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Simulating Growth of Transportation Networks

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Abstract The shapes that networks form are thought to be related to the performance of networks during this process. Using reduced travel time as an evaluation criterion, this study examines growth of road and rail transportation networks to determine the differences in growth patterns that are caused by various factors, including speed on transportation network.

Keywords transportation network; growth; speed; performance

1 Introduction

The growth patterns of the transportation networks vary widely from one city to another. For example, Japan's bullet train (*Shinkansen*) network forms a tree structure, while subway train network usually forms a grid structure. These differences can be attributed to many different factors, such as network performance due to technological innovation, urban population distribution, and the shape of cities.

This study focuses on these types of transportation network growth patterns. In addition to simulating different growth patterns due to transportation network speed and population distribution, and observing the resultant network features, this study will determine the mechanism that can be used to describe these differences. As well, optimum network performance and amount of maintenance, taking into account the trade-off between network speed and construction cost, will be considered.

Batty and Xie [2] simulated the areal expansion of a city and the linear development of a network using cellular automata. They constructed a model of city growth generated by random numbers, in which cells are selected at random from neighboring cells, taking into account the hierarchical level of the network. It was found that the denser the network is, the higher its hierarchical level. They suggested that the growth of a transportation network can be evaluated using local indicators. However, for trunk networks in particular, which form the axis that shape the skeleton of national and city-wide networks, it is important to construct networks based on evaluation indicators that measure overall efficiency, rather than on simply improving local indicators.

Urban network is designed not only by considering local accessibility but also mobility in the whole area. Network design models allow us to investigate what type of network is superior when we are taking overall accessibility into account. Magnanti and Wong [5], Minoux [6] and Ahuja et al. [1] provide many references on network design including applications. However, network design in the real world transportation network is difficult to solve because of its size. One of the possible alternatives is to consider evolution of network in which the optimal adding interval of transportation network is determined one by one. Hirayama et al. [3] proposed a model for extending networks that minimize inconvenience. This method was able to capture the shape features of the growth process. However, they did not discuss the possibility of obtaining diverse shapes due to differences in initial conditions. Given these findings, this study will examine transportation networks that are designed to minimize mean required travel time as a whole to clarify the features of the shapes and the effects of networks.

2 Simulation of Transportation Network Growth

We use the ideal city that is defined as a total of 169 demand points (points of access and egress on a high-speed transportation network), which are represented on a square grid. The transportation demand between an arbitrary pair of demand points is proportional to the population at each of the two demand points.

A general link is already constructed enabling straight-line travel between any two points, making it possible to travel in a straight line between any two points at speed 1. The number of general links is 312. A section that connects grid points vertically or horizontally is considered as a single high-speed link. A high-speed link enables travel in c units of time ($0 \leq c \leq 1$), where the time is normalized based on the time required using the main link. The number of high-speed links, l, varies from 1 to 312. The travel route between demand points is either the route using only the general links, or the route via high-speed links, whichever results in the shortest travel time. The shape of real cities is not regular, as implied by the above conditions. However, the aim of this study is to determine the fundamental characteristics of such a network. Thus, a simplified network in an ideal square city was used.

In the ideal city, only a single high-speed link was created at a time. Assume that the high-speed links are constructed in order, one by one, and define the "period" as the length of time required to construct a single such link. The "mean travel time" is defined as the mean travel time between all demand points. To determine the order of construction, a sequential optimum construction method, whereby high-speed links that minimize the value of mean demand time in each "period" are successively constructed, was used. Many transportation networks are, to a certain extent, constructed according to broad-ranging scheme, based on a prescribed plan. However, in this study, it was assumed that the transportation network is constructed one by one without any overarching plan.

In addition to illustrating the construction order and shapes in relation to the transportation network growth patterns defined by the above rules, in order to clarify their features, three quantitative indicators were created to evaluate growth pattern processes.

The first indicator is the rate of reduction in mean travel time. If the mean travel time at demand point i during the time in which l high-speed links are constructed (l periods) is t_{il}, the mean travel time for the whole network can be expressed by the following expression.

$$T_l = \sum_i \frac{t_{il}}{169} \tag{1}$$

Furthermore, the rate of reduction, r_l, in mean travel time when l high-speed links have been constructed (after l periods) relative to the mean travel time when there are no high-speed links is determined by

$$r_l = 1 - \frac{T_l}{T_0} \tag{2}$$

The second indicator is the number of closed paths, p. When the number of high-speed links in the growing transportation network is e, and the number of demand points connected by these high-speed links is v, then the number of closed paths formed by high-speed links can be expressed as

$$p = e - v + 1 \tag{3}$$

The closed path capacity utilization factor, α, can be calculated as the number of closed paths divided by the maximum number of closed paths, as shown below (Okudaira [8]; Honda [4]).

$$\alpha = \frac{p}{2v-5} = \frac{e-v+1}{2v-5} \tag{4}$$

The third indicator is the π indicator. This indicates the degree to which the graph of the transportation network resembles a circle. It is defined as

$$\pi = \frac{l}{d} \tag{5}$$

where l is the number of links that equals to the overall length of the transportation network, and d is the diameter of the graph, that is, the required time between the two demand points that are furthest apart (Okudaira [8]). Obviously π becomes one when the network is a chain of links. As the network is densely constructed with keeping its circular shape, π gets larger. However, if the network extends to a particular direction, π becomes small. If the lengths of networks are same, then the larger π means that the shape of the network resembles a circle.

These three indicators can be used to determine the characteristics of the network.

3 Transportation Network Growth Patterns

Here it is assumed that the population is the same at each demand point. Based on this assumption, the speed of the high-speed links, c, was varied from 0 to 1 in increments of 0.1 to determine the shape of the network as a function of l. Figure 1 shows the results.

Networks where the speed is high (i.e., c is small) show a tendency to try to expand outward from the center of the city, with repeated branching. As a result, the network shape has essentially a radial form: the higher the speed of the links, the fewer the number of loops present. The network had a tendency at high speeds to extend rapidly to every corner of its periphery. On the other hand, networks where the speed is slow (i.e., c is large) are shaped like regular grids. Once the grid-shaped routes that form the trunk lines are created, the network has a tendency to grow in a manner such that the grids are further subdivided into smaller grids. Generally, the lower the speed of the links, the smaller the size of the grid and the lower the effectiveness in reducing the mean travel time, even as the network grows by branching. Thus, growth by construction of shorter length routes from the center of the region is preferable to growth by branching.

Figure 2 illustrates the process of mean travel time reduction as a function of speed. For slow networks, having a speed of greater than 0.7, the reduction in mean travel time remains very small as high-speed links grow. In contrast, for fast networks, having a speed of less than 0.7, there is a large reduction in mean travel time as the number of high-speed links increases. Figure 3 shows that, for a reduction rate when $l = 312, r_{312}$, there is a linear improvement in the reduction rate for $c \leq 0.7$, while when $c > 0.7$, the reduction in mean travel time is small compared to when there are no high-speed links.

As well, the main features of the network shape, which are seen in Figure 1, are confirmed using the number of closed paths and the indicator π. Figure 4 shows the number of closed paths. The number of closed paths for networks of high speed is small, and most of the networks with high-speed links have a tree structure, due to branching. On the other hand, the number of closed paths in networks of low speed is high, and a regular structure that includes closed paths can be seen. Given the above, the network can be expected to be useful between two distant points when the speed is fast. When the speed is slow, the effectiveness of each high-speed link in reducing mean travel time is low, so that the influence of the high-speed links occurs only in the vicinity of where the links are constructed. In addition, the indicator π in Figure 5 reveals that, although the values vary according to the growth process, in general, networks of lower speed grow in a shape that is closest to round.

Similar trends can be observed in real networks. For example, most high-sped train networks, such as the *Shinkansen* and the TGV, have no closed paths and grow by means of branching. On the other hand, subway networks in cities are usually composed of numerous routes that intersect one another. The differences in patterns due to the speed in network growth models reflect the features of real transportation networks.

4 Trade-off between Speed and Cost

The construction of a road network or rail network is extremely expensive. As shown in Tables 1 and 2, the real costs of high-speed networks are generally more expensive than that of low-speed networks [9]. Monorails and new transit systems, which are usually constructed in crowded urbanized areas, are relatively expensive comparing with bullet trains because of higher land price or complicated structures. If there is a fixed budget to construct railways, the following options are available: constructing a high-speed, short distance network or a low-speed, long distance network. Both options present trade-offs. Thus, this section will examine the relationship between construction cost and the speed of a transportation network. This will lead to the determination of the appropriate speed for the transportation network.

First, the number of possible networks that can be constructed given the construction costs is considered. As Table 1 shows, the relationship between cost and speed is based on an average speed for expressways and national highways of 76.7 km/h and 32.8 km/h, with construction costs totaling 6.37 billion yen and 2.99 billion yen, respectively. Taking an average, it can be seen that a road with a speed of 1.17 km/h will cost 0.1 billion yen. By dividing this value by the speed, the number of networks per 0.1 billion yen can be determined. Table 3 summarizes the relationship between the speed and number

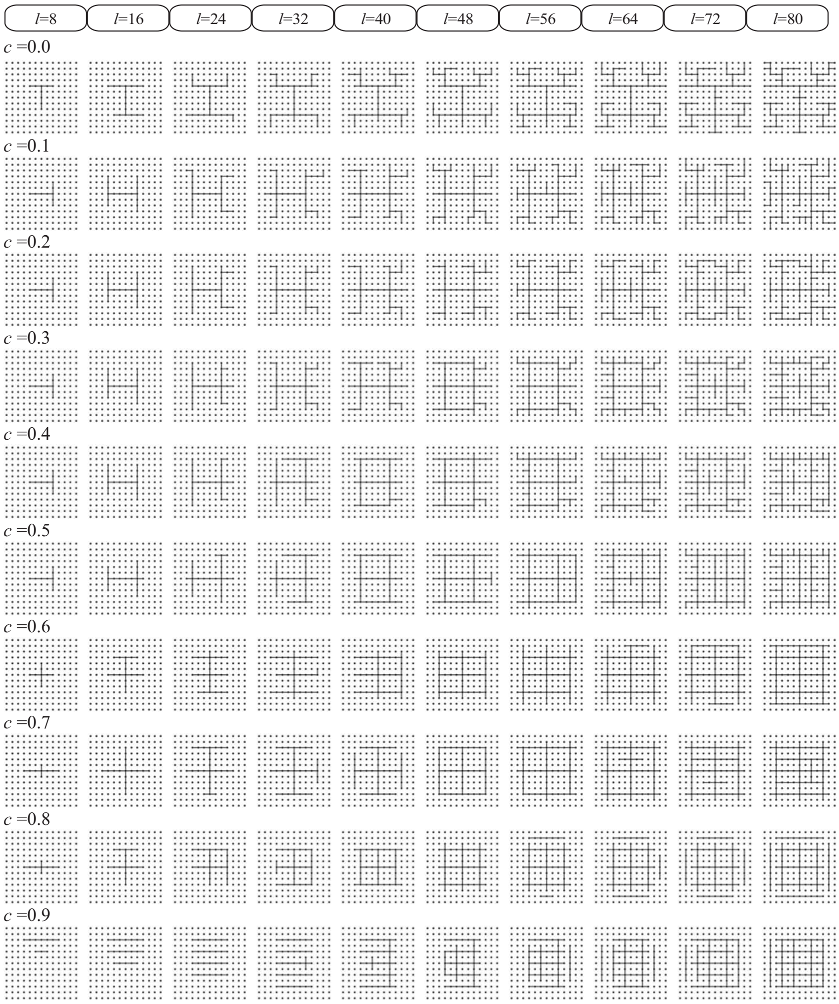

Figure 1: Speed and shape of network growth with high-speed links

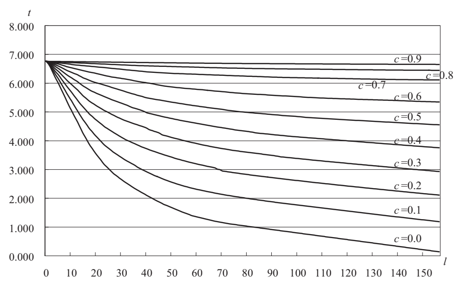

Figure 2: Mean travel time reduction process.

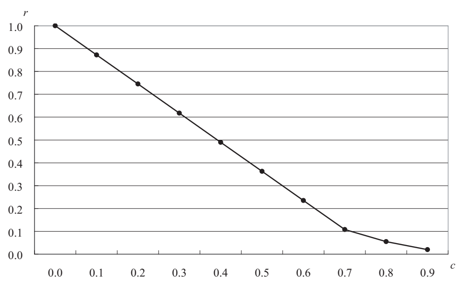

Figure 3: Reduction rate, r_{312}.

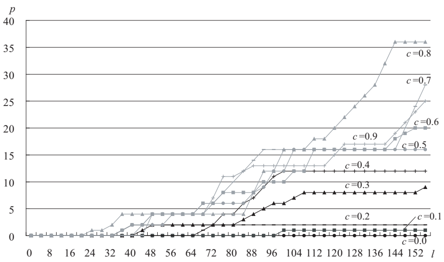

Figure 4: Number of closed paths, p.

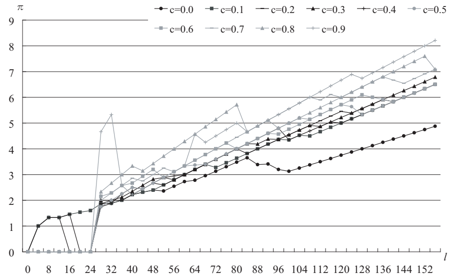

Figure 5: Indicator π.

Table 1: Road construction costs.

	average velocity (km/h)	cost (million yen/km)
expressway	76.7	0.64
national highway	32.8	0.30
prefecture highway	30.8	0.11

Table 2: Railway construction costs.

	average velocity (km/h)	cost (million yen/km)
bullet train	220	0.7
LRT	60-120	0.15-0.25
tram	60-70	0.10-0.20
new transit system	50-60	0.70-1.20
monorail	65-80	1.00-1.90
subway	80-100	2.50-3.50
guideway bus	60	0.30-0.40

Table 3: Construction cost by speed.

c	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
velocity	10.00	5.00	3.33	2.50	2.00	1.67	1.43	1.25	1.11
link/cost	0.12	0.23	0.35	0.47	0.59	0.70	0.82	0.94	1.05

of networks that can be constructed. The mean travel times for networks that could be constructed at a certain cost were compared, as construction costs increased. In this way, a combination of the desirable speed and shape that results in the minimum mean travel time can be determined. Furthermore, by using a base unit for the number of networks that can be constructed, changes in the available budget can be easily taken into consideration to determine the desirable combination of speed and the number of networks.

Figure 6 shows the relationship between total cost and mean travel time of transportation networks by speed. The lower the mean travel time, the more desirable networks are. It can be seen that, in the initial stage, a network of $c = 0.3$ reduces required travel time significantly. Links slower than this cannot reduce the required time, even if constructed over longer distance networks. Furthermore, it can be seen that networks cannot be effective unless they can provide transportation at a certain speed. After this, as construction costs increase, the network with the minimum required travel time is the faster network of

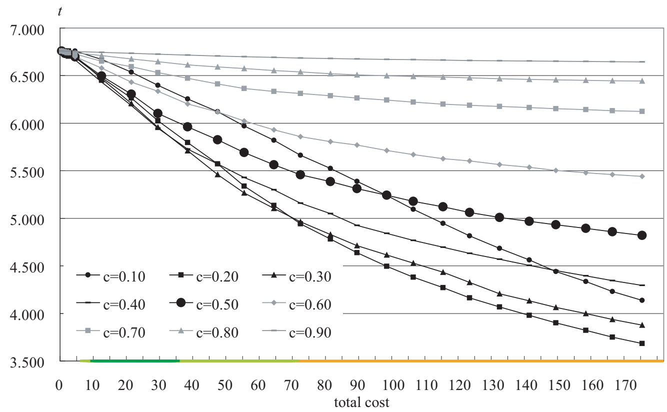

Figure 6: Relationship between total cost and mean travel time for different speeds.

$c = 0.2$. This is because, as the number of links increases, the effectiveness of the network at speed $c = 0.3$ in reducing travel time reaches a limiting value. Also, the length of the network at speed $c = 0.2$ grows sufficiently to fully demonstrate its usefulness. Based on these results, it can be concluded that networks need a certain length in order to display their effectiveness. Since only a small number of networks of speed $c = 0.1$, the highest speed, can be constructed due to the high construction costs, mean travel time cannot be minimized.

5 Conclusions

In this study, different models were constructed to describe the growth of transportation networks. Based on the network speed and population distribution, the growth processes and shapes of transportation networks were examined. Furthermore, the trade-offs between construction costs and travel speed were considered.

Using the transportation network growth models, it was shown that the shape of networks depends on the speed of high-speed links. High-speed networks tend to grow by means of repeated branching from a central trunk. On the other hand, low-speed networks tend to grow in the shape of a grid, with straight lines overlapping each other, without branching. The effect of speed differences on growth patterns can be illustrated by the contrasting features of the *Shinkansen* network, which has a tree structure, with urban subway networks, which have many closed paths. It was also shown that, taking into consideration the trade-offs between speed and construction costs, a desirable network speed and shape are primarily constrained by cost.

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