to \mathbb{R}_+$ and a set of sinks $S \subseteq V$. Since we consider the evacuation to the sinks of S, we assume without loss of generality that $\delta_D(s) = \varnothing$ and $b(s) = 0$ hold for every $s \in S$, and at least one sink is reachable from every vertex.

We define a *dynamic flow* $f\colon A \times \mathbb{Z}_+ \to \mathbb{R}_+$ as follows. For each $a \in A$ and $\theta \in \mathbb{Z}_+$, we denote by $f(a,\theta)$ the flow rate entering the tail of a at time step θ which reaches the head of a at the time step $\theta + \tau(a)$. We call f *feasible* if it satisfies the *capacity constraint*

$$f(a,\theta) \le c(a) \quad (\forall a \in A, \forall \theta \in \mathbb{Z}_+),$$

the *flow conservation*

$$\sum_{a\in\delta_D(x)}\sum_{\theta=0}^{\Theta} f(a,\theta) \le \sum_{a\in\rho_D(x)}\sum_{\theta=0}^{\Theta-\tau(a)} f(a,\theta)+b(x) \quad (\forall x\in V,\ \forall\Theta\in\mathbb{Z}_+),$$

and the *demand constraint*

$$\sum_{s\in S}\sum_{a\in\rho_D(s)}\sum_{\theta=0}^{\Theta-\tau(a)} f(a,\theta) = \sum_{x\in V} b(x) \quad (\exists\Theta\in\mathbb{Z}_+). \tag{1}$$

For each feasible dynamic flow f, we define the *evacuation time* of f by the minimum time step Θ satisfying (1). Then, the *evacuation problem* asks for finding the minimum evacuation time among all feasible dynamic flows as well as an optimal dynamic flow which attains the minimum evacuation time.

In the above definition of dynamic networks and dynamic flows, we do not consider the capacity of sinks. However, since sinks represent refuges, it is natural to consider that the total number of people which can evacuate to each sink is limited. Thus, in this paper, we are given a capacity function (for sinks) $l\colon S\to\mathbb{R}_+$ and a feasible dynamic flow must satisfy

$$\sum_{a\in\rho_D(s)}\sum_{\theta=0}^{\Theta} f(a,\theta) \le l(s) \quad (\forall s\in S,\ \forall\Theta\in\mathbb{Z}_+). \tag{2}$$

We call a feasible dynamic flow satisfying the capacity constraint (2) *c-feasible*. In this paper, we assume without loss of generality that there exists a c-feasible dynamic flow in $\mathcal{N}$.

2.2 A proposed method

In this subsection, we give an algorithm for evaluating capacities of refuges. An optimal c-feasible dynamic flow for the evacuation problem represents the most favourable situation. Thus, it is very natural to evaluate capacities of refuges by using an optimal c-feasible dynamic flow for the evacuation problem. In an optimal c-feasible dynamic flow, if the sum of people evacuated to a sink $s\in S$ is less than $l(s)$, the capacity of s is sufficiently large. On the other hand, for a sink $s\in S$ to which the sum of people evacuated is equal to $l(s)$, we consider that if the capacity of s is larger, more people may evacuate to s. Therefore, the capacity of this sink is considered to be not enough in this case. Namely, our algorithm can be described as Algorithm 1. (An algorithm for solving the evacuation problem will be given in the subsequent subsection.)

Algorithm 1

Input: a dynamic network $\mathcal{N}$
Output: evaluation for the capacities of the sinks of $\mathcal{N}$
1: Compute an optimal c-feasible dynamic flow for the evacuation problem in $\mathcal{N}$.
2: For each sink $s\in S$, if $\sum\{f^*(\theta,a)\mid\theta\in\{0,\ldots,\Theta^*\}, a\in\rho_D(s)\}<l(s)$, the capacity of s is sufficiently large, where Θ^* denotes the minimum evacuation time for the evacuation problem in $\mathcal{N}$. Otherwise, we consider that the capacity of s is not enough.

2.3 Solving the Evacuation Problem

In this subsection, we give an algorithm for solving the evacuation problem which was presented by Ford and Fulkerson [1, 2].

2.3.1 Time-expanded networks

Let $\mathscr{S} = (H, u, v, T)$ be a *static network* which consists of a directed graph $H = (N, L)$, a capacity function $u\colon L \to \mathbb{R}_+$, a supply function $v\colon S \to \mathbb{R}_+$ and a set of sinks $T \subseteq N$. Then, we call $\xi : L \to \mathbb{R}_+$ a *feasible flow* if it satisfies the *capacity constraint*

$$\xi(a) \le u(a) \quad (\forall a \in L)$$

and the *flow conservation*

$$\sum_{a \in \delta_H(x)} \xi(a) = \sum_{a \in \rho_H(x)} \xi(a) + v(x) \quad (\forall x \in N \setminus T).$$

In order to find an optimal dynamic flow for the evacuation problem, Ford and Fulkerson [1, 2] introduced the *time-expanded network* $\mathscr{D}(\Theta)$ which is a static network for a dynamic network $\mathscr{D}$ with a time horizon Θ (see Figure 2). The vertex set of $\mathscr{D}(\Theta)$ contains a vertex $x(\theta)$ for each $x \in V$ and $\theta \in \{0, \ldots, \Theta\}$. The arc set of $\mathscr{D}(\Theta)$ consists of two parts defined as follows. The first part contains an arc $a(\theta) = x(\theta)y(\theta + \tau(a))$ with a capacity $c(a)$ for each $a = xy \in A$ and $\theta \in \{0, \ldots, \Theta - \tau(a)\}$, and the second part contains an arc $x(\theta)x(\theta+1)$ with infinite capacity for each $x \in V \setminus S$ and $\theta \in \{0, \ldots, \Theta - 1\}$. The arcs of the second part are called *holdover arcs*. For each vertex $x \in V$, the supply of $x(0)$ is $b(x)$ and that of $x(\theta)$ is zero for every $\theta \in \{1, \ldots, \Theta\}$. We define the sink set of $\mathscr{D}(\Theta)$ as $\{s(\theta) \mid s \in S, \theta \in \{0, \ldots, \Theta\}\}$.

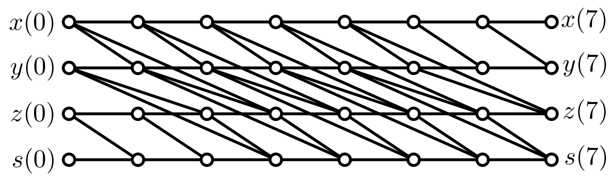

Figure 2: The structure of the time-expanded network of a dynamic network in Figure 1. All arcs are directed from left to right.

2.3.2 An algorithm for the evacuation problem

It is known [1, 2] that there exists a feasible dynamic flow in $\mathscr{N}$ whose evacuation time is less than or equal to Θ if and only if there exists a feasible flow in $\mathscr{N}(\Theta)$. Furthermore, it is known that we can check if there exists a feasible flow in $\mathscr{N}(\Theta)$ by computing a maximum flow in $\mathscr{N}(\Theta)$. (For the definition of a maximum flow, see [3].) Notice that the size of the time-expanded network $\mathscr{D}(\Theta)$ is proportional to Θ and thus is a pseudo-polynomial in the input size.

However, in this paper, we take into account the capacity constraint of sinks, i.e., c-feasible dynamic flows. Thus, we have to modify a time-expanded network. More precisely, we add a super sink s^* (we consider s^* as a single sink instead of vertices of

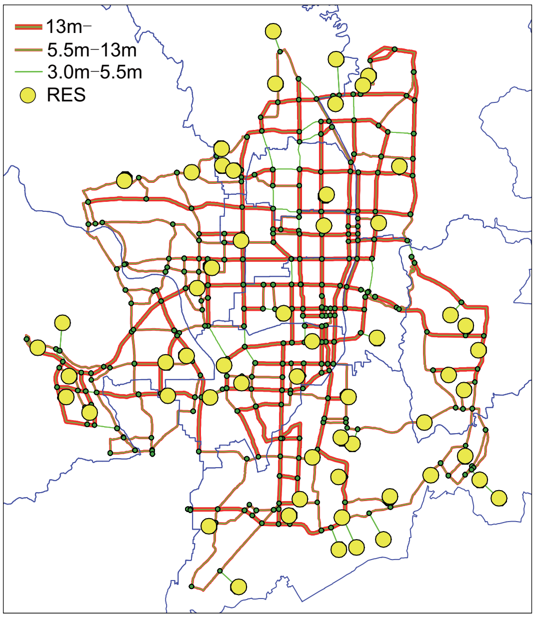

Figure 3: A network modeling Kyoto City.

T) and connect $s(\Theta)$ to s^* with an arc whose capacity is $l(s)$ for each $s \in S$. Then, it is not difficult to see that there exists a extended feasible dynamic flow in $\mathscr{N}$ whose evacuation time is less than or equal to Θ if and only if there exists a feasible flow in modified $\mathscr{N}(\Theta)$. Thus, we can check if there exists a feasible flow in modified $\mathscr{N}(\Theta)$ by computing a maximum flow in modified $\mathscr{N}(\Theta)$.

3 A Numerical Example

In this section, we apply our method to a numerical example modeling Kyoto City in Japan. In this example, regional evacuation sites (RESs) assigned by Kyoto City are considered as refuges.

3.1 Setting

Figure 3 shows the road network of Kyoto city. The gray points represent refuges (i.e., sinks). The number of vertices of this network is about four hundred, and the number of people exists in this area is about one-and-a-half million. We discretize the width of each road as follows. (i) More than 13 meter $\to$ 20 meter. (ii) 5.5 meter $\sim$ 13 meter $\to$ 9 meter. (iii) less than 5.5 meter $\to$ 4 meter. Here we explain how to determine a capacity and a transit time of each arc. We first set a unit time and a unit supply to be one second and one person, respectively. **Capacity.** Since it is known that we have to keep congestion less than or equal to 6 person per one square meter, we set the capacity of each arc to be

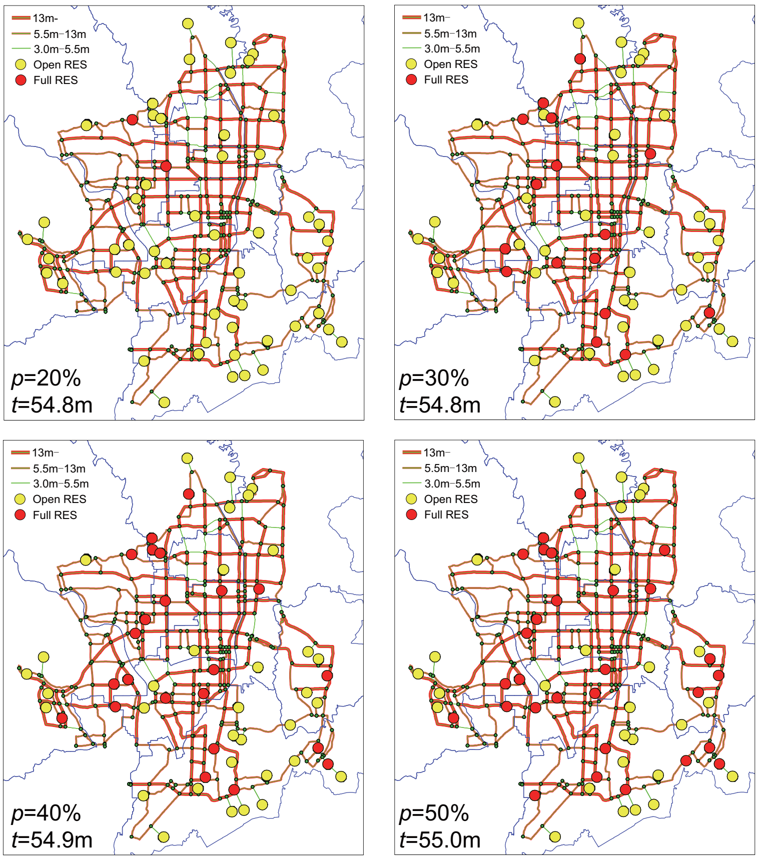

Figure 4: Illustration of situations where from 20% to 50% of the whole population evacuates to RESs.p denotes the ratio of evacuation people and t denotes the minimum evacuation time (minutes).

$6d$, where d denotes the width of this arc. **Transit time.** We assume that people can go forward one meter per second. Therefore, we set the transit time of each arc to be the length of this arc.

Supply. For each administrative division, we determine the supplies of vertices contained in this administrative division by uniformly distributing people exists in the division.

3.2 Results

Figures 4 and 5) illustrate which RESs become full when p% of people evacuates to RESs, where p ranges from 20 to 90. The dark gray circles represent the RESs which became full. RESs located in the northwest of Kyoto City become full when just 20% of people evacuate. This implies that there are not enough RESs in this area. Although a lot of RESs become full when 50% of people evacuate, RESs in the southeast still have some margins on capacity.

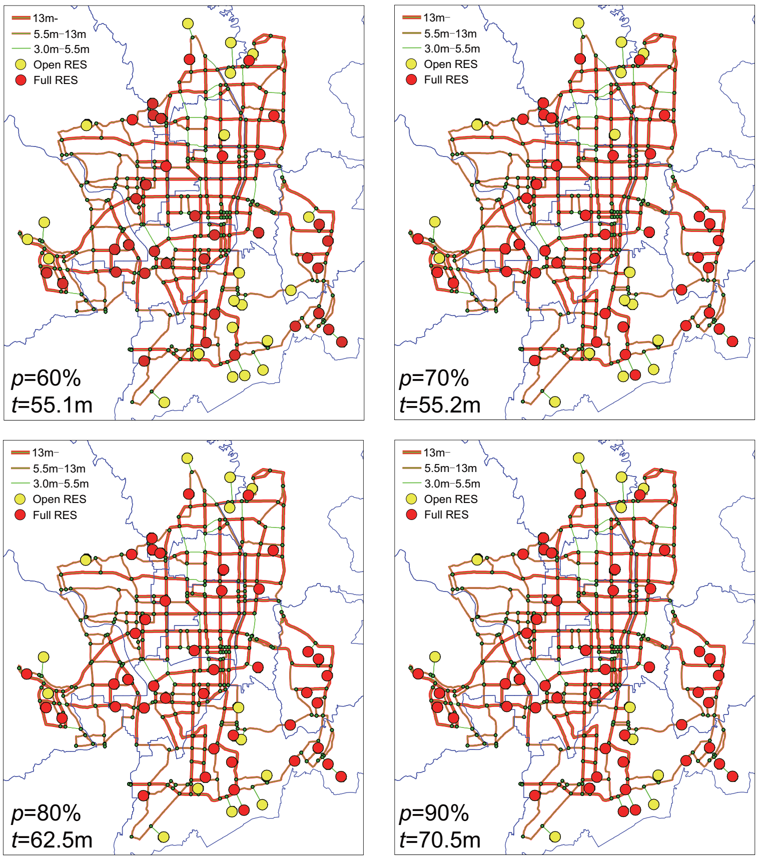

Figure 5: Illustration of situations where from 60% to 90% of the whole population evacuates to RESs.*p* denotes the ratio of evacuation people and *t* denotes the minimum evacuation time (minutes).

Until *p*=70%, the minimum evacuation time remains about 55 minutes. This means that most of the people can evacuate to their neighbour RESs if less than or equal to 70% of the whole people evacuate. Meanwhile, the minimum evacuation time begins to increase considerably when 80% of people evacuate. If it becomes 90%, the time increases to 70 minutes.

We can conclude that under the current assignment of RESs, most of the people can evacuate to their neighbour RESs even if a relatively large earthquake occurs, but there is still an umbalance of capacities or assignment of RESs such that the capacities of RESs in the northwest are not sufficient while those in the southeast are abundant.

4 Conclusion

In this paper, we considered the problem for checking whether capacities of RESs are sufficiently large, and proposed a new algorithm for this problem by using a dynamic network flow, and then we applied our method to a numerical example modeling Kyoto City in Japan. A next step in this research is to devise an algorithm for detecting bottleneck arcs in evacuation situations.

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