The Eighth International Symposium on Operations Research and Its Applications (ISORA'09)
Zhangjiajie, China, September 20–22, 2009

A Safety Stock Problem in Battery Switch Stations for Electric Vehicles

Honma Yudai[1] Kurita Osamu[2]

[1]Faculty of System Design, Tokyo Metropolitan University, Tokyo, 191-0065, Japan.
[2]Faculty of Science and Technology, Keio University, Kanagawa, 223-8522, Japan.

Abstract Electric vehicles (EVs) have attracted attention for the clean-energy society for decades. However, as defects of EVs, short battery life and limited range are pointed out. To overcome these problems, Better Place has proposed a business model of battery switch stations where batteries of EVs can be replaced. In this study, we formulate a safety stock problem in battery switch stations using queueing theory, and discuss the several condtion to manage the stations in stable.

Keywords electric vehicle, battery switch station, safety stock problem, queueing theory

1 Introduction

In this paper, we focus on *battery switch stations* which are facilities for *electric vehicles* (EVs) to replace their batteries. Particularly, we consider the safety stock problem in battery switch stations based on queueing theory. Our main objective is to clarify how the arrival rate of EVs and the charging time of batteries affect the safety stock in a station.

Since the twentieth century, we have been consuming massive amounts of fossil fuels. It is not an exaggeration to say that the evolution of our culture has been achieved by utilizing this energy resources [10]. However, rapid consumption in past a hundred years has caused several serious problems at the same time.

One of the most important issues is global warming. In recent years, an increase in the average surface temperature and a rising sea level has been observed, and their influences on the many physical and biological systems are of concern [3]. These climate changes over last 50 years are very likely to be attributable to human activities— especially carbon dioxide (CO_2) emissions by the combustion of fossil fuels. According to *Intergovernmental Panel on Climate Change*, projections in climate models result in an increase in a globally averaged surface temperature of 1.4 to 5.8°C over the period 1990 to 2100. In addition, the disruption of ecosystems, drought, and pest infestations are expected to increase [6].

Depletion of fossil-fuel resources is another problem that we are facing [8]. Specifically, the exhaustion of conventional oil has been warned by a great number of institutes and scientists. Although extremely pessimistic projections, for example that the production of oil will peak between 2010 and 2020 [7], are heard only infrequently, it is evident that these resources are finite and must be replaced eventually.

These facts are particularly pressing in the transportation sector, which still relies almost exclusively on oil [5]. That is to say, mitigation of CO_2 emissions and departure from dependence on oil is strongly expected in the sector. Recently, as a result of these factors, EVs are focused again as a next-generation technology. EV is a vehicle with electric motors which is powered by on-board batteries. EV is invented over a hundred ago, and has a long history as well as a gasoline vehicle. It owing to the simple structure of EV, namely to operate motors is simpler than to use gasoline engines. However, EV has also some disadvantages unfortunately, so they have not benn popularized until today.

First, the low energy density of batteries should be pointed out. Compared to the gasoline vehicles, EV is low running distance per charge. This is the biggest factor which disturbs the popularization of EVs. Nonetheless, battery performance has been developed recently such as the adoption of lithium-ion batteries. In fact, several automotive enterprises announce the release of new EVs for the public, and it will come to market in 2010 [2].

However, even though on-board batteries have been surely improved, the EV's running distance per charge is still about 100km to 200km. In addition, EVs also often have long recharge times compared to the refueling of gasoline vehicles which is relatively fast process. It means that we need the lots of public charging stations to drive a long distance travel using EVs. Furthermore, it should be considered how to manage charging stations. Under existing conditions, it need a long time to recharge, and therefore it is impractical to recharge batteries in a station such as gasoline station [4].

As an alternative, Project Better Place has proposed the system of "battery switch stations" [1]. This business model proposes to "replace" the batteries instead of "recharging". Obviously, replacing batteries is much faster than recharging them, so EVs can use a battery switch station like a typical refuel station. Better Place has already launched some test markets in Israel, Denmark and Hawaii. As described, the infrastructure of "battery switch stations" is enough practical idea to support EVs.

In this point of view, we focus on the battery switch station as a infrastructure to support EVs, and evaluate how to manage battery stock in each stations. Particularly, formulating the battery's stochastic process in a station using queueing theory, we discuss the theoretical relationship how the number of safety stock depends on the arrival rate of EVs and the charging time of batteries.

2 Formulation of a Battery Switch Station

In this section, we formulate the system of "battery switch station" using queueing theory.

2.1 The system of battery switch station

First, let us summarize a overview of battery switch station system. As previously mentioned, the EV's running distance per charge is less than 200km. Hence, to use EVs for long drive, it is essential to recharge on-board batteries at public stations. However, charging the batteries generally need long time, so it is much more difficult than refueling gasoline vehicles.

To ease this problem, Beter Place projected by Shai Agassi has proposed the business plan of public battery station where to "replace" batteries beside "recharge" them. By

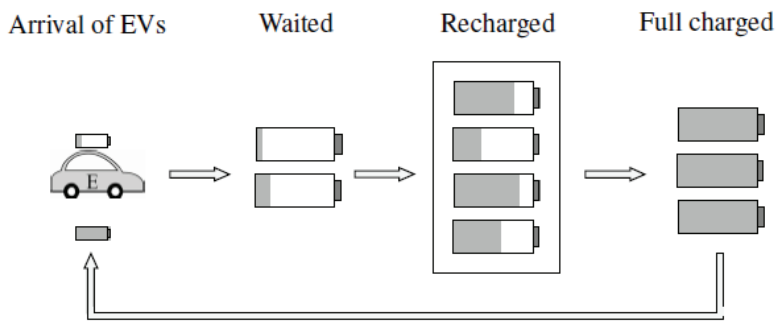

Figure 1: The flow of batteries in a station

replacing EV's empty batteries to full-charged batteries at the station, they can resume their drive in a few minutes. Since, this business model suppose to lease batteries beside to sell them, it is also expected that the price of EVs is dropped down.

Now, under the several assumptions, we consider how each battery switch station is managed. To begin with, we assume that all station recharge (withdrawn) empty batteries by themselves. It means that they do not recharge batteries in other places. Furthermore, it is also inhibited to import full-charged batteries from other place. Then, the flow of batteries in a station is described as Figure 1. (I) First, (withdrawn) empty batteries on EVs and full-charged batteries are replaced, Then, (II) empty batteries are waited to be recharged until an arbitrary recharging device become available, and (III) start to be recharged as soon as it is available. (IV) After recharging is finished, it will be a stock of full-charged batteries for next EVs.

As is clear from the above, it takes a long time that withdrawn batteries become the stock of full-charged batteries for next EVs. Hence, to supply the charged batteries to EVs steadily, it is inevitable to prepare enough stock of full-charged batteries at each station. In this study, we examine **the number of safety stock in battery switch station** based on queueing theory.

Now, there is an important feature about the number of batteries , which is quite important to formulate the stock problem in a battery station:

[Feature about the number of batteries in a station]
The total number of batteries in a station stays constant. even if various operations (such as replacing) is served in a station.

The feature stated above would be overlooked, but is easily-understood because all EVs just "replace" the batteries. If the number of full-charged batteries decreases one, then the number of empty batteries increases one. This feature is essential to formulate battery switch station in the following.

2.2 Stochastic process of battery stock

Let us now formulate the stochastic process of battery stock in the station based on queueing theory. To begin with, we explain how to define the state of the station.

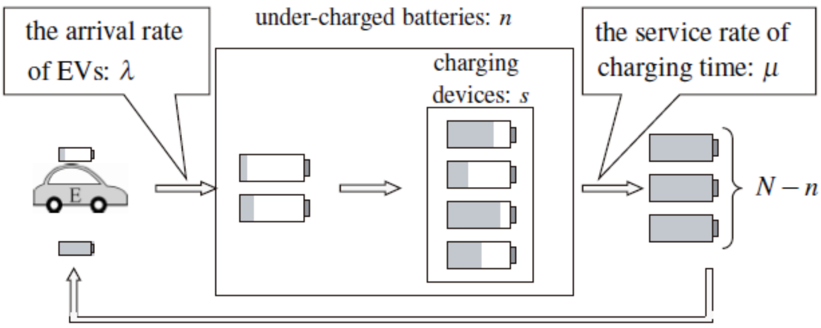

Figure 2: The formulation of battery switch station

As indicated in Figure 1, there are three phases of batteries, (i) waited to charge, (ii) charged on the device, and (iii) full-charged. In this study, we present the state of station using the number of batteries in each phase. we define that the total number of batteries in a station is N (= constant) and the number of charging device is s (it means that s batteries are simultaneously charged in the station). Furthermore, we define the "waited + recharged" phase as "under-charged phase", and n is the number of batteries in under-charged phase. We assume $s < N$ and $0 \leq n \leq N$. Then, the number of batteries in each phase is described as follows:

$$\{\text{waited}, \text{recharged}, \text{full-charged}\} = \begin{cases} \{0, n, N-n\} & (0 \leq n \leq s) \\ \{n-s, s, N-n\} & (s \leq n \leq N) \end{cases} \tag{1}$$

Note that N and s are both constant, we can confirm the state of battery switch station if n is determined.

The objective in this section is to describe the stochastic process of the station (it is equivalent to formulate the stochastic process of n). Suppose a battery switch station in which the total number of batteries is N (= constant) and the number of charging device is s. In addition, we assume that EV arrivals are the Poisson process whose arrival rate is λ, and charging time follows exponential distribution which service rate is μ (see Figure 2).

Now, all events which would happen in the station is the next three cases:

(a) EV's arrival $\rightarrow$ replaced with full-charged battery,
(b) EV's arrival $\rightarrow$ there is no full-charged battery, and cannot be replaced,
(c) a battery is full-charged.

In each case, we can summarize as follows:

(a) EV's arrival $\rightarrow$ replaced with full-charged battery.

In this case, the number of under-charged batteries increases one, and that of full-charged batteries decreases one. To happen this event during time length Δt, it has to be satisfied that an EV arrives (the probability is $\lambda \Delta t$) and full-charged batteries is stocked.

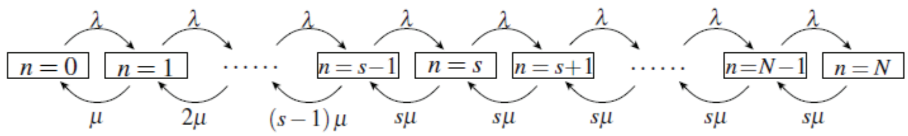

Figure 3: The transition diagram of n (the number of under-charged batteries)

(b) EV's arrival $\rightarrow$ there is no full-charged battery, and cannot be replaced

If there is no full-charged battery, then EV cannot replace the battery. Since replacement does not happen, also the state of the battery station is not changed. In this case, $n = N$ is satisfied.

(c) A battery is full-charged

In this case, the number of under-charged batteries decreases one, and that of full-charged batteries increases one. To happen this event during time length Δt, it is needed that an arbitrary battery on the recharging device is full-charged (the probability is $\min\{n, s\} \times \mu \Delta t$).

As a result, the transition diagram of the battery switch station is summarized as Figure 3.

2.3 The stationary distribution of n

Now, we calculate the stationary distribution of the number of batteries. from Figure 3, the balance equations are as follows:

$$\lambda p_0 = \mu p_1 \qquad (n = 0), \tag{2}$$

$$(\lambda + n\mu)\, p_n = \lambda p_{n-1} + (n+1)\, \mu p_{n+1} \qquad (1 \le n < s), \tag{3}$$

$$(\lambda + s\mu)\, p_n = \lambda p_{n-1} + s\mu\, p_{n+1} \qquad (s \le n < N), \tag{4}$$

$$s\mu p_N = \lambda p_{N-1} \qquad (n = N), \tag{5}$$

where p_n is

$$\begin{aligned} p_n &\stackrel{\text{def}}{=} [\text{the probability that the number of under-charged batteries is } n] \\ &= [\text{the probability that the number of full-charged batteries is } N - n]. \end{aligned} \tag{6}$$

Hence, as is clear from the balance equations, This stochastic process is equivalent to M/M/s/N queueing model [9]. From (2) to (5), we derive

$$p_0 = \left(\sum_{n=0}^{s} \frac{(s\rho)^n}{n!} + \frac{s^s}{s!} \sum_{n=s+1}^{N} \rho^n \right)^{-1} \tag{7}$$

$$p_n = \begin{cases} \dfrac{(s\rho)^n}{n!} p_0 & (n \le s) \\ \\ \dfrac{s^s}{s!} \rho^n p_0 & (s \le n \le N) \end{cases} \tag{8}$$

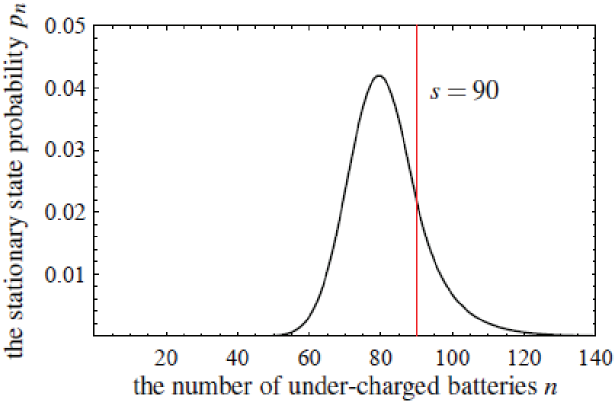

Figure 4: The stationary distribution of n

as stationary state probability, where $\rho = \lambda/s\mu$ and $\rho < 1$ should be satisfied for stability. The condition $\rho < 1$ means that the minimum number of recharging device s is determined by the arrival-service rate, that is $s > \lambda/\mu$. The example of stationary distribution is shown in Figure 4 ($\lambda = 1/3$, $\mu = 1/240$, $s = 90$, $N = 140$). We suppose [minute] as unit time in parameters.

3 Evaluation of Loss Probability and Safety Stock

In this section, we evaluate the loss probability and the safety stock of batteries using the battery station model discussed in the previous section.

3.1 Calculation of loss probability

First, we calculate the loss probability, which EVs cannot replace the battery because of the loss of full-charged batteries in the station. The preceding event would happen when the number of full-charged batteries $N - n$ is 0. Hence, the loss probability is obtained by the probability of $n = N$, namely

$$p_N = \frac{s^s}{s!}\rho^N p_0 = \frac{s^s}{s!}\rho^N \left(\sum_{n=0}^{s} \frac{(s\rho)^n}{n!} + \frac{s^s}{s!} \sum_{n=s+1}^{N} \rho^n \right)^{-1}. \tag{9}$$

The examples of the loss probability when N and s are changed is described in Figure5·6 ($\lambda = 1/3$, $\mu = 1/240$). Please note that $\lambda/\mu = 80 < s \leq N$ has to be satisfied in these examples. From the Figures, we can confirm that the loss probability p_N deceases with the increases of the total number of batteries N and the number of recharging devices s.

3.2 Calculation of safety stock

Next, we discuss how to determine the safety stock of batteries in a station. In particular, we examine the conditions to reduce the loss probability to a certain targeted level.

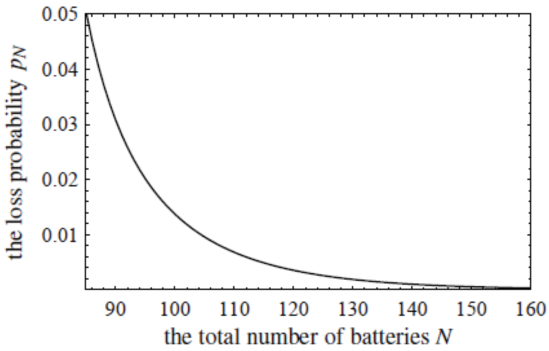

Figure 5: The loss probability in various N ($s = 85$)

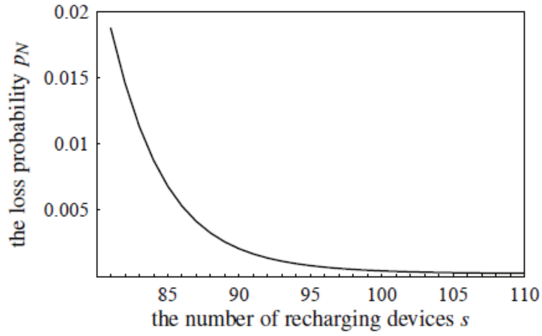

Figure 6: The loss probability in various s ($N = 110$)

To reduce the loss probability below δ, the following inequality is required from (9):

$$p_{N^*} = \frac{s^s}{s!}\rho^{N^*}\left(\sum_{n=0}^{s}\frac{(s\rho)^n}{n!} + \frac{s^s}{s!}\sum_{n=s+1}^{N^*}\rho^n\right)^{-1} \leq \delta. \tag{10}$$

where N^* expresses the number of safety stock of batteries in a station. This is the relational expression between the safety stock N^*, traffic intensity ρ, the total number of batteries N, and the number of recharging devices s.

Here, we examine how the increase of recharging time rate μ affects the safety stock N^*. In the following, we fix the arrival rate as $\lambda = 1/3$, and set the number of recharging device s to be the traffic intensity $\rho \fallingdotseq 0.9$. Then, gradually increasing the average recharging time $1/\mu$, we numerically solve the safety stock N^* to maintain $p_{N^*} \leq \delta = 0.001$ (see Figure 7). From the Figure 7, we can confirm that the safety stock N^* increases approximately in proportion to the recharging time (however, be cautious that it is not linear).

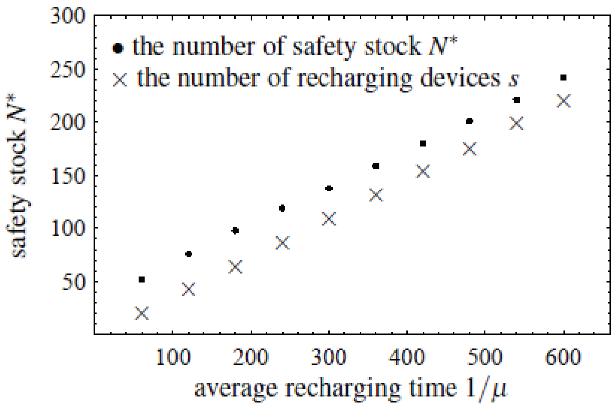

Figure 7: the number of safety stock N^* in various recharging time ($\rho \fallingdotseq 0.9$)

4 Conclusion

In this study, we focused on battery switch station which are infrastructures to support electric vehicles, and discussed the several issues to manage the station in terms of safety stock of batteries. Particularly, we formulated the stochastic process of battery management in a station using queueing theory, and clarified the theoretical relationship between the arrival rate of EVs, recharging time, the number of recharging devices, and the safety stock of batteries. This study mainly focused on the simplicity of modeling, it was assumed that each station managed independently. In reality, stations managed by same organization might share batteries or information. It will be an important future work to examine the optimal management system which incorporates multiple stations.

References

[1] Better Place(2009) Better Place Web Page. (http://www.betterplace.com/)

[2] S. Funase(2004) *Hashire! denki jidosha*, Tsukiji-Shokan. (in Japanese)

[3] A. Gore(2006) An inconvenient truth: the planetary emergency of global warming and what we can do about it, Rodale.

[4] S. Izuka(2006) *Nenryo denchisya*·denki jidosha no kanohsei, Grand Prix Publishing.

[5] International Energy Agency(2004) World energy outlook 2004, OECD.

[6] Intergovernmental Panel on Climate Change(2001) Climate change 2001: synthesis report, Cambridge University Press.

[7] J.J. MacKenzie(2000) Oil as a finite resource: When is global production likely to peak?, World Resources Institute.

[8] P. Roberts(2004) The end of oil: On the edge of a perilous new world, Houghton Mifflin.

[9] R. W. Wollff(1992) Stochastic modeling and the theory of queues, Simon and Schuster.

[10] D. Yergin(1992) The prize: The epic quest for oil, money, and power, Simon and Schuster.